Lesson 8-2: Geometric Mean - Altitude Rule

AGENDA:

- Check HW 8.1
- Go over Bridge
- Notes 8.1 with Applications and Guided Practice

HOMEWORK:

- Text p. 521 #9, 12 (only solve for X)15, 25(only solve for X), 27, 28, 42, 48
- CR#7 is Due Friday 3/17

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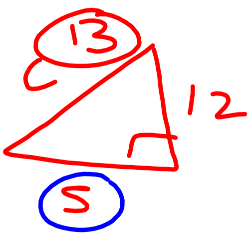
PROBLEM SET 8-1R: Draw a picture to justify your work. Show all work.

1. Which set of numbers could be the lengths of sides of a right triangle?
   a. 4, 6, $\sqrt{40}$
   b. 2, 6, $\sqrt{40}$
   c. 2, 18, 20
   d. 4, 36, 40

2. The ratio of the lengths of the legs of a right triangle is 5: 12. What is the ratio of the length of the shorter leg to that of the hypotenuse?
   a. 13 : 5 b. $\sqrt{119}$: 5 c. 5 : 13 d. 5 : $\sqrt{119}$

5:3



$5^2+12^2=c^2$

$25+144=c^2$

$169=c^2$

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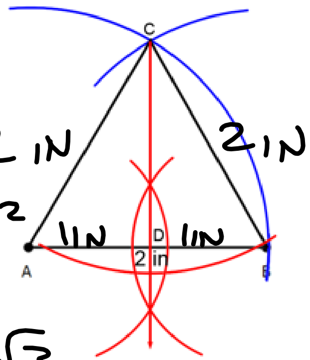
12. Given the construction of equilateral triangle ABC with a side length of 2 inches with the altitude $\overline{CD}$ where D is the point of intersection with the base $\overline{AB}$



Fill in the measures of the following:

i. $m\measuredangle ADC =$ 90 ° and $m\measuredangle BDC =$ 90 °

ii. $m\measuredangle A =$ 60 ° and $m\measuredangle B =$ 60 °

iii. $m\measuredangle ACD =$ 30 ° and $m\measuredangle BCD =$ 30 °

iv. AC = 2 inches = BC

v. AD = 1 inches = BD

vi. CD = $\sqrt{3}$ inches (use the Pythagorean Theorem to answer in simplest radical form)

$1^2 + x^2 = 2^2$

$x^2 = 3$

$x = \pm\sqrt{3}$

A ———————— B

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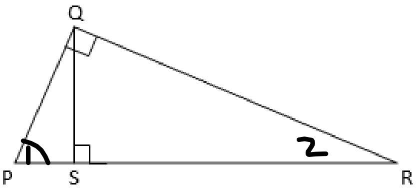
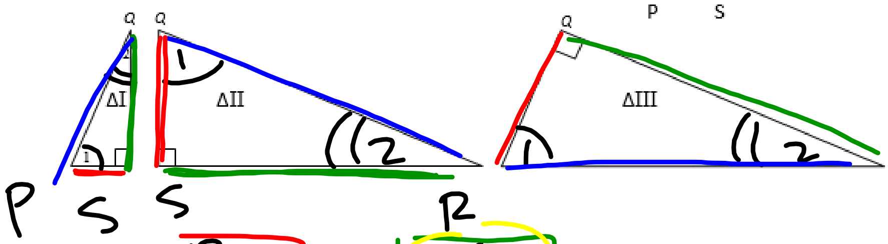
Geometry Name: ______________ Section: ________ Date: ________

HW: Bridge to Unit 8

Right Triangle Investigation – using similarity and proportions from Unit 7



1. Since you know that $\measuredangle PQR$ is a right angle, what is the relationship between $\measuredangle P$ and $\measuredangle R$? ______________



$$\frac{PS}{QS} = \frac{QS}{SR}$$

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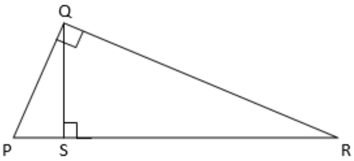
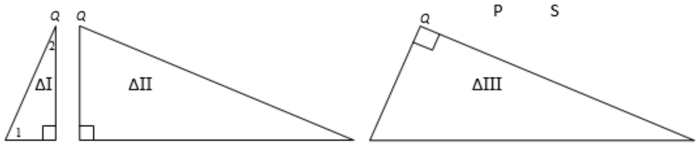
Right Triangle Investigation – using similarity and proportions from Unit 7

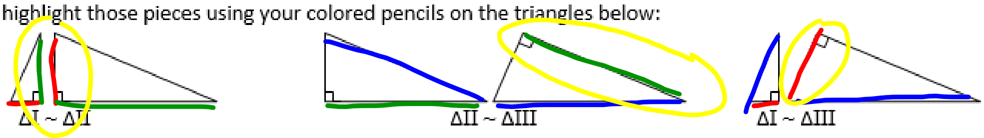
1. Since you know that $\measuredangle PQR$ is a right angle, what is the relationship between $\measuredangle P$ and $\measuredangle R$? ____________________





12. Explain how you might be able to solve a problem to find QS if all you know is that PS = 5and SR = 6. Show your algebraic work as well as how you decided that you could use this approach. (Consider that you only know two pieces of information, but a proportion has 4 pieces of information. Why were you able to solve this?)

13. Look back at number 8. See the highlighting? That is what you used to do #12. Find the most important proportion within each triangle similarity that uses a duplicate piece of information and highlight those pieces using your colored pencils on the triangles below:



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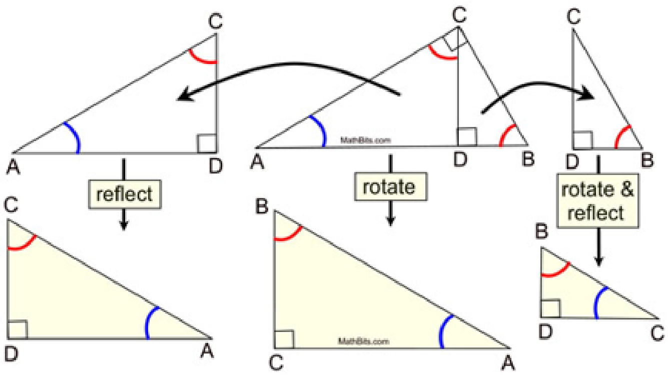
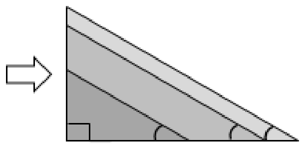
Geometry LAB Name: ____________________ Section: __________ Date: ________

Lesson 8-2 Notes: Similarity in Right Triangles and the Geometric Mean – Altitude Rule

Are ALL right triangles similar? No, but remember our AA~ criteria....

Remember our bridge discoveries... Drawing the altitude of the original right triangle from the right angle vertex to the hypotenuse created three similar triangles because the angle measures meet the AA~ criteria and can be repositioned through rigid motions:





Similarity Statement $\Delta CDA \sim \Delta BCA \sim \Delta BDC$

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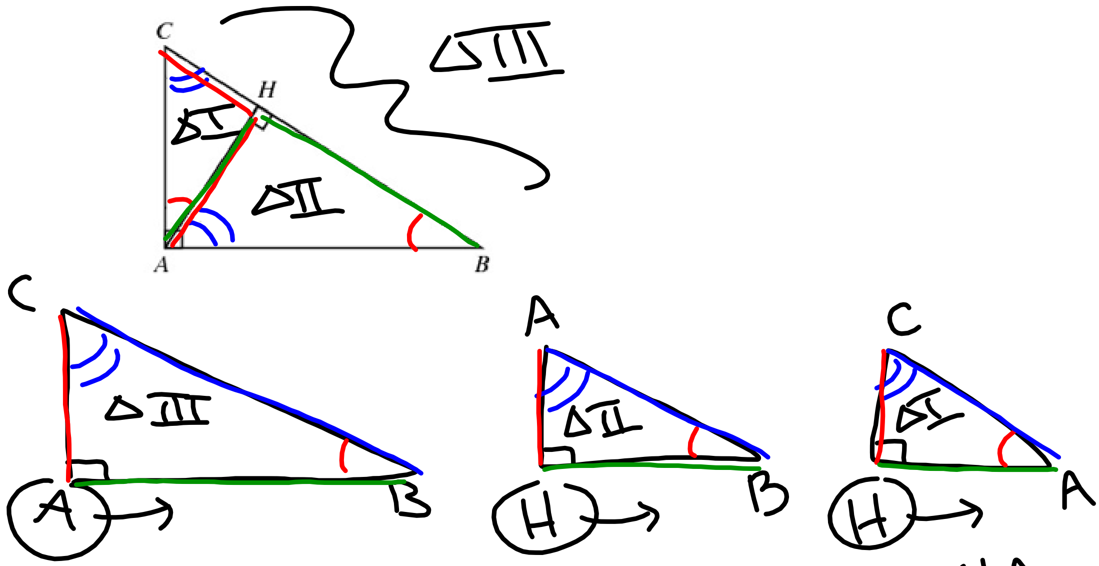
Practice

a) Draw the three similar right triangles.
b) Color the corresponding angles in all the drawings.
c) Write the similarity statement and label the triangles I, II, III.
d) Color the corresponding sides in all the drawings for triangles I and II.

My Legend
1 Short Leg
2 Long Leg
3 Hypotenuse



$\triangle ABC \sim \triangle HBA \sim \triangle HAC$

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Geometric Mean

Consider the proportion:

$$\frac{2}{10} = \frac{10}{50}$$

MEAN

EXTREMES

- What do you notice about the number 10?
- What should we call the other two numbers then?

Whenever the means in a proportion are equal, that number is said to be the **geometric mean** between the two extremes. In the example above, 10 is the geometric mean between 2 and 50.

Write and solve each indicated proportion:

1. x is the geometric mean between 5 and 20.

$$\frac{5}{x} = \frac{x}{20} \qquad x^2 = 100 \qquad x = \pm\sqrt{100} \qquad \boxed{\pm 10}$$

- What should we do about the negative square root in this case? CONSIDER IT
- Remember to simplify your radicals.

2. x is the geometric mean between 3 and 18.

$$\frac{3}{x} = \frac{x}{18} \qquad x^2 = 54 \qquad x = \pm\sqrt{54} = \pm\sqrt{\cancel{9}}\sqrt{6} \qquad x = \boxed{\pm 3\sqrt{6}}$$

3. 8 is the geometric mean between 4 and x.

$$\frac{4}{8} = \frac{8}{x} \qquad 4x = 64 \qquad x = 16$$

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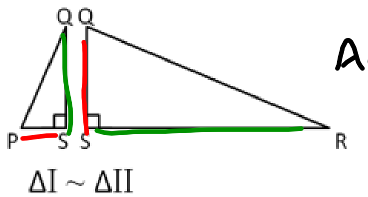
Geometry Mean Relationship 1 – Altitude Rule



Which is the one side that plays a role in two different triangles? ALTITUDE

What roles does it play? SHORT & LONG

The significant proportion between the triangles is

$$\frac{\Delta I}{\Delta II}: \frac{SHORT}{SHORT} = \frac{LONG}{LONG}$$

The DOUBLE ROLE side becomes the GEOMETRIC MEAN because it is used TWICE in the proportion.

The **ALTITUDE** of the original right triangle is the geometric mean between the **2 PARTS** of the hypotenuse of a right triangle (the proportion produced by the two smaller right triangles short & long legs).

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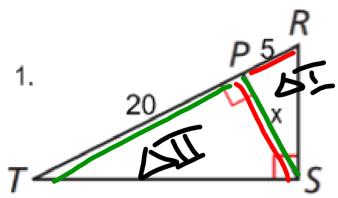
Practice: Use the altitude rule to find x, filling in the word description and writing a proportion.

1.



$$\frac{\Delta I}{\Delta II} \quad \boxed{\frac{SHORT}{SHORT}} = \boxed{\frac{LONG}{LONG}}$$

$$\frac{5}{x} = \frac{x}{20}$$

Word description: The ALTITUDE is the geometric mean between SHORT ΔII and LONG ΔI.

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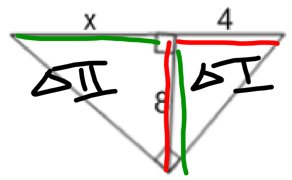
2.



$$\frac{\Delta I}{\Delta II} : \boxed{\frac{\text{SHORT}}{\text{SHORT}}} = \boxed{\frac{\text{LONG}}{\text{LONG}}}$$

Word description: The ALTITUDE (8) is the geometric mean between LONG ΔI and SHORT Δ2.

$$\frac{4}{8} = \frac{8}{x}$$

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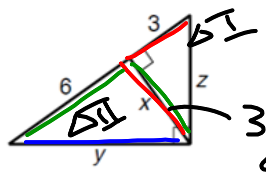
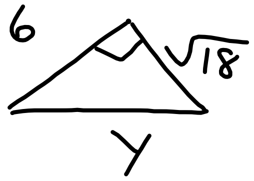
Using the Geometric Mean and the Pythagorean Theorem

3) Find x and y.



$3\sqrt{2}$ OR $\sqrt{18}$



$$\frac{\Delta I}{\Delta II} : \boxed{\frac{\text{SHORT}}{\text{SHORT}}} = \boxed{\frac{\text{LONG}}{\text{LONG}}}$$

$$\frac{3}{x} = \frac{x}{6}$$

$x^2 = 18$

$x = \pm\sqrt{18}$

$x = \sqrt{9}\sqrt{2}$ (3)

$\boxed{x = 3\sqrt{2}}$

$a^2 + b^2 = c^2$

$6^2 + (\sqrt{18})^2 = y^2$

$36 + 18 = y^2$

$54 = y^2$

$\pm\sqrt{54} = y$

$(3\sqrt{2})^2$

$3\sqrt{9}\sqrt{6} = y$

$\boxed{3\sqrt{6} = y}$

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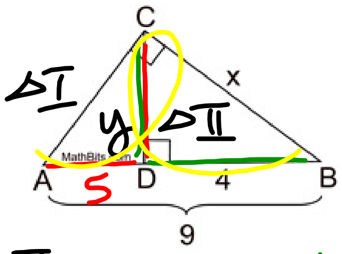
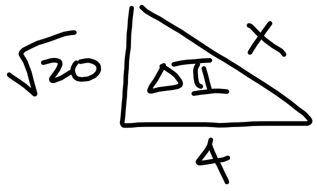
4) Find x.





$$\frac{\Delta I}{\Delta II}: \quad \frac{s}{y} = \frac{y}{4}$$

SHORT LONG

$$y^2 = 20$$
$$y = \pm\sqrt{20}$$
$$y = \sqrt{4}\sqrt{5}$$
$$y = 2\sqrt{5}$$

$$a^2 + b^2 = c^2$$
$$4^2 + (\sqrt{20})^2 = x^2$$
$$16 + 20 = x^2$$
$$36 = x^2$$
$$\pm\sqrt{36} = x$$
$$\boxed{x = 6}$$

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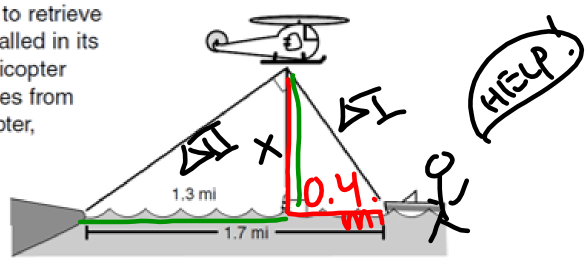
Using the Geometric Mean in Word Problems

The Coast Guard has sent a rescue helicopter to retrieve passengers off a disabled ship. The ship has called in its position as 1.7 miles from shore. When the helicopter passes over a buoy that is known to be 1.3 miles from shore, the angle formed by the shore, the helicopter, and the disabled ship is 90°. Determine what the altimeter would read to the nearest foot when the helicopter is directly above the buoy.



$$\frac{\Delta I}{\Delta II}: \quad \frac{0.4}{x} = \frac{x}{1.3}$$

$$x^2 = (0.4)(1.3)$$

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Attachments

Bridge to 8.docx