

Lesson 4-1: Angle Relationships in Triangles

AGENDA:

- Triangle Exploration
- Guided Notes

Homework: Day 2

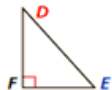

- p 228 17,19,21, 23, 29-32
- Cumulative Review #3 due 11/17

4.2 Notes: Angle Relationships in Triangles

$$\underline{90^\circ} + \underline{D} + \underline{E} = 180^\circ$$

Recall the Triangle Sum Theorem: the measures of the angles in a triangle sum to 180° . A **corollary** is a theorem whose proof follows directly from another theorem. Here are two corollaries to the triangle sum theorem.

Corollaries

COROLLARY	HYPOTHESIS	CONCLUSION
4-2-2 The acute angles of a right triangle are complementary.		$\angle D$ and $\angle E$ are complementary. $m\angle D + m\angle E = 90^\circ$
4-2-3 The measure of each angle of an equiangular triangle is 60° .		$m\angle A = m\angle B = m\angle C = 60^\circ$

$$D + E = 90^\circ$$

compl.

You can still always use Triangle Sum Theorem for calculations, but this is an important relationship.

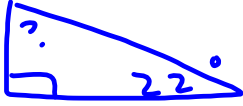
$$\underline{x} + \underline{x} + \underline{x} = 180^\circ$$

$$3x = 180^\circ$$

$$x = 60^\circ$$

$\Delta \text{ SUM } x + 22^\circ + 90^\circ = 180^\circ$

Ex 1) One of the acute angles in a right triangle measures 22° . What is the measure of the other acute angle?

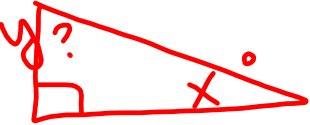


ACUTE \angle 'S
IN RT Δ
COMP:

$$22^\circ + x = 90^\circ$$

$$x = 68^\circ$$

Ex 2) One of the acute angles in a right triangle measures x° . In terms of x , what is the measure of the other angle?

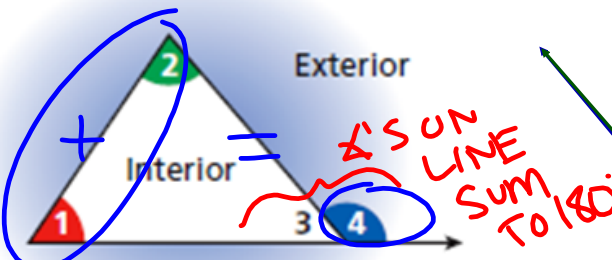
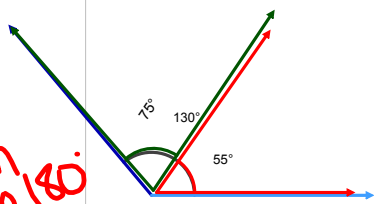


$\Delta \text{ SUM: } x^\circ + 90^\circ + y = 180^\circ$

$$x + y = 90^\circ$$

$$y = (90 - x)^\circ$$

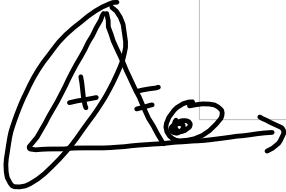
Visually:

$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ \Delta \text{ sum}$

$m\angle 3 + m\angle 4 = 180^\circ$

Symbolically:

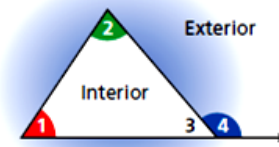
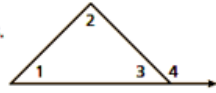


$$m\angle 1 + m\angle 2 + m\angle 3 - m\angle 3 + m\angle 4$$

Theorem 4-2-4 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

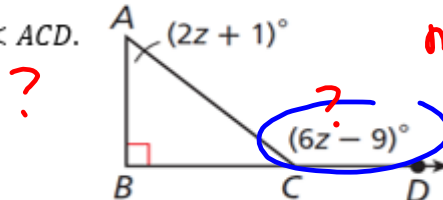
$$m\angle 4 = m\angle 1 + m\angle 2$$



Proof:

- Since the interior angle measures of a triangle sum to 180° , then $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$.
- Since $\angle 3$ & $\angle 4$ form a linear pair and linear pairs of angles are supplementary, then $m\angle 3 + m\angle 4 = 180^\circ$ by the definition of supplementary angles.
- By the **SUBTRACTION** property of equality, $m\angle 1 + m\angle 2 = 180^\circ - m\angle 3$ and $m\angle 4 = 180^\circ - m\angle 3$.
- Therefore by **SUBSTITUTION**, $m\angle 1 + m\angle 2 = m\angle 4$.

Ex. 3) a. Find $m\angle ACD$.



$$m\angle ACD = 6(25) - 9 = 141^\circ$$

Δ EXT \angle THM: $m\angle ACD = m\angle A + m\angle B$

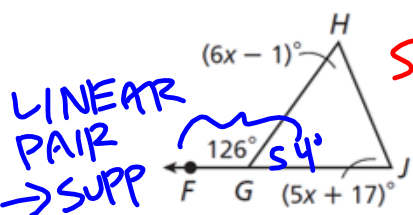
$$6z - 9 = (2z + 1)^\circ + 90^\circ$$

$$6z - 9 = 2z + 91$$

$$4z = 100$$

$$z = 25$$

b. Determine x . Is there another approach if you have the degree measure of the exterior angle?



Δ SUM:

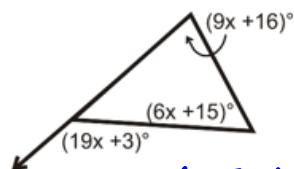
$$\underline{5x+17} + \underline{6x-1} + \underline{54} = 180^\circ$$

$$11x + 70 = 180$$

$$11x = 110$$

$$x = 10$$

c. Practice: Determine the value of x .



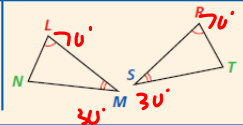
Δ EXT & THM:

$$\underline{19x+3} = \underline{9x+16} + \underline{6x+15}$$

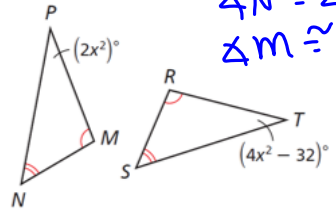
$$19x+3 = 15x+31$$

$$4x = 28$$

$$\boxed{x=7}$$

THEOREM	HYPOTHESIS	CONCLUSION
If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.		$\angle N \cong \angle T$

Ex. 6) Find $m\angle P$ and $m\angle T$.



$\angle N \cong \angle S$
 $\angle M \cong \angle R$

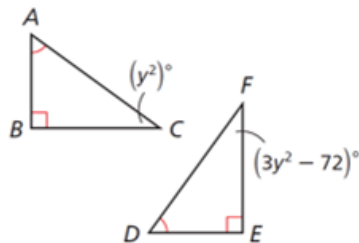
$\} \implies \angle P \cong \angle T$
 3rd ANGLE THEOREM

$2x^2 = 4x^2 - 32$
 $-4x^2 \quad -4x^2$

 $-2x^2 = -32$
 $\div -2 \quad \div -2$
 $x^2 = 16$
 $x = \pm 4$

$m\angle P = 2(-4)^2 = 2(16) = 32^\circ$
 $m\angle T = 4(-4)^2 - 32$
 $= 4(16) - 32$
 $= 64 - 32$
 $= 32^\circ$

b. Find $m\angle C$ and $m\angle F$



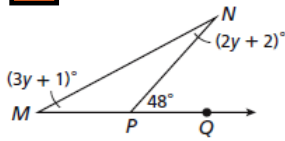
The measure of one of the acute angles in a right triangle is given. What is the measure of the other acute angle?

6. 20.8° 7. y°

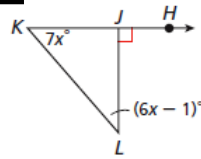
8. $24\frac{2}{3}$

Find each angle measure.

9. $m\angle M$

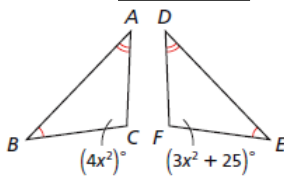


10. $m\angle I$

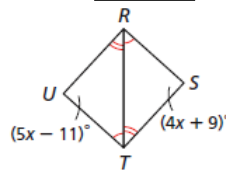


11. In $\triangle ABC$, $m\angle A = 65^\circ$, and the measure of an exterior angle at C is 117° . Find $m\angle B$ and the $m\angle BCA$.

12. $m\angle C$ and $m\angle F$



13. $m\angle S$ and $m\angle U$



14. For $\triangle ABC$ and $\triangle XYZ$, $m\angle A = m\angle X$ and $m\angle B = m\angle Y$. Find the measures of $\angle C$ and $\angle Z$ if $m\angle C = 4x + 7$ and $m\angle Z = 3(x + 5)$