

Lesson 4-1: Classifying Triangles

AGENDA:

- Bridge Completion Check & Discussion of Back Page
- Triangle Exploration
- Guided Notes

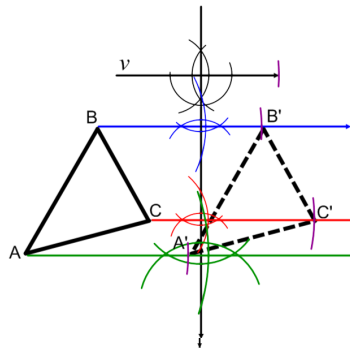
Homework:

- p 219 12-19, 23-25
- Cumulative Review #3 due 11/17

Congruency through Transformation Exploration

Look back to Unit 1 for help

Determine and write the transformation that mapped the pre-image $\triangle ABC$ to its image $\triangle A'B'C'$ in the table at right. Explain how you know.



1) What construction steps were used in this transformation?

2) How do the construction steps justify the transformation?

3) State the corresponding sides and angles between the pre-image and the image in the table.

4) Is this an isometric transformation? Explain your thinking.

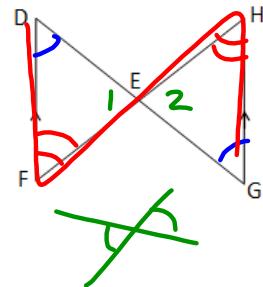
Function Notation	
Sequence of corresponding sides (Are they congruent?)	
Sequence of corresponding angles (Are they congruent?)	
Isometry? (Yes/No)	
Congruency Statement	$\triangle ABC \cong \triangle \underline{\hspace{2cm}}$

Congruent Angles in Triangles Investigation

Bridge A

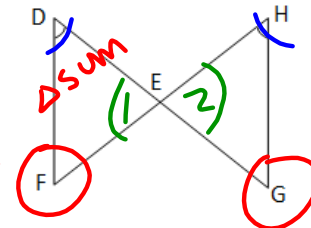
A) Given the drawing at right with $\overline{DF} \parallel \overline{GH}$, state three sets of congruent angles and the relationship you used to determine that they are congruent:

- a. Angle Pair: $\angle D \cong \angle G$
Reason: $\parallel \rightarrow$ ALT INT \angle 'S \cong
- b. Angle Pair: $\angle F \cong \angle H$
Reason: $\parallel \rightarrow$ ALT INT \angle 'S \cong
- c. Angle Pair: $\angle 1 \cong \angle 2$
Reason: VERTICAL \angle 'S ARE \cong



B) Given the drawing with $\angle D \cong \angle H$ instead, would $\angle F \cong \angle G$? Explain your reasoning below:

2 PAIRS $\cong \angle$ 'S IN 2 Δ 'S \rightarrow 3RD \angle THM

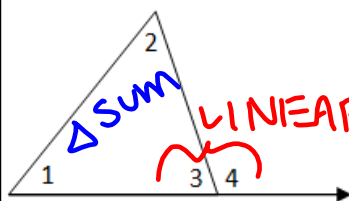


C) Explain why the congruent angle pairs for questions A and B are different.

NO \parallel IN PART B
WATCH YOUR CORRESPONDENCE

Exterior Angle Discovery

Use the drawing of the triangle with one side extended into a ray to complete the following questions:



a. $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$. Why?

Δ SUM THM

b. $m\angle 3 + m\angle 4 = 180^\circ$. Why?

LINEAR PAIR \rightarrow SUPP \angle 'S ON A LINE SUM TO 180

c. Determine a relationship between the $m\angle 1$, $m\angle 2$, and the $m\angle 4$.

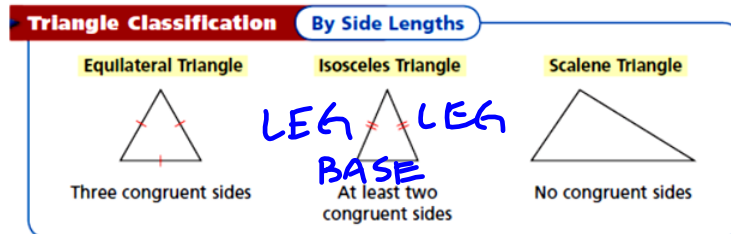
Algebraic steps:

$$m\angle 1 + m\angle 2 = m\angle 4$$

$$m\angle 1 + m\angle 2 + m\angle 3 = m\angle 3 + m\angle 4$$

Triangle Exploration

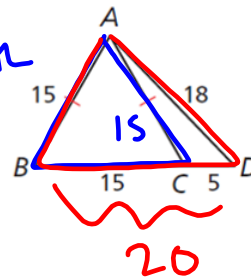
- What do you remember?
- What can you discover?



Ex 1) Classify each triangle by its side lengths.

A. $\triangle ABC$ $3 \cong$ SIDES \rightarrow EQUILATERAL $\triangle ABC$

B. $\triangle ABD$ NO \cong SIDES \rightarrow SCALENE $\triangle ABD$



EX 2 Find the side lengths of the triangle.

a)

$5x - 0.2$
 $7.5 - 0.2$
 7.3
 $4x - 1.3$
 $x + 3.2 = 4.7 = LK$
 $J\bar{K} \cong L\bar{K}$
 $JK = LK$
 $4x - 1.3 = x + 3.2$
 $3x = 4.5$
 $x = 1.5$
 $JK = 4.7$

b)

$6y$
 $4y + 12$
 $5y + 6$
 36
 36
 36
 $6y = 5y + 6$
 $y = 6$

Triangle Classification By Angle Measures

Acute Triangle	Equiangular Triangle	Right Triangle	Obtuse Triangle
Three acute angles	Three congruent acute angles	One right angle	One obtuse angle

Ex 3) Classify each triangle by its angle measures.

A. $\triangle EHG$

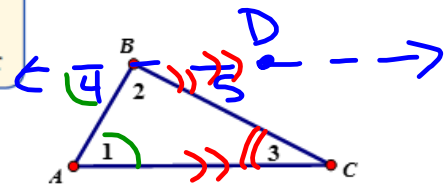
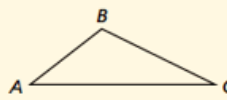
$\perp \rightarrow RT \triangle \rightarrow RT \triangle EHG$

B. $\triangle EFH$

OBTUSE $\triangle \rightarrow 120^\circ > 90^\circ$
 $\triangle EFH$ IS OBTUSE

Theorem 4-2-1 Triangle Sum Theorem

The sum of the angle measures of a triangle is 180° .
 $m\angle A + m\angle B + m\angle C = 180^\circ$

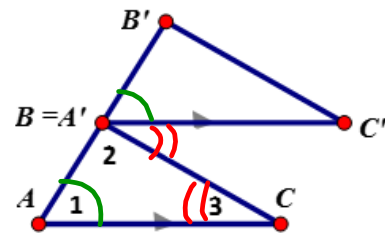


Proof: Given triangle ABC, prove $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$ algebraically.

Statements	Reasons
1. $\overline{BD} \parallel \overline{AC}$	1. Only 1 line can be drawn parallel to a given line through a given point
2. $\angle 1 \cong \angle 4$ $\angle 3 \cong \angle 5$	2. Parallel lines \rightarrow Alternate interior angles congruent
3. $m\angle 1 = m\angle 4$ $m\angle 3 = m\angle 5$	3. \cong measure $\leftrightarrow \cong$ figures
4. $m\angle 4 + m\angle 2 + m\angle 5 = 180$	3. Angles on a line sum to 180°
5. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	4. Substitution

Proof: Given triangle ABC, prove $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$ using a transformational approach.

- Translate $\triangle ABC$ by vector \overrightarrow{AB} to form straight segment $\overline{ABB'}$. Translations preserve \angle measures, thus $m\angle 1 = m\angle C'B'B'$
- Angles on a line sum to 180° so $m\angle 2 + m\angle C'BC' + m\angle C'BB' = 180^\circ$.
- Since translations occur along parallel and congruent vectors, then $\overline{AC} \parallel \overline{BC'}$ making the alternate interior angles congruent: $m\angle 3 = m\angle C'BC'$
- By making two substitutions into the equation, $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$.



Ex 4) Determine if the following triangles containing the given angle(s) are obtuse, right, or acute.

a. 81° and 68°

$$\Delta \text{sum: } \underline{81^\circ} + \underline{68^\circ} + \underline{X} = 180^\circ$$

$$X = 31^\circ \text{ ACUTE } \Delta$$

ALL ACUTE Δ 'S

d. Two angles of 45°

$$\underline{45^\circ} + \underline{45^\circ} + \underline{X} = 180^\circ$$

$$X = 90^\circ$$

RT Δ \rightarrow RT Δ .

Determine if the following triangles containing the given angle(s) are obtuse, right, or acute.

b. Two angles of 34°

$$\underline{34^\circ} + \underline{34^\circ} + \underline{X} = 180^\circ$$

$$68^\circ + X = 180^\circ$$

$$X = 112^\circ \text{ OBTUSE } \Delta$$

\rightarrow OBTUSE Δ

c. Two angles of 78°

$$\underline{78^\circ} + \underline{78^\circ} + \underline{X} = 180^\circ$$

$$156^\circ + X = 180^\circ$$

$$X = 24^\circ$$

3 ACUTE
 Δ 'S \rightarrow ACUTE
 Δ

Determine if the following triangles containing the given angle(s) are obtuse, right, or acute.

e.

$(8x-1) + (3x+4) + (3x+4) = 180$
 $14x + 12 = 180$
 $14x = 168$
 $x = 12$

$8(12) - 1 = 95^\circ$
 $3(12) + 4 = 40^\circ$
 $3(12) + 9 = 45^\circ$

OBTUSE Δ B/C 1 OBTUSE \times

Ex 4) Solve for the missing variable, writing a reason for filling in any angle measure or writing an equation.

a.

LINEAR PAIR \rightarrow SUPP \angle 'S
 OR CONSEC ADJ
 \angle 'S ON A LINE
 SUM TO 180°

Δ SUM: $70^\circ + 50^\circ + p = 180$
 $p = 60^\circ$

Solve for the missing variable, writing a reason for filling in any angle measure or writing an equation.

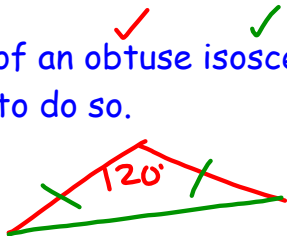
b.

$61 + 73 + y = 180$
 $\quad \quad \quad -134$
 $y = 46$

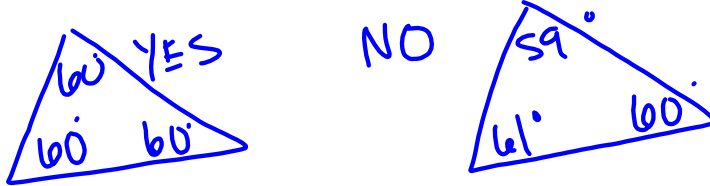
$\Delta \text{ SUM: } x + x + 46 = 180$
 $2x = 134$
 $x = 67$

Think and Discuss:

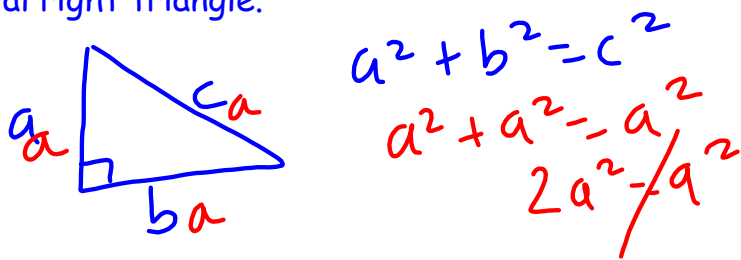
1. Sketch an example of an obtuse isosceles triangle, or explain why it is not possible to do so.



2. Is every acute triangle equiangular. Support your answer with a sketch.



3. Use the Pythagorean Theorem to explain why you cannot draw an equilateral right triangle.



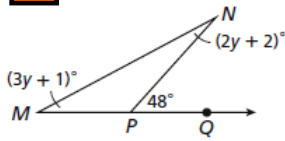
The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

6. 20.8° 7. y°

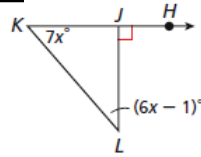
8. $24\frac{2}{3}$

Find each angle measure.

9. $m\angle M$

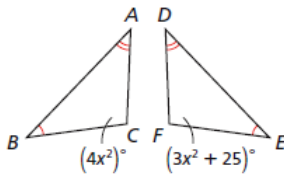


10. $m\angle I$

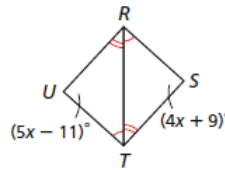


11. In $\triangle ABC$, $m\angle A = 65^\circ$, and the measure of an exterior angle at C is 117° . Find $m\angle B$ and the $m\angle BCA$.

12. $m\angle C$ and $m\angle F$



13. $m\angle S$ and $m\angle U$



14. For $\triangle ABC$ and $\triangle XYZ$, $m\angle A = m\angle X$ and $m\angle B = m\angle Y$. Find the measures of $\angle C$ and $\angle Z$ if $m\angle C = 4x + 7$ and $m\angle Z = 3(x + 5)$