Lesson 4-1: Classifying Triangles and Angle Relationships

<u>AGENDA:</u>

- Bridge Completion Check & Discussion of Back Page
- Triangle Exploration
- Guided Notes

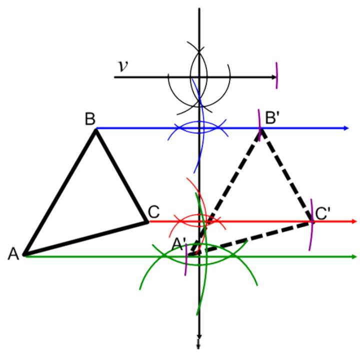
<u>Homework:</u>

- **p 219 12-19, 23-25**
  p 228 17,19,21, 23, 36
- Cumulative Review #3 due 11/17

<u>**Congruency through Transformation Exploration**</u>

Look back to Unit 1 for help

Determine and write the transformation that mapped the pre-image ΔABC to its image $\Delta A'B'C'$ in the table at right. Explain how you know.



1) What construction steps were used in this transformation?

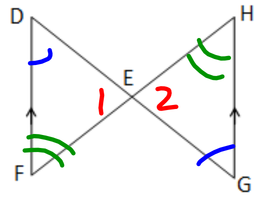
2) How do the construction steps justify the transformation?

Function Notation	
Sequence of corresponding sides (Are they congruent?)	
Sequence of corresponding angles (Are they congruent?)	
Isometry? (Yes/No)	
Congruency Statement	$\Delta ABC \cong \Delta$________

3) State the corresponding sides and angles between the pre-image and the image in the table.

4) Is this an isometric transformation? Explain your thinking.

Congruent Angles in Triangles Investigation Bridge



A) Given the drawing at right with $\overline{DF} \parallel \overline{GH}$, state three sets of congruent angles And the relationship you used to determine that they are congruent:

a. Angle Pair: $\measuredangle D \cong \measuredangle G$

Reason: $\parallel \rightarrow$ ALT INT $\measuredangle$'S $\cong$

b. Angle Pair: $\measuredangle 1 \cong \measuredangle 2$

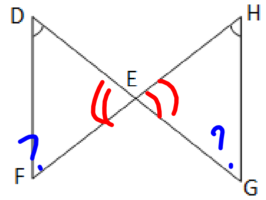
Reason: VERTICAL $\measuredangle$ PAIRS ARE $\cong$

c. Angle Pair: $\measuredangle F \cong \measuredangle H$

Reason: $\parallel \rightarrow$ ALT INT $\measuredangle$'S $\cong$

B) Given the drawing with $\measuredangle D \cong \measuredangle H$ instead, would $\measuredangle F \cong \measuredangle G$? Explain your reasoning below:

3RD $\measuredangle$ THM



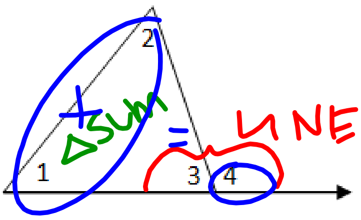
C) Explain why the congruent angle pairs for questions A and B are different.

WATCH CORRESPONDENCE

NO $\parallel$ IN PART B

Exterior Angle Discovery

Use the drawing of the triangle with one side extended into a ray to complete the following questions:



a. $m\measuredangle 1 + m\measuredangle 2 + m\measuredangle 3 = 180°$. Why?

△ SUM THM

b. $m\measuredangle 3 + m\measuredangle 4 = 180°$. Why?

LINEAR PAIR → SUPP $\measuredangle$'S

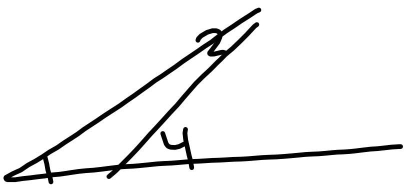
c. Determine a relationship between the $m\measuredangle 1$, $m\measuredangle 2$, and the $m\measuredangle 4$.

Algebraic steps:

$m\measuredangle 1 + m\measuredangle 2 + \cancel{m\measuredangle 3} = \cancel{m\measuredangle 3} + m\measuredangle 4$

SUBSTITUTION

SUBTR PROP EQ $m\measuredangle 1 + m\measuredangle 2 = m\measuredangle 4$



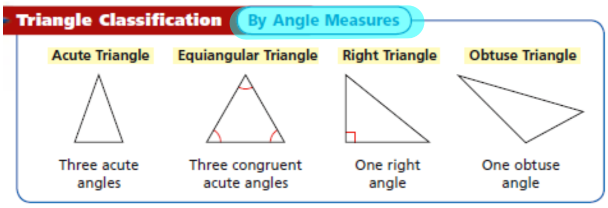
Triangle Exploration

- What do you remember?

- What can you discover?

4.1 Classifying Triangles and Angle Relationships

Triangles can be classified by their angle measures and side lengths.

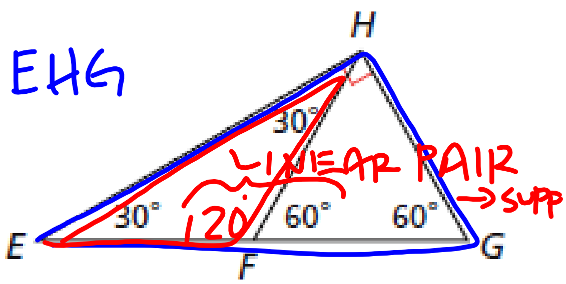


EX 1

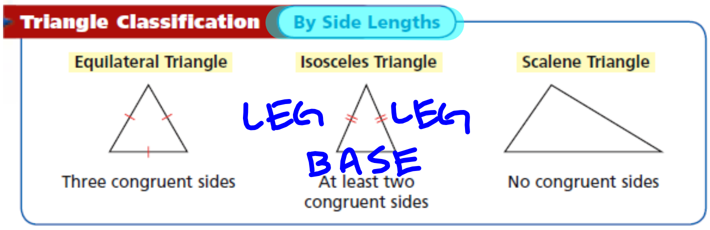
Classify each triangle by its angle measures.

A ΔEHG

| → RT∡ → RT△ EHG



B ΔEFH | OBTUSE ∡ → OBTUSE △EFH



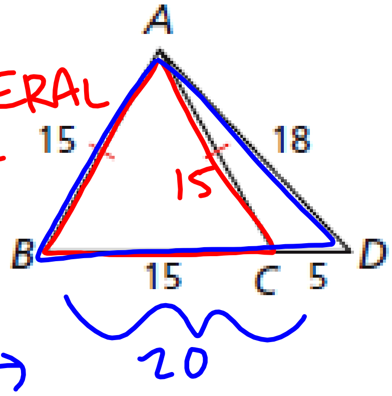
EX 2

Classify each triangle by its side lengths.

A $\triangle ABC$

3 $\cong$ SIDES → EQUILATERAL

$\triangle ABC$



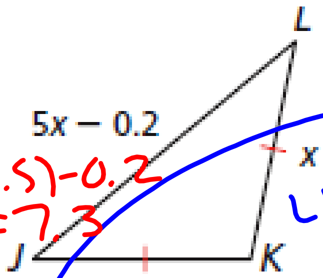
B $\triangle ABD$ $15 \neq 18 \neq 20$

NO $\cong$ SIDES →

$\triangle ABD$ SCALENE

EX 3 Find the side lengths of the triangle.

a)



$5(1.5)-0.2$

$JL = 7.3$

$LK = 4.7$

$JK = 4.7$

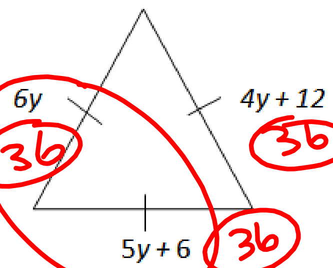
$\overline{JK} \cong \overline{LK}$

$4x - 1.3 = x + 3.2$

$3x = 4.5$

$x = 1.5$

b)



36

36

36

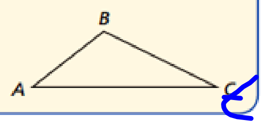
$6y = 5y + 6$

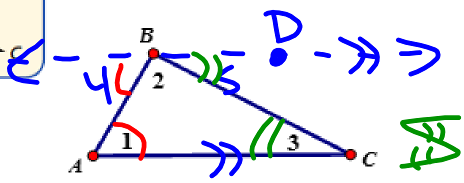
$y = 6$

Theorem 4-2-1 **Triangle Sum Theorem**

The sum of the angle measures of a triangle is 180°.

$m\angle A + m\angle B + m\angle C = 180°$



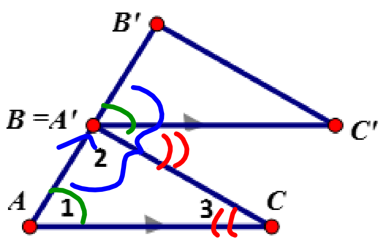


Proof: Given triangle ABC, prove $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ algebraically.

Statements	Reasons
1. $\overline{BD} \parallel \overline{AC}$	1. Only 1 line can be drawn parallel to a given line through a given point
2. $\measuredangle 1 \cong \measuredangle 4$ $\measuredangle 3 \cong \measuredangle 5$	2. Parallel lines → Alternate interior angles congruent
3. $m\measuredangle 1 = m\measuredangle 4$ $m\measuredangle 3 = m\measuredangle 5$	3. = measure ↔ ≅ figures
4. $m\measuredangle 4 + m\measuredangle 2 + m\measuredangle 5 = 180$	3. Angles on a line sum to 180°
5. $m\measuredangle 1 + m\measuredangle 2 + m\measuredangle 3 = 180$	4. Substitution

Proof: Given triangle ABC, prove $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ using a transformational approach.

- Translate ΔABC by vector $\overrightarrow{AB}$ to form straight segment $\overline{ABB'}$. Translations preserve $\measuredangle$ measures, thus $m\angle 1 = m\angle C'BB'$
- Angles on a line sum to 180° so $m\angle 2 + m\angle CBC' + m\angle C'BB' = 180°$.
- Since translations occur along parallel and congruent vectors, then $\overline{AC} \parallel \overline{BC'}$ making the alternate interior angles congruent: $m\angle 3 = m\angle CBC'$
- By making two substitutions into the equation, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$.



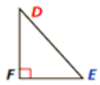
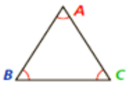
A **corollary** is a theorem whose proof follows directly from another theorem.

Here are two corollaries to the triangle sum theorem.

$\_\_ + \_\_ + \_\_ = 180°$

$\_\_ + \_\_ + 90° = 180°$

Corollaries

COROLLARY	HYPOTHESIS	CONCLUSION
4-2-2 The acute angles of a right triangle are complementary.		$\angle D$ and $\angle E$ are complementary. $m\angle D + m\angle E = 90°$
4-2-3 The measure of each angle of an equiangular triangle is 60°.		$m\angle A = m\angle B = m\angle C = 60°$

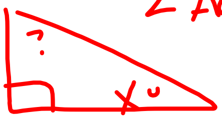
$x + x + x = 180°$

$3x = 180°$

$x = 60°$

Ex 4) One of the acute angles in a right triangle measures x°. In terms of *x*, what is the measure of the other angle?

2 ACUTE ∡'S IN A RT △ → COMP



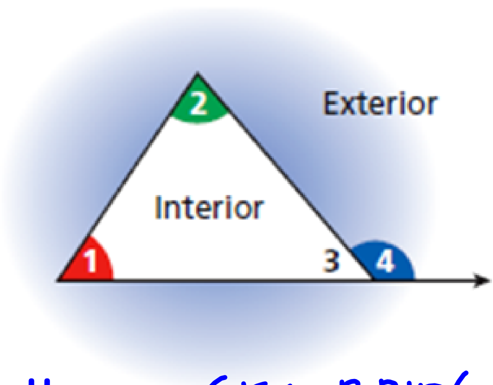
$m\measuredangle + x° = 90°$

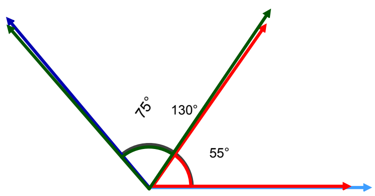
$m\measuredangle = (90 - x)°$

$\triangle$sum: $m\measuredangle + x° + 90° = 180°$

$m\measuredangle = (90 - x)°$

Visually:



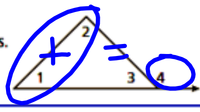


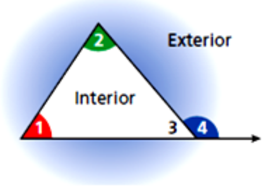
Symbolically: SEE BRIDGE

Theorem 4-2-4 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

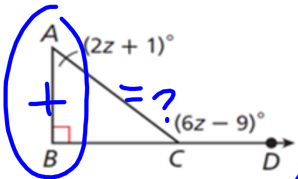
$m\angle 4 = m\angle 1 + m\angle 2$





Ex. 5)

a) Find $m < ACD$.



$m\measuredangle ACD = \underline{m\measuredangle A} + \underline{m\measuredangle B}$

$(6z-9) = (2z+1)^\circ + 90^\circ$

$6z - 9 = 2z + 91$

$4z = 100$

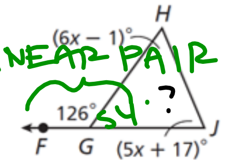
$z = 25$

$m\measuredangle ACD = 6(25) - 9$

$= 141^\circ$

WATCH WHICH THEY ASK FOR!

b) Find $m < J$. Find another approach if you have the degree measure of the exterior angle:



LINEAR PAIR → SUPP ∡'S

$m\measuredangle J = 67^\circ$

Δ sum:

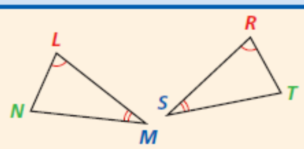
$(6x-1)^\circ + (5x+17)^\circ + 54^\circ = 180$

$11x + 70 = 180$

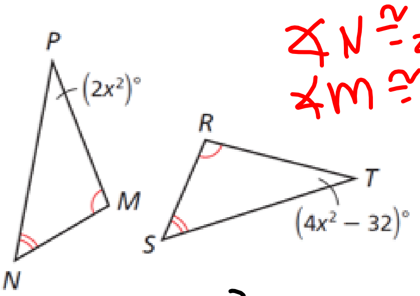
$11x = 110$

$x = 10$

Theorem 4-2-5 Third Angles Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.		$\angle N \cong \angle T$

Ex. 6) Find $m < P$ and $m < T$.



$\measuredangle N \cong \measuredangle S$
$\measuredangle M \cong \measuredangle R$ } → $\measuredangle P \cong \measuredangle T$

BY 3RD ANGLE THEOREM

$2x^2 = 4x^2 - 32$

$-2x^2 \quad -2x^2$

$-2x^2 = -32$

$\overline{-2} \quad \overline{-2}$

$\sqrt{x^2} = \pm\sqrt{16}$

$\boxed{x = \pm 4}$

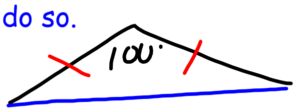
$m\measuredangle P = 2(-4)^2 = 32^\circ$

$m\measuredangle T = 4(-4)^2 - 32$

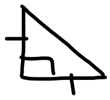
$= 32^\circ$

Think and Discuss:

1. Sketch an example of an obtuse isosceles triangle, or explain why it is not possible to do so.



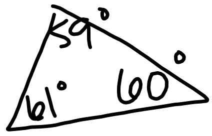
ISOS RT



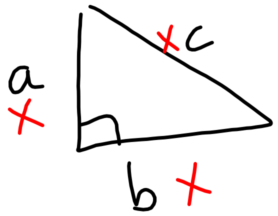
2. Is every acute triangle equiangular. Support your answer with a sketch.

NO

COUNTEREX :



3. Use the Pythagorean Theorem to explain why you cannot draw an equilateral right triangle.



$a^2 + b^2 = c^2$

$x^2 + x^2 = x^2$

$2x^2 \neq x^2$

$x = 0$

NO $\triangle$

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The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

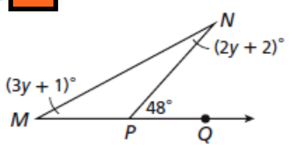
6. 20.8°

7. $y°$

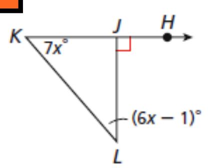
8. $24\frac{2}{3}°$

Find each angle measure.

9. m$\angle M$

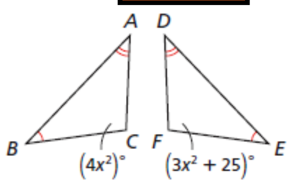


10. m$\angle L$

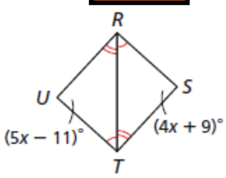


11. In $\triangle ABC$, m$\angle A = 65°$, and the measure of an exterior angle at C is 117°. Find m$\angle B$ and the m$\angle BCA$.

12. m$\angle C$ and m$\angle F$



13. m$\angle S$ and m$\angle U$



14. For $\triangle ABC$ and $\triangle XYZ$, m$\angle A$ = m$\angle X$ and m$\angle B$ = m$\angle Y$. Find the measures of $\angle C$ and $\angle Z$ if m$\angle C = 4x + 7$ and m$\angle Z = 3(x + 5)$.