Lesson 4-3: SAS Triangle Congruency Criteria

Agenda:

- Check & Review Homework
- Exploration & Notes

Homework:

- Problem Set in Notes
- Reminder: CR#3 due 11/17

Homework 4-2 p235-6 #13-18, 20, 27(No Lab), 31

Given: Polygon $CDEF \cong$ polygon $KLMN$. Identify the congruent corresponding parts.

13. $\overline{DE} \cong$ __?__ $\overline{LM}$

14. $\overline{KN} \cong$ __?__ $\overline{CF}$

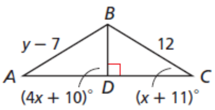
15. $\angle F \cong$ __?__ $\angle N$

16. $\angle L \cong$ __?__ $\angle D$

Given: $\triangle ABD \cong \triangle CBD$. Find each value.

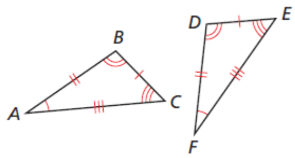
17. $m\angle C$
$31°$

18. y
19



31. Which congruence statement correctly indicates that the two given triangles are congruent?

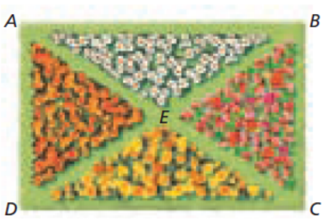
(A) $\triangle ABC \cong \triangle EFD$ (C) $\triangle ABC \cong \triangle DEF$
(B) $\triangle ABC \cong \triangle FDE$ (D) $\triangle ABC \cong \triangle FED$



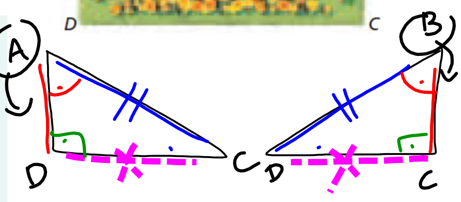
20. Hobbies In a garden, triangular flower beds are separated by straight rows of grass as shown.

Given: $\angle ADC$ and $\angle BCD$ are right angles. ≅
$\overline{AC} \cong \overline{BD}$, $\overline{AD} \cong \overline{BC}$
$\angle DAC \cong \angle CBD$

Prove: $\triangle ADC \cong \triangle BCD$

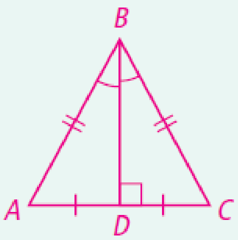


20. 1. $\angle ADC$ and $\angle BCD$ are rt. $\angle$s. (Given)
A 2. $\angle ADC \cong \angle BCD$ (Rt. $\angle \cong$ Thm.)
A 3. $\angle DAC \cong \angle CBD$ (Given)
A 4. $\angle ACD \cong \angle BDC$ (Third $\angle$s Thm.)
S 5. $\overline{AC} \cong \overline{BD}$, $\overline{AD} \cong \overline{BC}$ (Given) S
6. $\overline{DC} \cong \overline{DC}$ (Reflex. Prop. of $\cong$)
7. $\triangle ADC \cong \triangle BCD$ (Def. of $\cong$ $\triangle$s)



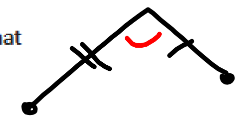
27. Draw a diagram and then write a proof.
Given: $\overline{BD} \perp \overline{AC}$. D is the midpoint of $\overline{AC}$. $\overline{AB} \cong \overline{CB}$, and $\overline{BD}$ bisects $\angle ABC$.
Prove: $\triangle ABD \cong \triangle CBD$

27.

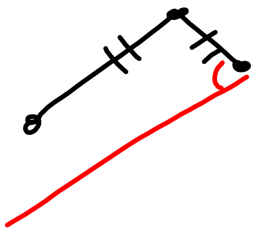


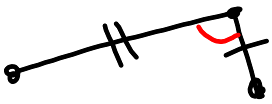
1. $\overline{BD} \perp \overline{AC}$ (Given)
2. $\angle ADB$ and $\angle CDB$ are rt. $\angle$s (Def. of $\perp$)
3. $\angle ADB \cong \angle CDB$ (Rt. $\angle \cong$ Thm.)
4. $\overline{BD}$ bisects $\angle ABC$. (Given)
5. $\angle ABD \cong \angle CBD$ (Def. of bisect)
6. $\angle A \cong \angle C$ (Third $\angle$s Thm.)
7. $\overline{AB} \cong \overline{CB}$ (Given)
8. $\overline{BD} \cong \overline{DB}$ (Reflex. Prop. of $\cong$)
9. D is the mdpt. of $\overline{AC}$. (Given)
10. $\overline{AD} \cong \overline{CD}$ (Def. of mdpt.)
11. $\triangle ABD \cong \triangle CBD$ (Def of $\cong$ $\triangle$s)

Side-Angle-Side Triangle Congruence Criteria (SAS): **Given two triangles $\triangle ABC$ and $\triangle A'B'C'$ so that $AB = A'B'$ (Side), $m\angle A = m\angle A'$ (Angle), $AC = A'C'$ (Side). Then the triangles are congruent.**

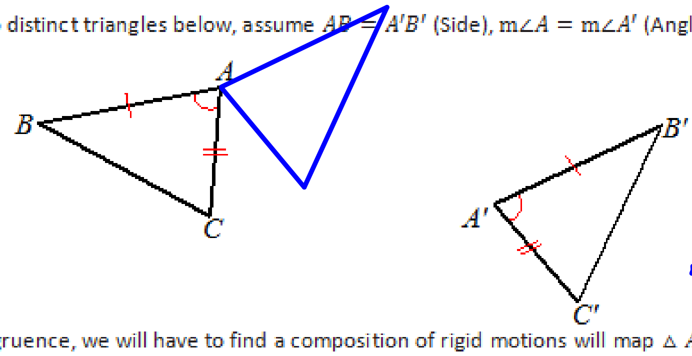


The steps below show the most general case for determining a congruence between two triangles that satisfy the SAS criteria. Note that not all steps are needed for every pair of triangles. For example, sometimes the triangles will already share a vertex. Sometimes a reflection will be needed, sometimes not. It is important to understand that we can always use the steps below—some or all of them—to determine a congruence between the two triangles that satisfies the SAS criteria





Proof: Provided the two distinct triangles below, assume $AB = A'B'$ (Side), $m\angle A = m\angle A'$ (Angle), $AC = A'C'$ (Side).



GOAL:
A' → A
B' → B
C' → C

By our definition of congruence, we will have to find a composition of rigid motions will map $\triangle A'B'C'$ to $\triangle ABC$. We must find a congruence F so that $F(\triangle A'B'C') = \triangle ABC$. First, use a translation T to map a common vertex.

Which two points determine the appropriate vector? A' & A

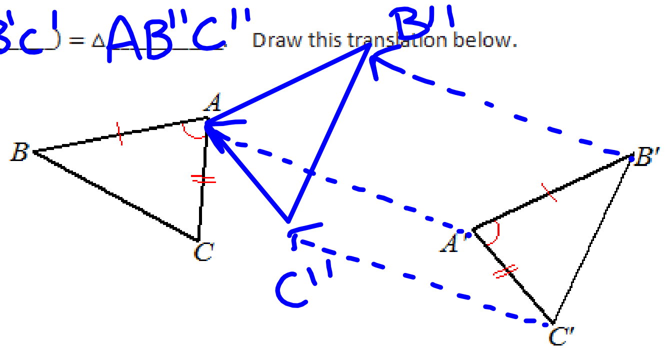
Can any other pair of points be used? NO Why or why not?

GIVEN ∡A ≅ ∡A'

State the vector in the picture below that can be used to translate $\triangle A'B'C'$: $\overrightarrow{A'A}$

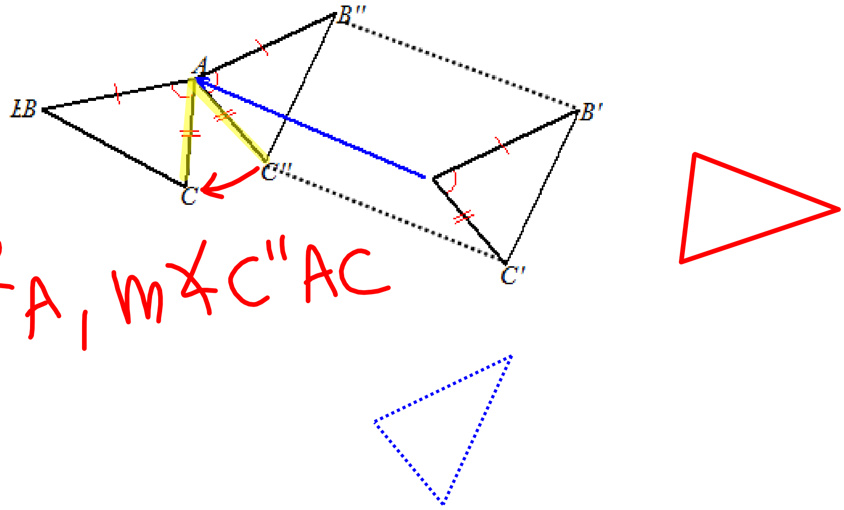
After the translation above, show $T_{\overrightarrow{A'A}}(\triangle A'B'C')$ shares one vertex with $\triangle ABC$, A. In fact, we can say

$T_{\overrightarrow{A'A}}(\triangle A'B'C') = \triangle AB''C''$ Draw this translation below.



A' → A ✓
B' → B''
C' → C''

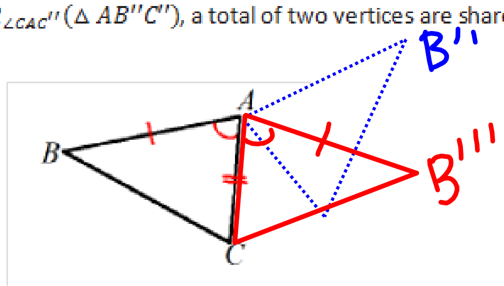
Next, use a clockwise rotation $R_{\angle CAC''}$ to bring the sides $\overline{AC''}$ to $\overline{AC}$ (or counterclockwise rotation to bring $\overline{AB''}$ to $\overline{AB}$). angle of rotation.



$R_{A,\, m\angle C''AC}$

rotation of appropriate measure will map $\overrightarrow{AC''}$ to $\overrightarrow{AC}$, but how can we be sure that vertex C'' maps to C

ter the rotation $R_{\angle CAC''}(\triangle AB''C'')$, a total of two vertices are shared with $\triangle ABC$, A and C. Draw this rotation below.



$A \to A$ ✓

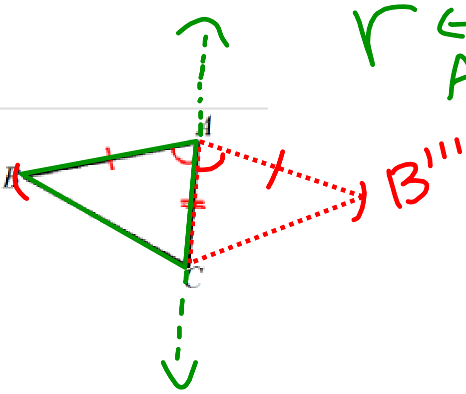
$C'' \to C$ ✓

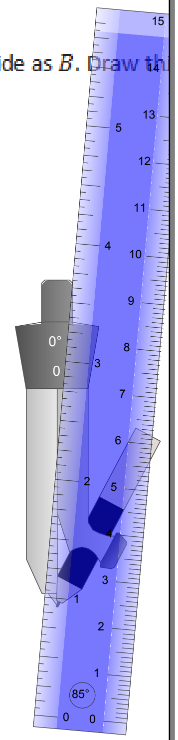
$B'' \to B'''$

Finally, if B''' and B are on opposite sides of the line that joins AC, a reflection $r_{\overleftrightarrow{AC}}$ brings B''' to the same side as B. Draw th[e] reflection below.

$r_{\overleftrightarrow{AC}}$

$A \to A$
$C \to C$
$B''' \to B$



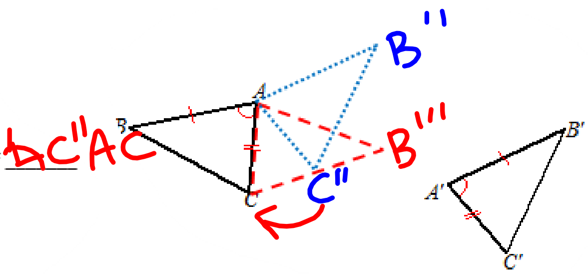


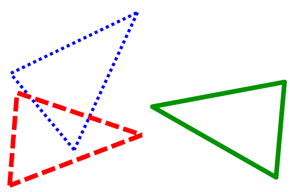
⊥ BIS OF $B'''B$

Another way to visually confirm this is to draw the marks of the ⊥ BIS OF $B'''B$ construction for $\overline{AC}$.

SUMMARIZE: Write the sequence of transformations that maps $\triangle A'B'C'$ onto $\triangle ABC$ such that $\triangle A'B'C' \cong \triangle ABC$:

1. Translate Δ $A'B'C'$ by vector $\overrightarrow{A'A}$ to map A' to A, B' to B'', C' to C''.
2. Rotate Δ $AB''C''$ around A by the measure of $\angle C''AC$ to map A to A, B'' to B''', and C'' to C.
3. Reflect Δ $AB'''C$ into line $\overleftrightarrow{AC}$ to map A to A, C to C, and B''' to B.



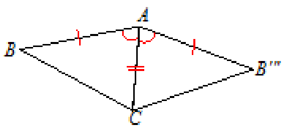
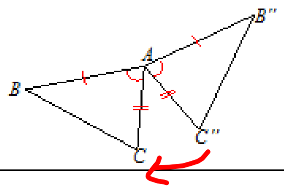


Written as a composition of transformations: $r_{\overline{AC}}(R_{A,m\angle C''AC}(T_{\overline{A'A}}(\Delta A'B'C'))) \to \Delta ABC$

We have now shown a sequence of rigid motions that takes $\triangle A'B'C'$ to $\triangle ABC$ with the use of just three criteria from each triangle: two sides and an included angle. Given any two distinct triangles, we could perform a similar proof. **Note that when using the Side-Angle-Side triangle congruence criteria as a reason in a proof, you need only state the congruence and "by SAS ≅ SAS".**

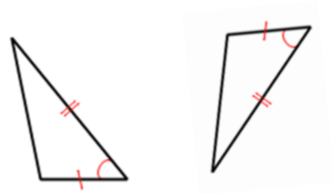
Example 1

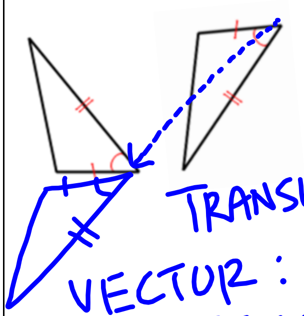
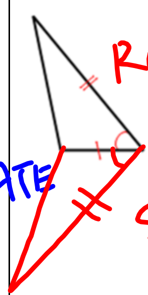
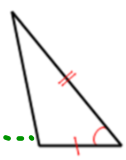
What if we had the SAS criteria for two triangles that were not distinct? Consider the following two cases. How would the transformations needed to demonstrate congruence change?

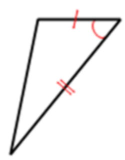
Case	Diagram	Transformations Needed
Shared Side		REFLECT
Shared Vertex		① ROTATION ② REFLECTION

Exercises 1–4

1. **Given: Triangles with a pair of corresponding sides of equal length and a pair of included angles of equal measure.** Sketch and label three phases of the sequence of rigid motions that prove the two triangles to be congruent.



1st Transformation	2nd Transformation	3rd Transformation
 TRANSLATE VECTOR: ≅∡'S VERTICES	 ROTATION TO GET SHARED SIDE	 REFLECTION OVER SHARED SIDE

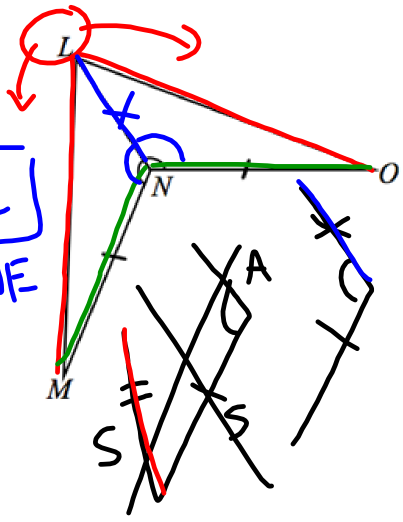


Justify whether the triangles meet the SAS congruence criteria; **explicitly state which pairs of sides or angles are congruent and why.**

2. **Given** $\angle LNM \cong \angle LNO$, $\overline{MN} \cong \overline{ON}$.
Do $\triangle LMN$ and $\triangle LON$ meet the SAS criteria?

YES

S	A	S
$\overline{MN} \cong \overline{ON}$	$\angle LNM \cong \angle LNO$	$\overline{NL} \cong \overline{NL}$
GIVEN	GIVEN	REFLEXIVE



$\triangle LMN \cong \triangle LON$ BY SAS ≅ SAS

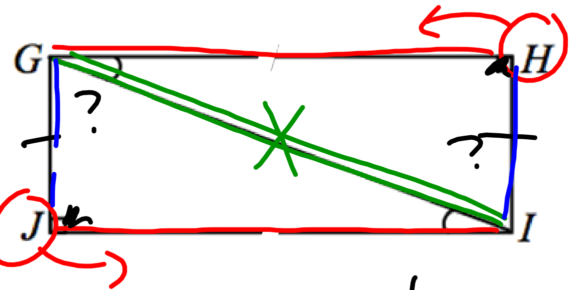
Rigid motion(s) to map $\triangle LMN$ onto $\triangle LON$: REFLECTION INTO $\overleftrightarrow{LN}$

$L \to L$
$N \to N$
$M \to O$

3. Given: $\angle HGI \cong \angle JIG$, $\overline{IH} \cong \overline{GJ}$.
Do $\triangle HGI$ and $\triangle JIG$ meet the SAS criteria?

NO

S	A	S
REFLEXIVE	NO INFO	GIVEN

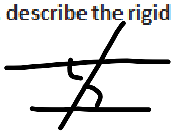


MUST INCLUDE ∡!

Problem Set

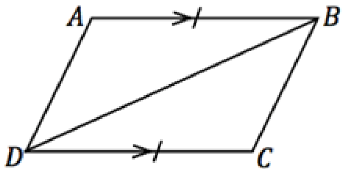
Justify whether the triangles meet the SAS congruence criteria; explicitly state which pairs of sides or angles are congruent and why. If the triangles do meet the SAS congruence criteria, describe the rigid motion(s) that would map one triangle onto the other.

$\measuredangle$'s



1. **Given:** $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD}$

Do $\triangle\, ABD$ and $\triangle\, CDB$ meet the SAS criteria?



Rigid motions (3) to map ΔABD onto ΔCDB: