Agenda- 6-10

Quadrilateral Proofs in the Coordinate Plane

- Check HW 6.9
- Guided Notes 6-10

HW

- Problem Set 6-10
- Midterm remediation Worksheets - Due 2/10

TAKE OUT LESSON SUMMARY

Geometry Lab Name: ____________ Section: ____________ Due: ____________

Lesson 6-10: Quadrilateral Proofs in the Coordinate Plane – Parallelograms and Trapezoids

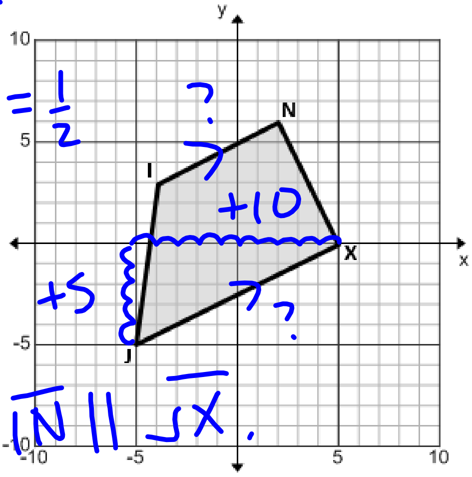
Use the coordinate plane tools of slope, distance, and midpoint in order to prove relationships among sides and angles that satisfy conditions for quadrilaterals (fill in your Lesson Summaries).

- Slope: prove two sides are PARALLEL (or $\perp$).
- Distance: prove two sides are CONGRUENT
- Midpoint: locate or prove a point is the MIDPOINT of a segment (→ segment bisector).

Complete in LESSON SUMMARY

Ways to prove a quadrilateral is a parallelogram:		
Theorem/Condition	Diagram	Formula
Show 2 pairs of opposite sides **Parallel**	→ **Parallelogram**	4 SLOPES
Show 2 pairs of opposite sides **Congruent**	→ **Parallelogram**	4 DISTANCE
Show 1 pair of opposite sides **Both Parallel and Congruent**	→ **Parallelogram**	2 SLOPES } SAME 2 DISTANCE } PAIR
Show 2 pairs of opposite angles **Congruent**	→ **Parallelogram**	**n/a**
Show diagonals **Bisect Each Other**	→ **Parallelogram**	2 MIDPOINTS (COMMON)
Show an angle is supplementary to **Both Its Consecutive Angles**	3 SUPP → **Parallelogram**	**n/a**

1. The vertices of quadrilateral JINX are J(-5,-5), I(-4,3), N(2,6), X(5,0). Prove by means of coordinate geometry that the JINX is a trapezoid.



$$m_{\overline{IN}} = \frac{\Delta y}{\Delta x} = \frac{6-3}{2-(-4)} = \frac{3}{6} = \frac{1}{2}$$

$$m_{\overline{JX}} = \frac{\Delta y}{\Delta x} = \frac{5}{10} = \frac{1}{2}$$

SINCE THEIR SLOPES ARE EQUAL, THEN $\overline{IN} \parallel \overline{JX}$.

$\therefore$ JINX IS A TRAPEZOID.

2. Prove using coordinate geometry that the quadrilateral PQRS with vertices P(-8,-6), Q(-5,-1), R(1,-5), S(-2,-10) is a parallelogram by each of the following methods:

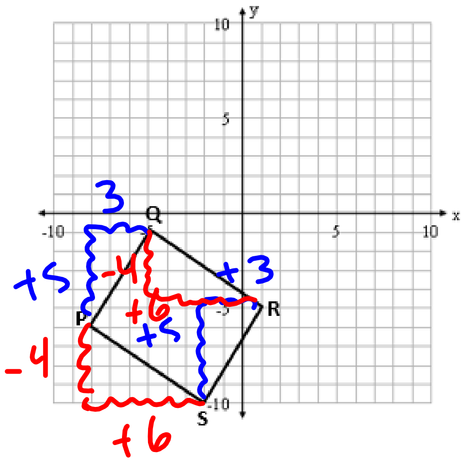
A. **Definition – BOTH sets of opposite sides are parallel**

$m\overline{PQ} = \frac{5}{3}$

$m\overline{RS} = \frac{5}{3}$

$m\overline{QR} = -\frac{2}{3}$

$m\overline{PS} = -\frac{2}{3}$



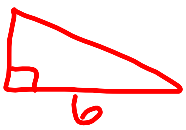
Since $m\overline{PQ} = \frac{5}{3} = m\overline{SR}$, then $\overline{PQ} \parallel \overline{SR}$.

Since $m\overline{QR} = -\frac{2}{3} = m\overline{PS}$, then $\overline{QR} \parallel \overline{PS}$.

Since both sets of opposite sides are parallel, then quadrilateral PQRS is a PARALLELOGRAM.

B. **BOTH sets of opposite sides are congruent**

PS & QR



$a^2+b^2=c^2$

$4^2+6^2=c^2$

$16+36=c^2$

$52=c^2$

$\pm\sqrt{52}=c$

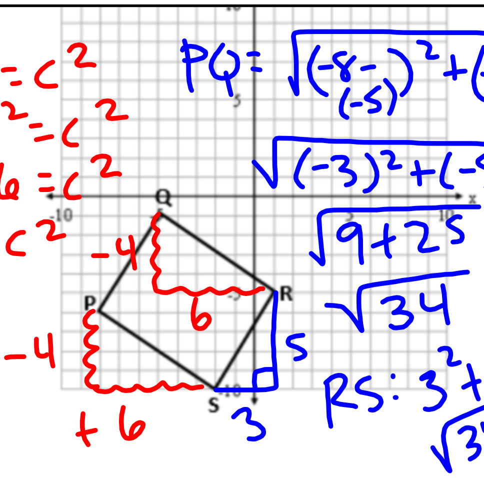
$2\cancel{\sqrt{4}}\sqrt{13}$

PQ: $\sqrt{(-8-(-5))^2+(-6-(-1))^2}$

$\sqrt{(-3)^2+(-5)^2}$

$\sqrt{9+25}$

$\sqrt{34}$



RS: $3^2+5^2=c^2$

$\sqrt{34}$

Since $PQ = \sqrt{34} = SR$, then $\overline{PQ} \cong \overline{SR}$

Since $QR = 2\sqrt{13} = PS$, then $\overline{QR} \cong \overline{PS}$

Since both sets of opposite sides are congruent, then quadrilateral PQRS is a PARALLELOGRAM

C. **One set of opposite sides is BOTH parallel and congruent**

Since $m\overline{PQ} = \frac{5}{3} = m\overline{SR}$, then $\overline{PQ} \parallel \overline{SR}$

Since $PQ = \sqrt{34} = SR$, then $\overline{PQ} \cong \overline{SR}$

Since one set of opposite sides is both parallel and congruent, then quadrilateral PQRS is a parallelogram.

D. Diagonals bisect each other

MIDPOINT $\overline{PR} = \left(\frac{-8+1}{2}, \frac{-6+-5}{2}\right) = \left(-\frac{7}{2}, -\frac{11}{2}\right)$

MIDPOINT $\overline{QS} = \left(\frac{-5+-2}{2}, \frac{-1+-10}{2}\right) = \left(-\frac{7}{2}, -\frac{11}{2}\right)$

Since the midpoint of diagonal $\overline{QS}$ is $\left(-\frac{7}{2}, -\frac{11}{2}\right)$, and the midpoint of diagonal $\overline{PR}$ is $\left(-\frac{7}{2}, -\frac{11}{2}\right)$, then the diagonals have a common midpoint and thus BISECT EACH OTHER.
Therefore, quadrilateral PQRS is a parallelogram.