Lesson 4-4R 4-5L: Isosceles Triangles

Agenda:

- Check & Review Homework
- Exploration & Notes

Homework:

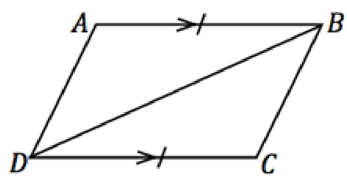
- Problem Set in Notes AND
- p. 277 # 13, 14, 16, 20, 26
- Reminder: CR#3 due 11/17

Problem Set

Justify whether the triangles meet the SAS congruence criteria; explicitly state which pairs of sides or angles are congruent and why. If the triangles do meet the SAS congruence criteria, describe the rigid motion(s) that would map one triangle onto the other.

1. Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD}$

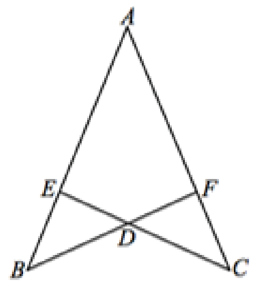
   Do $\triangle ABD$ and $\triangle CDB$ meet the SAS criteria?



Rigid motions (3) to map ΔABD onto ΔCDB:

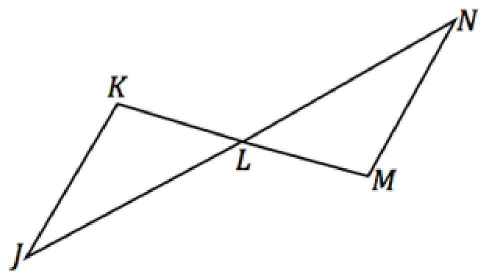
2. Given: $\overline{BF} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$

Explain why $\triangle BED$ and $\triangle CFD$ do not meet the SAS criteria:



3. **Given: $\overline{KM}$ and $\overline{JN}$ bisect each other.**

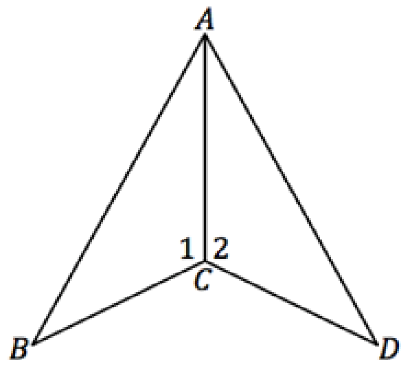
Do $\triangle JKL$ and $\triangle NML$ meet the SAS criteria?



Rigid motion to map $\triangle JKL$ onto $\triangle NML$:

4. Given: $\angle 1 \cong \angle 2$, $\overline{BC} \cong \overline{DC}$

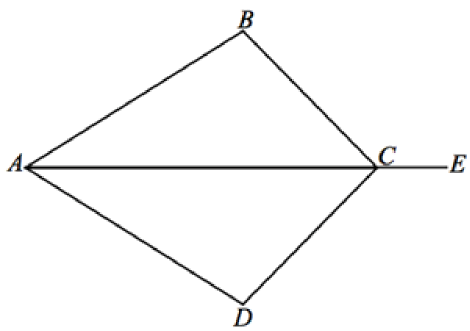
Do $\triangle ABC$ and $\triangle ADC$ meet the SAS criteria?



Rigid motion to map $\triangle ABC$ onto $\triangle ADC$:

5. Given: $\overline{AE}$ bisects angle $\angle BCD$, $\overline{BC} \cong \overline{DC}$

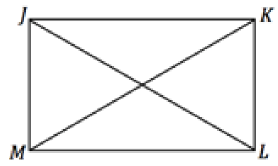
Do $\triangle CAB$ and $\triangle CAD$ meet the SAS criteria?



Rigid motion to map $\triangle CAB$ onto $\triangle CAD$:

6. **Given:** $\overline{JM} \cong \overline{KL}$, $\overline{JM} \perp \overline{ML}$, $\overline{KL} \perp \overline{ML}$

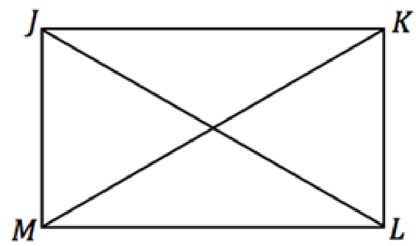
Do $\triangle JML$ and $\triangle KLM$ meet the SAS criteria?



Rigid motions (2) to map $\triangle JML$ onto $\triangle KLM$:

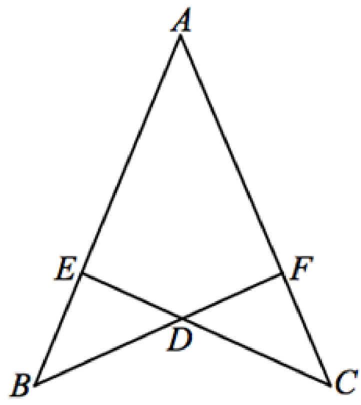
7. **Given:** $\overline{JM} \cong \overline{KL}$, $\overline{JM} \perp \overline{ML}$, $\overline{KL} \perp \overline{ML}$

Do $\triangle JML$ and $\triangle KLM$ meet the SAS criteria?



8. Given: $\overline{BF} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$

Do $\triangle BED$ and $\triangle CFD$ meet the SAS criteria?



LEGS LEGS VERTEX ∠

BASE

DEFN OF ISOS △

Name:_______________ Date:_______________ Section:_______________

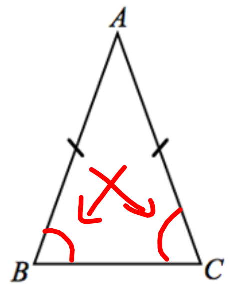
Regents Geometry & Lab Notes 4-4R/4-5L: Isosceles Triangles (Textbook 4-8)

Exploratory Challenge

The Isosceles Triangle Theorem states that the base angles of an isosceles triangle are congruent. We are going to prove this theorem in two ways: (1) by using transformations and (2) by using SAS triangle congruence criteria.

➔ Label the congruent angles in the figure. Write the reason for this congruency.

ISOS △ THM



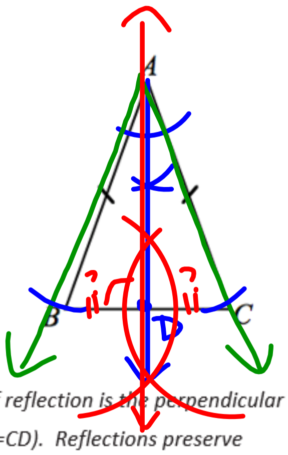
Now we will prove that the base angles of an isosceles triangle are always congruent.

ISOS △ → BASE ∠'S ≅

Prove Base Angles of an Isosceles are Congruent: Transformations

Given: Isosceles $\triangle ABC$, with $AB = AC$

Prove: $m\angle B = m\angle C$ (show that rigid motions will map point B to point C and C to B).



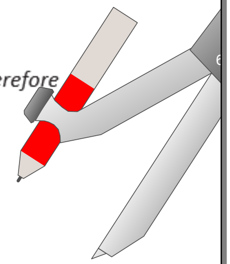
1) ***Construct the angle bisector $\overrightarrow{AD}$ of $\angle A$, where D is the intersection of the bisector and $\overline{BC}$.***

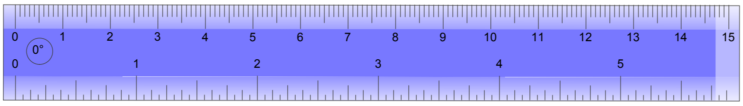
Since reflections preserve ∡ MEASURE *and $m\measuredangle BAD =$ $m\measuredangle CAD$ due to the construction and definition of an angle bisector, then $\overrightarrow{AB}$ maps to* AC*. Given $AB = AC$, then $\overline{AB}$ maps to* AC *resulting in B mapping to* C *and C mapping to* B.

2) ***Construct the line of reflection that would map point B to point C and C to B.***

Since we constructed the line of reflection through D, D maps to ITSELF*. Since the line of reflection is the perpendicular bisector of $\overline{BC}$, we know that $\overline{BD} \cong$* CD *by the definition of a segment bisector (so BD=CD). Reflections preserve distance so $\overline{BD}$ maps to* CD *and vice versa.*

3) *We have proven that $\measuredangle ABD$ will map onto $\measuredangle$* ACD*, and reflections preserve angle measure therefore $m\measuredangle ABD = m\measuredangle$* ACD.



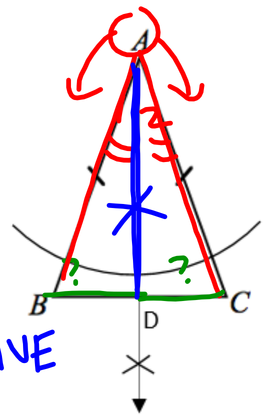


Prove Base Angles of an Isosceles are Congruent: SAS≅

Given: Isosceles $\triangle ABC$, with $\overline{AB} \cong \overline{AC}$

Prove: $\angle B \cong \angle C$

We will use the constructed angle bisector $\overrightarrow{AD}$ of $\angle A$, where D is the intersection of the bisector and $\overline{AD}$ as an auxiliary line (ray) towards our SAS criteria.



S: $\overline{AB} \cong \overline{AC}$ — GIVEN

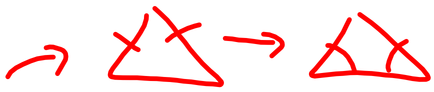
A: $\overrightarrow{AD}$ BISECTS ∡A — CONSTRUCTION

∡1 ≅ ∡2 — ∡ BISECTOR → 2 ≅ ∡'S

S ≅: $\overline{AD} \cong \overline{AD}$ — REFLEXIVE

$\triangle ABD \cong \triangle ACD$ BY SAS≅SAS

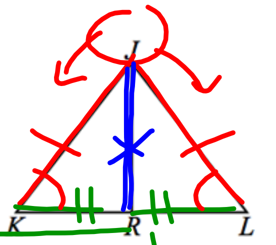
∡B ≅ ∡C CORRESPONDING ∡'S IN ≅ △S ARE CONGRUENT



Examples:

1. Given: $\overline{JK} \cong \overline{JL}$; $\overline{JR}$ bisects $\overline{KL}$

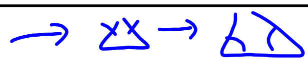
   Prove: $\Delta JKR \cong \Delta JLR$



S: $\overline{JK} \cong \overline{JL}$ — GIVEN → A: ΔJKL IS ISOS — DEFN OF ISOS Δ → $\angle K \cong \angle L$ — ISOS Δ → BASE ∠'S ≅

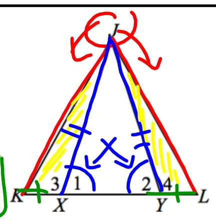
S: $\overline{JR}$ BISECTS $\overline{KL}$ — GIVEN → $\overline{KR} \cong \overline{LR}$ — SEG BISECTOR → 2 ≅ SEGS

$\Delta JKR \cong \Delta JLR$ BY SAS ≅ SAS



2. Given: $\overline{JX} \cong \overline{JY}$; $\overline{KX} \cong \overline{LY}$

   Prove: $\Delta JKX \cong \Delta JLY$



S: $\overline{JX} \cong \overline{JY}$ — GIVEN → A: ΔJXY IS ISOS — DEFN OF ISOS Δ → $\angle 1 \cong \angle 2$ — ISOS Δ THM

∠1 & ∠3, ∠2 & ∠4 LINEAR PAIR — DEFN LINEAR PAIR → ∠1 SUPP ∠3, ∠2 SUPP ∠4 — LINEAR PAIR → SUPP ∠'S

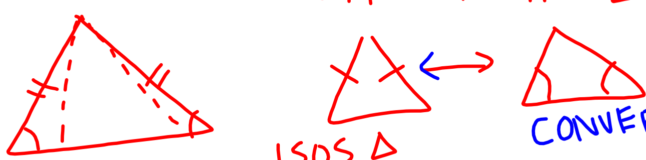
$\angle 3 \cong \angle 4$ — CONGRUENT SUPPLEMENTS THM

S: $\overline{KX} \cong \overline{LY}$ — GIVEN

$\Delta JKX \cong \Delta JLY$ BY SAS ≅ SAS

Explain why you think $\angle K \cong \angle L$: CORRESPONDING ∠'S IN ≅ Δ'S ARE ≅

Classify ΔJKL by its sides. ISOS Why? CORR SIDES IN ≅ Δ'S ARE ≅



ISOS Δ THM ⟷ CONVERSE

Prove Congruent Base Angles → Isosceles Triangle (Converse of Isosceles Triangle Theorem): Transformations

3. Given: $\triangle ABC$, with $m\angle CBA = m\angle BCA$

Prove: $\overline{BA} \cong \overline{CA}$

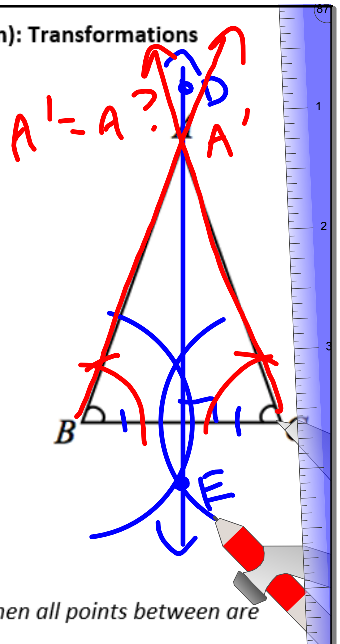
A. *Construct the auxiliary line $\overleftrightarrow{DE}$ that is the perpendicular bisector of $\overline{BC}$. $\overleftrightarrow{DE}$ is also the line of* REFLECTION *which maps point B to* C *and vice versa.*

B. *We need to still prove that A maps to A under this reflection since we don't know for sure whether A is on the line of reflection.*

1) *Copy $\measuredangle CBA$ onto $\measuredangle BCA$ to construct these congruent reflected angles. Since angle measures are preserved under reflections and $m\angle CBA =$ $m\angle BCA$ as given, then $m\angle CBA' = m\angle BCA'$ resulting in $\overrightarrow{CA}$ mapping to $\overrightarrow{CA'}$ and $\overrightarrow{BA}$ mapping to $\overrightarrow{BA'}$. Since the only point of intersection of the two rays is A or A', then A must map to A' on $\overleftrightarrow{DE}$.*

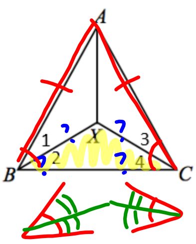
2) *Thus we have proven that A is on the line of reflection $\overleftrightarrow{DE}$.*

C. *Since reflections preserve distance and B maps to C, C maps to B, and A maps to A, then all points between are mapped such that* AB ≅ AC. *This proves the* CONVERSE *of the Isosceles Triangle Theorem.*



4. Given: $\overline{AB} \cong \overline{AC}$; $\overline{BX}$ bisects $\measuredangle ABC$; $\overline{CX}$ bisects $\measuredangle ACB$

Prove: ΔXBC is isosceles



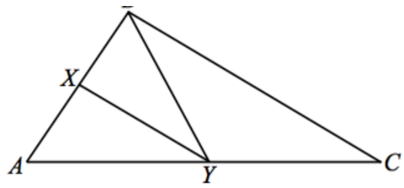
Statement	Reason
$\overline{AB} \cong \overline{AC}$	GIVEN
ΔABC is isosceles	DEFN OF ISOS Δ
$\overline{BX}$ bisects $\measuredangle ABC$; $\overline{CX}$ bisects $\measuredangle ACB$	GIVEN
$\measuredangle ABC \cong \measuredangle ACB$	ISOS Δ → BASE ∡'S ≅
$\measuredangle 1 \cong \measuredangle 2$; $\measuredangle 3 \cong \measuredangle 4$	DEFN ∡ BISECTOR or ∡ BIS → 2 ≅ ∡'S
$\measuredangle 2 \cong \measuredangle 4$	HALVES OF ≅ ∡'S ARE ≅
ΔXBC is isosceles	CONVERSE OF ISOS Δ THM; ≅ BASE ∡'S IN A Δ → ISOS Δ

B

Challenge for Extra Credit (copy onto notebook paper)

Given: $\triangle ABC$, with $\overline{XY}$ is the angle bisector of $\angle BYA$, and $\overline{BC} \parallel \overline{XY}$

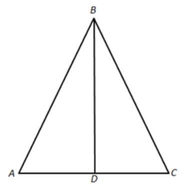
Prove: $\overline{YB} \cong \overline{YC}$



Problem Set 4-4R/4-5L – *Remember to also do the textbook problems*

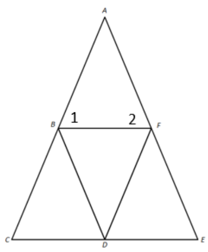
1. Given: $\overline{AB} \cong \overline{BC}; \overline{AD} \cong \overline{DC}$

   Prove: $\Delta BAD \cong \Delta BCD$



2. Given: ΔCAE is isosceles; $\overline{BF} \parallel \overline{CE}$

   Prove: ΔBAF is isosceles



Statements	Reasons
1. ΔCAE is isosceles	1. Given
2. ____________________	2. Isosceles Triangle Theorem
3. $\overline{BF} \parallel \overline{CE}$	3. Given
4. $\measuredangle 1 \& \measuredangle C$ and $\measuredangle 2 \& \measuredangle E$ are ______________________________ angles	4. Defn of _________________________________ angles
5. $\measuredangle 1 \cong \measuredangle C; \measuredangle 2 \cong \measuredangle E$	5. ____
6. $\measuredangle 1 \cong \measuredangle$ _____	6. Substitution
7. ΔBAF is isosceles	7. ____