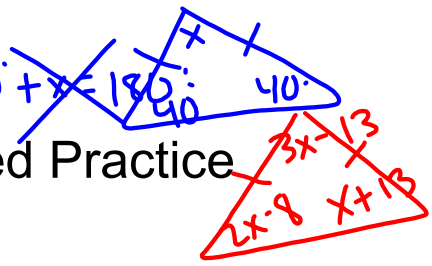
Lesson 4-5: ASA and SSS Congruency Criteria

Agenda:

$40^\circ + x + 40^\circ = 180^\circ$

$40^\circ + x = 180^\circ$



- Review 4-4R Homework
- Mini Quiz - Surprise!
- Exploration, Notes, Guided Practice

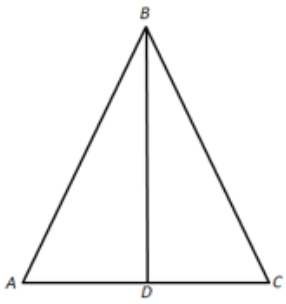
Homework:

- Problem Set in Notes
- ~~Pg. 257 11,12,17~~ Already in notes
- CR Due Thursday

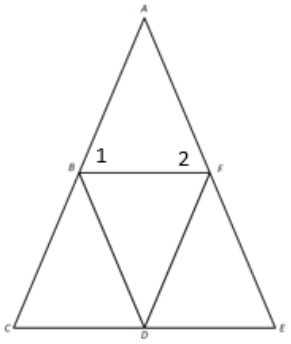
Problem Set 4-4R/4-5L – *Remember to also do the textbook problems*

1. Given: $\overline{AB} \cong \overline{BC}; \overline{AD} \cong \overline{DC}$

   Prove: $\Delta BAD \cong \Delta BCD$



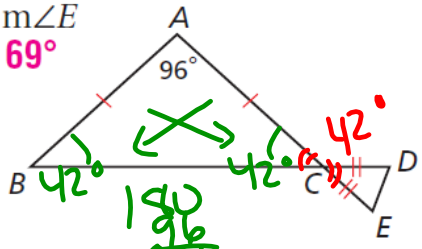
2. Given: ΔCAE is isosceles; $\overline{BF} \parallel \overline{CE}$
Prove: ΔBAF is isosceles



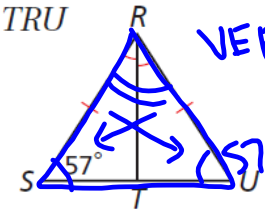
Statements	Reasons
1. ΔCAE is isosceles	1. Given
2. _____	2. Isosceles Triangle Theorem
3. $\overline{BF} \parallel \overline{CE}$	3. Given
4. $\angle 1$ & $\angle C$ and $\angle 2$ & $\angle E$ are _____ angles	4. Defn of _____ angles
5. $\angle 1 \cong \angle C; \angle 2 \cong \angle E$	5. _____
6. $\angle 1 \cong \angle$ ____	6. Substitution
7. ΔBAF is isosceles	7. _____

Find each angle measure.

13. $m\angle E$
69°

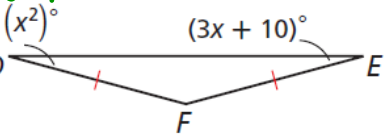


14. $m\angle TRU$
33°

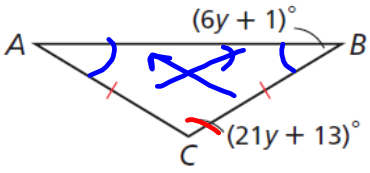


VERTEX ∠
180 − 2(57)°
÷ 2

15. $m\angle F$
130° or 172°

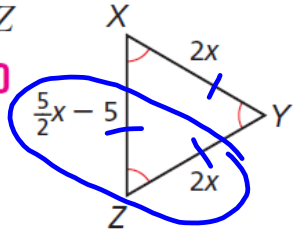


16. $m\angle A$
31°



$m\angle A + m\angle B + m\angle C = 180°$
$(6y+1) + (6y+1) + (21y+13)$ ∆ Sum

20. XZ
20



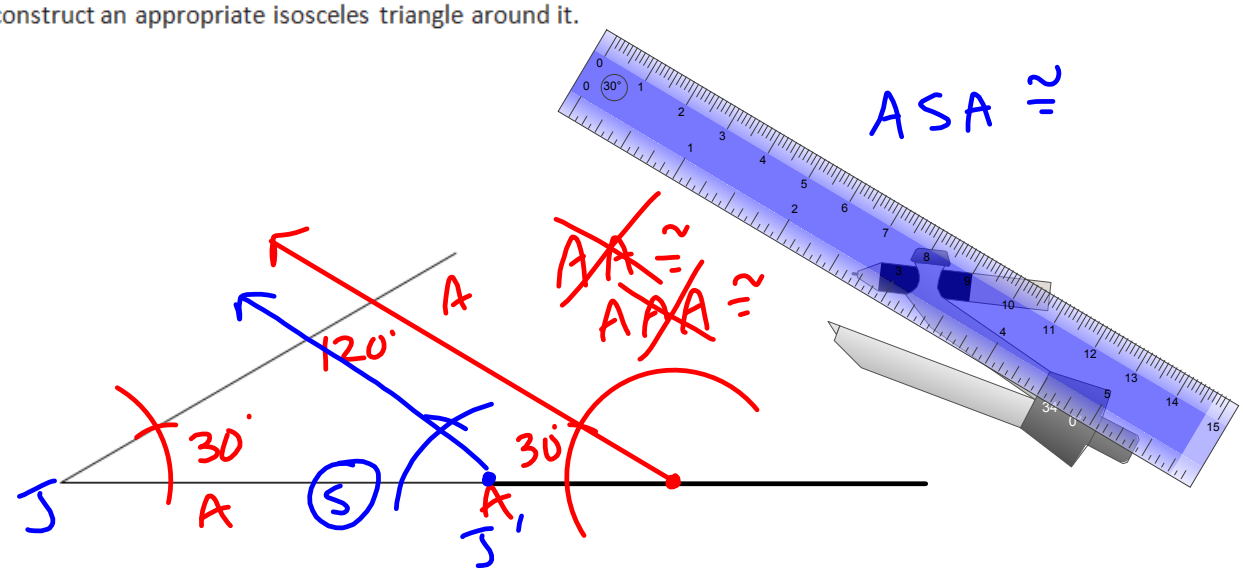
26. No; if a base ∠ is obtuse, then the other base ∠ would also have to be obtuse since they are ≅. The sum of the measures of the ∡s of the △ cannot be greater than 180°.

26. Critical Thinking Can a base angle of an isosceles triangle be an obtuse angle? Why or why not?

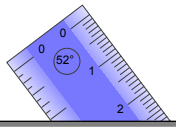
Classwork

Opening Exercise

Use the provided 30° angle as one base angle of an isosceles triangle. Use a compass and straight edge to construct an appropriate isosceles triangle around it.

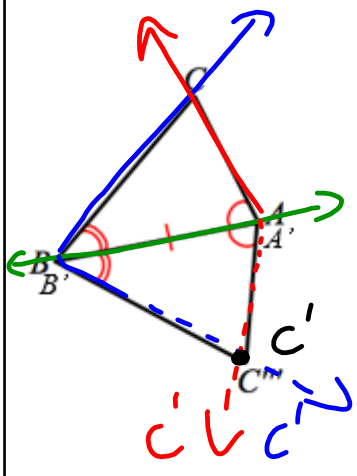


Compare your constructed isosceles triangle with a neighbor's. Does the use of a given angle measure guarantee that all the triangles constructed in class have corresponding sides of equal lengths? NO Why can't congruent triangles be guaranteed by only using angle measure? NEED A SIDE LENGTH TO DETERMINE SIZE



Angle-Side-Angle Triangle Congruence Criteria (ASA≅ASA): Given two triangles ABC and $A'B'C'''$. If $m\angle CAB = m\angle C'''B'$ (Angle), $AB = A'B'$ (Side), and $m\angle CBA = m\angle C'''A'$ (Angle), then the triangles are congruent.

Proof goal: map each of the corresponding vertices, working to prove C maps to C'''.



Since $\overleftrightarrow{AB}$ *is the line of reflection A maps to* A *and B maps to* B.

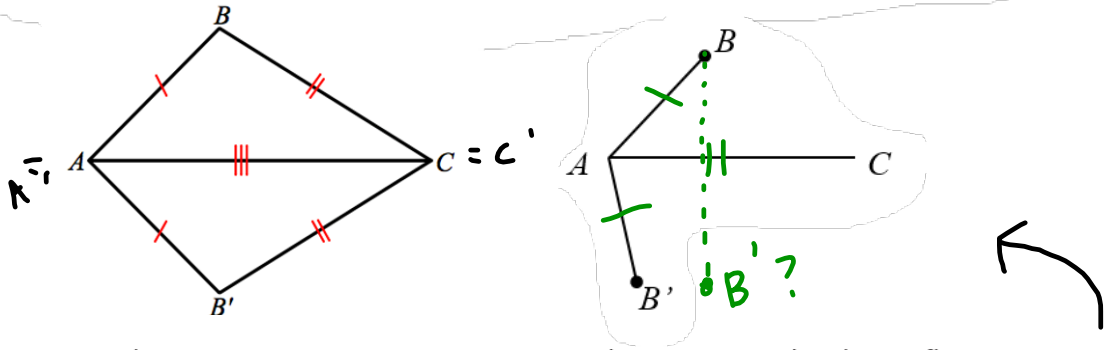
Since reflections preserve angle measure, then $\overline{BC}$ *maps to* $\overrightarrow{BC'}$ *and* $\overline{AC}$ *maps to* $\overrightarrow{AC'}$.

Since two rays intersect in exactly 1 *point and C is on both* $\overrightarrow{BC'}$ *and* $\overrightarrow{AC'}$, *then C' is the same as* C''', *meaning C maps to* C'''.

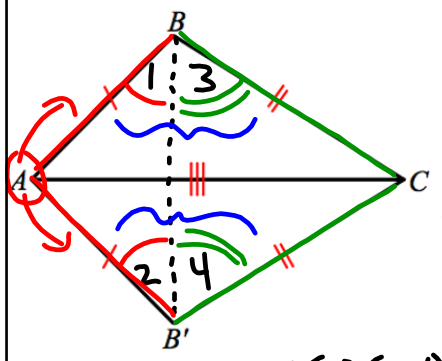
Therefore, ΔABC *maps to* $\Delta A'B'C'''$ *proving that the triangles are congruent.*

The gist: the angle measures (given as congruent) direct the rays; the congruent side locks in the points of intersection.

Side-Side-Side Triangle Congruence Criteria (SSS≅SSS): Given two triangles ABC and $A'B'C'$. If $AB = A'B'$ (Side), $AC = A'C'$ (Side), and $BC = B'C'$ (Side) then the triangles are congruent.



Since no angle measure is given, you cannot assume that B maps to B' under a reflection. Counterexample:



B'

1) Draw in $\overline{BB'}$ since two points determine a line segment

2) Prove $\Delta ABC \cong \Delta AB'C$ by SAS≅SAS:

S	A	S
$\overline{AB} \cong \overline{A'B'}$ GIVEN		$\overline{BC} \cong \overline{B'C}$ GIVEN
$\Delta ABB'$ is isosceles DEFN OF ISOS Δ		$\Delta CBB'$ is isosceles
$\measuredangle 1 \cong \measuredangle 2$ ISOS Δ THM	$m\measuredangle 1 = m\measuredangle 2$, $m\measuredangle 3 = m\measuredangle 4$ = MEAS $\leftrightarrow \cong$	$\measuredangle 3 \cong \measuredangle 4$ ISOS Δ $\rightarrow$ BASE $\measuredangle$'S $\cong$

$m\measuredangle 1 + m\measuredangle 3 = m\measuredangle 2 + m\measuredangle 4$ ADD PROP OF EQ

$m\measuredangle ABC = m\measuredangle AB'C$ $\measuredangle$ ADDITION POSTULATE

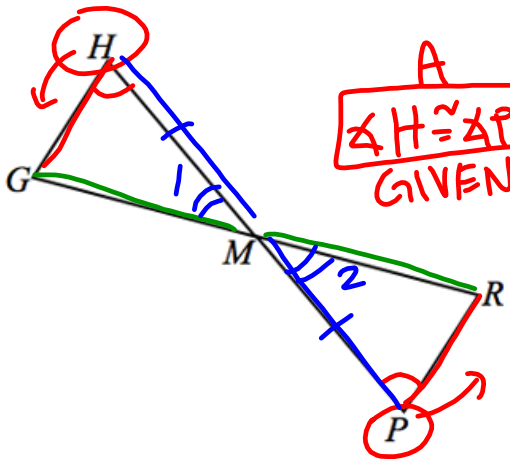
$\measuredangle ABC \cong \measuredangle AB'C$ = MEAS $\leftrightarrow \cong$

Therefore $\Delta ABC \cong \Delta AB'C$ by SAS≅SAS

Exercises

1. Given: M is the midpoint of $\overline{HP}$, $\angle H \cong \angle P$.

Prove: $\triangle HGM \cong \triangle PRM$ by ASA ≅ ASA



SSS ≅ ✗
SAS ≅ ✗
ASA ≅ ✓

A
$\angle H \cong \angle P$
GIVEN

S
M IS MIDPOINT OF HP
GIVEN

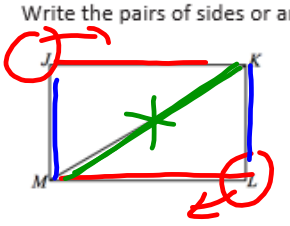
A
$\angle 1 \cong \angle 2$
VERTICAL ∠'S ARE ≅

$\overline{MH} \cong \overline{MR}$
DEFN OF MIDPOINT

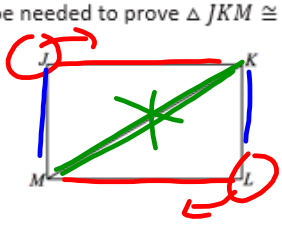
$\triangle HGM \cong \triangle PRM$ BY ASA ≅ ASA

2. Given: $JKLM$ with $\overline{JK} \parallel \overline{LM}$, $\overline{JK} \cong \overline{LM}$, $\overline{MJ} \cong \overline{KL}$, $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{LM}$ (note: these are properties of a rectangle)

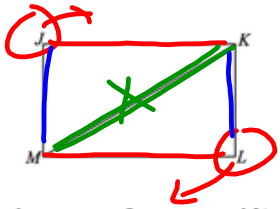
Write the pairs of sides or angles that would be needed to prove $\triangle JKM \cong \triangle LMK$ using each set of criteria:



S S S≅



S A S≅



A S A≅

Color all the correspondence first

Given: $JKLM$ with $\overline{JK} \parallel \overline{LM}$, $\overline{JK} \cong \overline{LM}$, $\overline{MJ} \cong \overline{KL}$, $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{LM}$ (note: these are properties of a rectangle)

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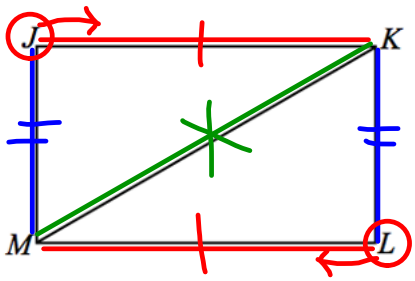
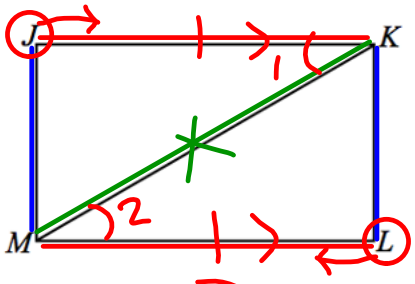


S $\overline{JK} \cong \overline{LM}$ S $\overline{KM} \cong \overline{MK}$ S≅ $\overline{MJ} \cong \overline{KL}$

Given: $JKLM$ with $\overline{JK} \parallel \overline{LM}$, $\overline{JK} \cong \overline{LM}$, $\overline{MJ} \cong \overline{KL}$, $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{LM}$ (note: these are properties of a rectangle)

Write the pairs of sides or angles that would be needed to prove $\triangle JKM \cong \triangle LMK$ using each set of criteria:

∠ PAIRS



S $\overline{JK} \cong \overline{LM}$ A $\angle 1 \cong \angle 2$ S≅ $\overline{KM} \cong \overline{MK}$

Given: $JKLM$ with $\overline{JK} \parallel \overline{LM}$, $\overline{JK} \cong \overline{LM}$, $\overline{MJ} \cong \overline{KL}$, $\overline{JK} \perp \overline{JM}, \overline{KL} \perp \overline{LM}$ (note: these are properties of a rectangle)

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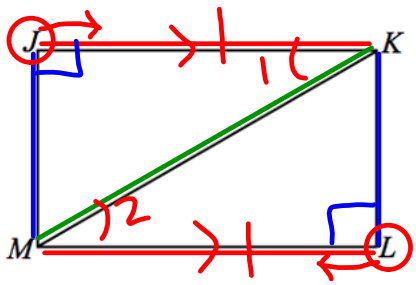
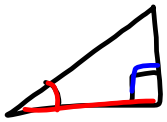


∥ → ALT INT ∠'s ≅

⊥ → RT ∠'s → ≅ ∠'s



A

∠1 ≅ ∠2

S

$\overline{JK} \cong \overline{LM}$

A≅

∠J ≅ ∠L

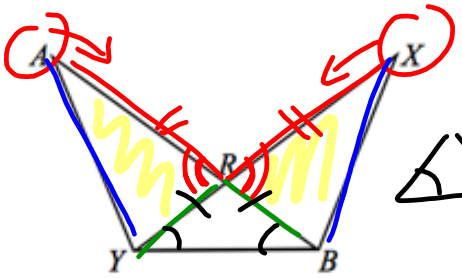
Identify whether each of the following can be proven by SSS≅SSS, SAS≅SAS, or ASA≅ASA:

3. Given: $\measuredangle RYB \cong \measuredangle RBY$, $\overline{AR} \cong \overline{XR}$

Prove: $\Delta ARY \cong \Delta XRB$

SSS

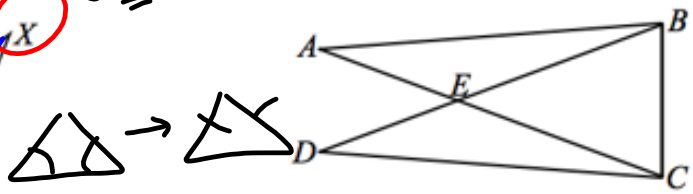
SAS



Criteria: SAS ≅ SAS

4. Given: $\angle A \cong \angle D$, $\overline{AE} \cong \overline{DE}$.

Prove: $\Delta AEB \cong \Delta DEC$

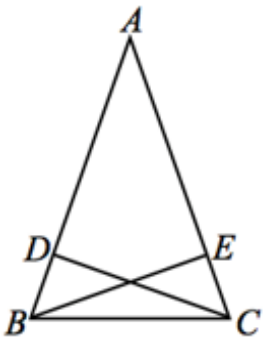


Criteria: _____

Identify whether each of the following can be proven by SSS≅SSS, SAS≅SAS, or ASA≅ASA:

5. Given: $\overline{AB} \cong \overline{AC}$, $\overline{BD} \cong \overline{CE}$

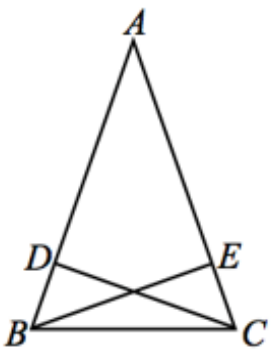
Prove: $\Delta BDC \cong CEB$



Criteria: _____

6. Given: $\overline{AB} \cong \overline{AC}$, $\overline{BD} \cong \overline{CE}$

Prove: $\Delta ABE \cong ACD$



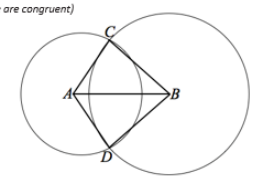
Criteria: _____

Problem Set 4-5R/4-7L **Use separate paper if you need more room.**

Use your knowledge of triangle congruence criteria to write proofs for each of the following problems.

1. Given: Circles with centers A and B intersect at C and D. *(Hint: radii of a circle are congruent)*

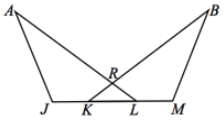
Prove: $\Delta CAB \cong \Delta DAB$.



What specific rigid motion would map ΔCAB onto ΔDAB? _____

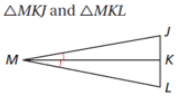
2. Given: $\angle J \cong \angle M$, $\overline{KR} \cong \overline{LR}$; $\overline{JK} \cong \overline{ML}$

Prove: $\triangle AJL \cong \triangle BMK$

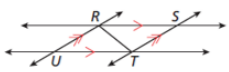


For 3&4 determine whether you can use ASA to prove the triangles are congruent. If possible, write out the proof. If not, explain why it is not possible.

3. $\triangle MKJ$ and $\triangle MKL$



4. $\triangle RST$ and $\triangle TUR$



Extra Credit:

Given: $\angle w \cong \angle x$ and $\angle y \cong \angle z$

Prove: $\triangle ABE \cong \triangle ACE$