5-2L Angle Bisectors and Incenter

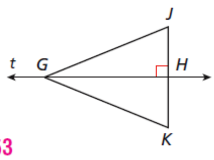
AGENDA
Check & Review Homework 5-1
Lesson 5.2 - Guided Notes, Practice Problems, Construction

HOMEWORK:
- p. 311-312: #9,10,11,~~20~~,28,29,31
- Proof A
- Construction Project – Incenter
- Keep up with lesson summaries - Do Incenter tonight
- Finish CR#4 due 12/14/16

Note: End of Interim is coming soon so back/missing work is due asap!

5.1 HW Answers

Use the diagram for Exercises 12–14.

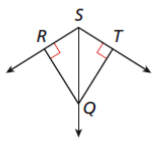


12. Given that line t is the perpendicular bisector of $\overline{JK}$ and $GK = 8.25$, find GJ. **8.25**

13. Given that line t is the perpendicular bisector of $\overline{JK}$, $JG = x + 12$, and $KG = 3x - 17$, find KG. **26.5**

14. Given that $GJ = 70.2$, $JH = 26.5$, and $GK = 70.2$, find JK. **53**

Use the diagram for Exercises 15–17.



15. Given that $m\angle RSQ = m\angle TSQ$ and $TQ = 1.3$, find RQ. **1.3**

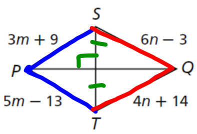
16. Given that $m\angle RSQ = 58°$, $RQ = 49$, and $TQ = 49$, find $m\angle RST$. **116°**

17. Given that $RQ = TQ$, $m\angle QSR = (9a + 48)°$, and $m\angle QST = (6a + 50)°$, find $m\angle QST$. **54°**

Write an equation in point-slope form for the perpendicular bisector of the segment with the given endpoints.

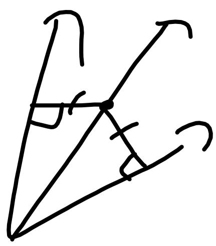
19. $E(-4, -7)$, $F(0, 1)$ **20.** $X(-7, 5)$, $Y(-1, -1)$ **21.** $M(-3, -1)$, $N(7, -5)$

22. $\overline{PQ}$ is the perpendicular bisector of $\overline{ST}$. Find the values of m and n.
$m = 11$; $n = 8.5$



19. $y + 3 = -\frac{1}{2}(x + 2)$
20. $y - 2 = x + 4$
21. $y + 3 = \frac{5}{2}(x - 2)$

$3m+9=5m-13$ $\quad$ $6n-3=4n+14$



19-22

Write an equation in point-slope form for the perpendicular bisector of the segment with the given endpoints.

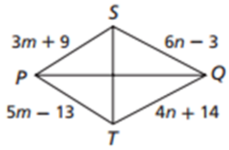
19. $E(-4, -7), F(0, 1)$ **20.** $X(-7, 5), Y(-1, -1)$ **21.** $M(-3, -1), N(7, -5)$

19. Step 1 Graph $\overline{EF}$.
The $\perp$ bisector of $\overline{EF}$ is $\perp$ to $\overline{EF}$ at its midpoint
Step 2 Find the midpoint of $\overline{EF}$.
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
midpoint of $\overline{EF} = \left(\frac{-4+0}{2}, \frac{-7+1}{2}\right) = (-2, -3)$
Step 3 Find the slope of the perpendicular bisector.
slope $= \frac{y_2 - y_1}{x_2 - x_1}$
slope of $\overline{EF} = \frac{1-(-7)}{0-(-4)} = \frac{8}{4} = 2$
Since the slopes of $\perp$ lines are opposite reciprocals, the slope of the $\perp$ bisector is $-\frac{1}{2}$.
Step 4 Use point-slope form to write an equation.
The $\perp$ bisector of $\overline{EF}$ has slope $-\frac{1}{2}$ and passes through $(-2, -3)$.
$y - y_1 = m(x - x_1)$
$y - (-3) = -\frac{1}{2}[x - (-2)]$
$y + 3 = -\frac{1}{2}(x + 2)$

20. Step 1 Graph $\overline{XY}$.
The $\perp$ bisector of $\overline{XY}$ is $\perp$ to $\overline{XY}$ at its midpoint
Step 2 Find the midpoint of $\overline{XY}$.
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
midpoint of $\overline{XY} = \left(\frac{-7+(-1)}{2}, \frac{5+(-1)}{2}\right) = (-4, 2)$
Step 3 Find the slope of the perpendicular bisector.
slope $= \frac{y_2 - y_1}{x_2 - x_1}$
slope of $\overline{XY} = \frac{-1-5}{-1-(-7)} = \frac{-6}{6} = -1$
Since the slopes of $\perp$ lines are opposite reciprocals the slope of the $\perp$ bisector is 1.
Step 4 Use point-slope form to write an equation.
The $\perp$ bisector of $\overline{XY}$ has slope $-\frac{1}{2}$ and passes through $(-2, -3)$.
$y - y_1 = m(x - x_1)$
$y - 2 = 1[x - (-4)]$
$y - 2 = x + 4$

21. Step 1 Graph $\overline{MN}$.
The $\perp$ bisector of $\overline{MN}$ is $\perp$ to $\overline{MN}$ at its midpoint
Step 2 Find the midpoint of $\overline{MN}$.
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
midpoint of $\overline{MN} = \left(\frac{-3+7}{2}, \frac{1+(-5)}{2}\right) = (2, -3)$
Step 3 Find the slope of the $\perp$ bisector.
slope $= \frac{y_2 - y_1}{x_2 - x_1}$
slope of $\overline{MN} = \frac{-5-(-1)}{7-(-3)} = \frac{-4}{10} = -\frac{2}{5}$
Since the slopes of $\perp$ lines are opposite reciprocals, the slope of the bisector is $\frac{5}{2}$.
Step 4 Use point-slope form to write an equation.
The bisector of $\perp$ has slope $-\frac{1}{2}$ and passes through $(-2, -3)$.
$y - y_1 = m(x - x_1)$
$y - (-3) = \frac{5}{2}(x - 2)$
$y + 3 = \frac{5}{2}(x - 2)$

22. $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{ST}$. Find the values of m and n.
$m = 11$; $n = 8.5$

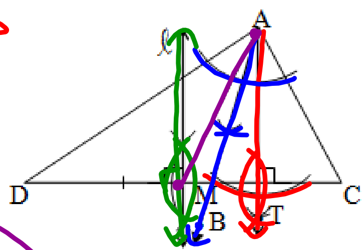


22.

$PS = PT$	$QS = QT$
$3m + 9 = 5m - 13$	$6n - 3 = 4n + 14$
$9 = 2m - 13$	$2n - 3 = 14$
$22 = 2m$	$2n = 17$
$11 = m$	$n = 8.5$

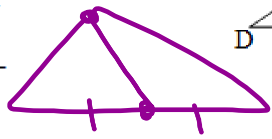
Warm Up

Are medians, perpendicular bisectors, altitudes, and angle bisectors the same segment, ray, or line for all triangles? Given ΔACD, precisely name the



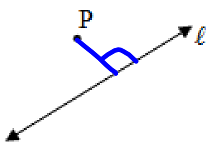
Angle Bisector $\overrightarrow{AB}$ Altitude $\overrightarrow{AT}$

Perpendicular Bisector ℓ Median $\overline{AM}$



Consider an Angle Bisector:

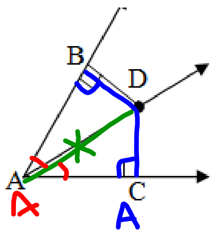
How do you measure the distance from a point to a line?



$\perp$

Given: $\overrightarrow{AD}$ bisects $\measuredangle BAC$; $\overline{BD} \perp \overline{AB}; \overline{DC} \perp \overline{AC}$

Prove: $\triangle ABD \cong \triangle ACD$ and then $\overline{DB} \cong \overline{DC}$



A | A | S

$\overline{BD} \perp \overline{AB}; \overline{DC} \perp \overline{AC}$ GIVEN

↓

$\measuredangle ABD$ & $\measuredangle ACD$ are right $\measuredangle$'s $\perp \rightarrow$ RT $\measuredangle$'S

↓

$\measuredangle ABD \cong \measuredangle ACD$ RT $\measuredangle$'S ARE $\cong$

$\overrightarrow{AD}$ bisects $\measuredangle BAC$ GIVEN

↓

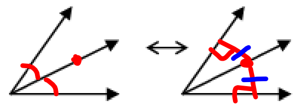
$\measuredangle BAD \cong \measuredangle CAD$ $\measuredangle$ BISECTOR $\rightarrow$ 2 $\cong$ HALVES

$\overline{AD} \cong \overline{AD}$ REFLEXIVE

$\triangle ABD \cong \triangle ACD$ by AAS $\cong$ AAS

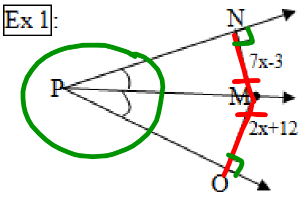
$\overline{DB} \cong \overline{DC}$ by CPCTC $\cong \triangle$'S $\rightarrow$ CORR SIDES $\cong$

Thm 5-1-3: **Angle Bisector Theorem**
If a point is on the bisector of an angle, then it is EQUIDISTANT from the sides of the angle.



Thm 5-1-4: Converse of Angle Bisector Theorem
If a point is in the interior of an angle and is equidistant from both sides of the angle, then the point is on the $\measuredangle$ BISECTOR

Ex 1:



$\overrightarrow{PM}$ BISECTS $\measuredangle NPO$ & $\perp$

What is MN?

$mN = mO$

$7x-3 = 2x+12$

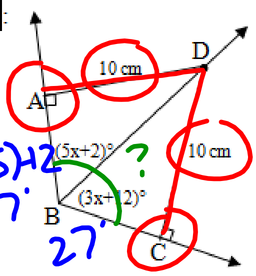
$5x = 15$

$x = 3$

$mN = 7(3) - 3$

$mN = 18$

Ex 2:



$5(5)+2$

27°

27°

D EQUIDISTANT TO $\overrightarrow{AD}$ & $\overrightarrow{BC}$ & $\perp$ → $\measuredangle$ BIS

Find $m\measuredangle ABC$.

$m\measuredangle ABD = m\measuredangle CBD$

$5x+2 = 3x+12$

$2x = 10$

$x = 5$

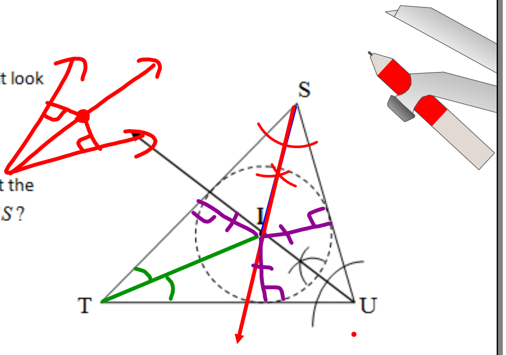
$m\measuredangle ABC = 27^\circ + 27^\circ$

$m\measuredangle ABC = 54^\circ$

Exploration of Concurrency of Angle Bisectors

Using the drawing,

a. Draw a segment connecting point **I** to vertex S. What does it look like the segment might do to angle S? Make a conjecture:



b. Confirm your conjecture by using your compass to construct the bisector of angle ~~T~~ S. IS $\overline{SI}$ coincident with the bisector of $\measuredangle S$?

c. Where do you think the third angle bisector would be? *(Hint: What does any of the three segments connected to* **I** *become?)*

d. What other segments can you draw that might be significant? Why? (Hint: consider what we just discussed about points on angle bisectors and where the equidistance is). RADII FROM I ⊥ TO ΔSIDES → ∡ SIDE RAYS

e. Where is the circle located? INSIDE → INSCRIBED

f. What could you call point I in relation to the circle?
CENTER INCENTER = I

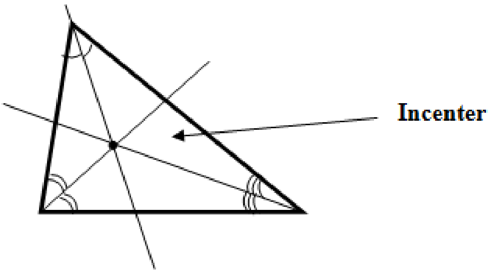
Defn: When 3 or more lines intersect in the same point, they are CONCURRENT The point of intersection is called the point of CONCURRENCY

The 4 special points of concurrency in triangles that we will study are:

Point of Concurrency	Lines that meet to form the point
Incenter *	Angle Bisectors TODAY
Circumcenter	Perpendicular bisectors
Orthocenter	Altitudes
Centroid	Medians

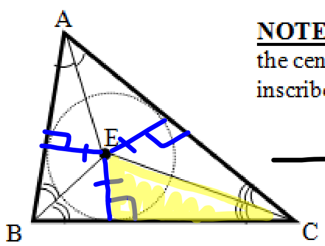
Incenter

Defn: The point of concurrency of the angle bisectors of a triangle is called the INCENTER of the triangle.

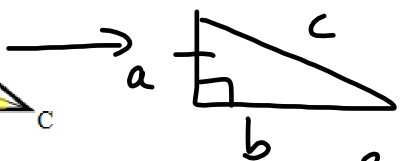


Theorem 5-2-2
The incenter of a triangle is equidistant from the SIDES of the triangle.

Remember, these segments (congruent radii of the inscribed circle) must be $\perp$ to the sides!



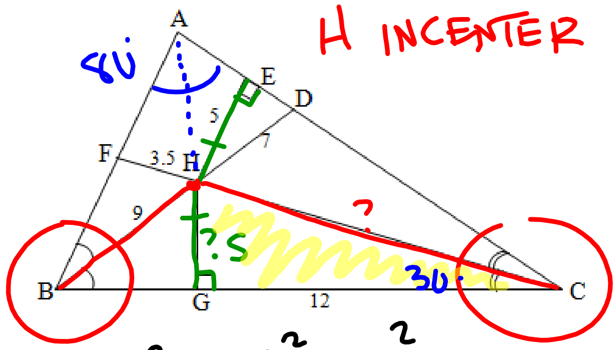
NOTE: The incenter of a triangle is the center of the circle that would be inscribed inside the triangle.



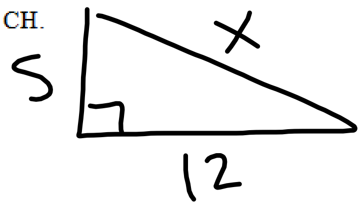
$a^2 + b^2 = c^2$

Example 1:

A) In the following triangle, what is the length of $\overline{GH}$?



B) Find the length CH.



$$5^2 + 12^2 = c^2$$
$$25 + 144 = c^2$$
$$169 = c^2$$
$$\pm\sqrt{169} = c$$
$$\boxed{13 = CH}$$

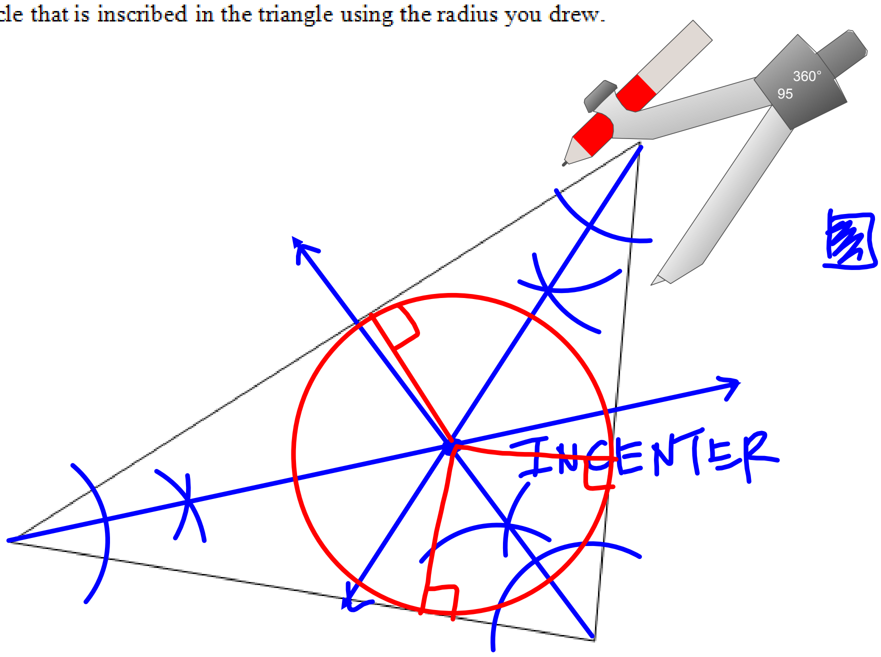
Example 2:

City planners want to locate a fountain equidistant from three straight roads that enclose a park. Explain how they can find the location.

CONSTRUCT 3 ∡ BISECTORS TO LOCATE THE INCENTER → EQUIDISTANT TO ROADS

Construction:

1. Construct the incenter of the triangle using at least two angle bisectors.
2. Draw in a radius (perpendicular to the triangle side) using your universal angle maker.
3. Construct the circle that is inscribed in the triangle using the radius you drew.



Exit Pass:

- Do number 1 on the Practice side of the construction project rubric – construct the angle bisector.
- Remember, your homework is
  - in the text book
  - in the Packet Supplement (Proof A)
  - to keep up with the lesson summaries (complete Incenter including constructing/sketching the incenter and label it P), and
  - to start the construction project.