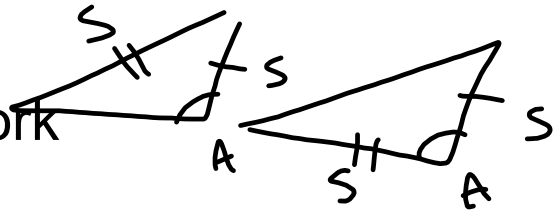
Lesson 4-7L: ASA and SSS Congruency Criteria

Agenda:



- Review 4-6L Homework
- Mini Quiz - Surprise!
- Exploration, Notes, Guided Practice

Homework:

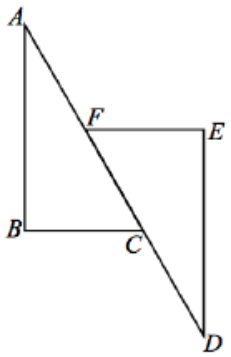
- Problem Set in Notes
- ~~Pg. 257 11,12,17~~ Already in notes
- CR Due tomorrow

PRACTICE – You write the given (finish for homework if necessary)

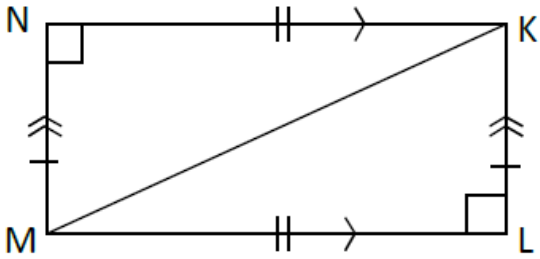
Using the illustration at right, decide what information you would give in order for a classmate to be able to prove that $\Delta ABC \cong \Delta DEF$ using $\text{SAS} \cong \text{SAS}$ differently for each example. Be more creative than *all* straightforward "givens" for each piece of information and decorate your drawing each time. Then write the proof.

3) Using $\measuredangle BCA$ & $\measuredangle EFD$ as included angles

Given: Actual Proof:



Problem Set 4-6L



1. Given the diagram, prove $\Delta MNK \cong \Delta KLM$ using

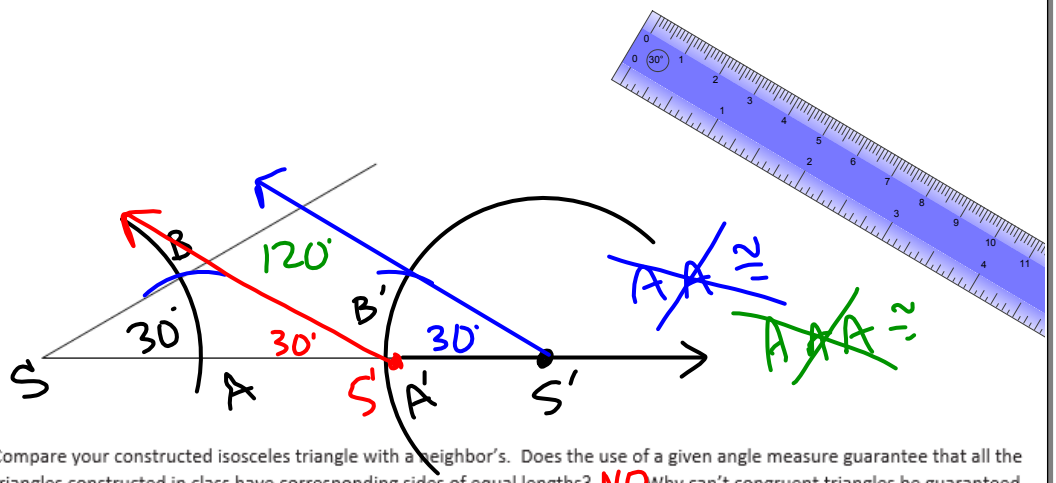
A) $SAS \cong SAS$ with included $\measuredangle N$ & $\measuredangle L$

B) $SAS \cong SAS$ with included $\measuredangle NKM$ & $\measuredangle LMK$

Classwork

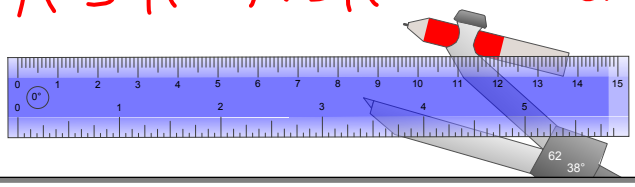
Opening Exercise

Use the provided 30° angle as one base angle of an isosceles triangle. Use a compass and straight edge to construct an appropriate isosceles triangle around it.



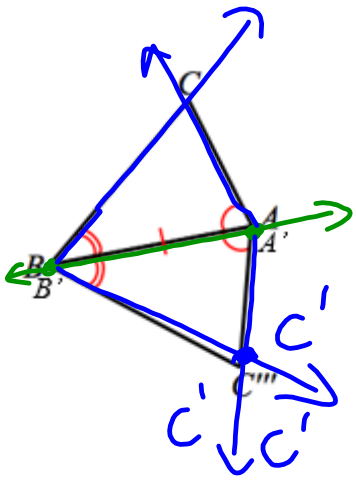
Compare your constructed isosceles triangle with a neighbor's. Does the use of a given angle measure guarantee that all the triangles constructed in class have corresponding sides of equal lengths? NO Why can't congruent triangles be guaranteed by only using angle measure? NEED SIDE LENGTH TO DETERMINE SIZE

ASA ≅ ASA



Angle-Side-Angle Triangle Congruence Criteria (ASA≅ASA): Given two triangles ABC and $A'B'C'''$. If $m\angle CAB = m\angle C'''B'$ (Angle), $AB = A'B'$ (Side), and $m\angle CBA = m\angle C'''A'$ (Angle), then the triangles are congruent.

Proof goal: map each of the corresponding vertices, working to prove C maps to C'''.



Since $\overleftrightarrow{AB}$ *is the line of reflection A maps to* A *and B maps to* B

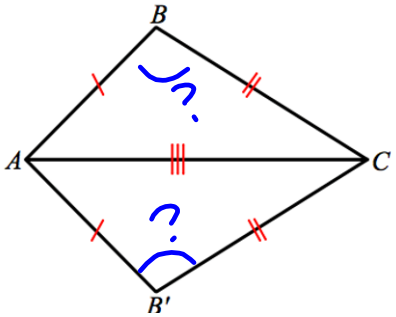
Since reflections preserve angle measure, then $\overrightarrow{BC}$ *maps to* $\overrightarrow{BC'}$ *and* $\overrightarrow{AC}$ *maps to* $\overrightarrow{AC'}$

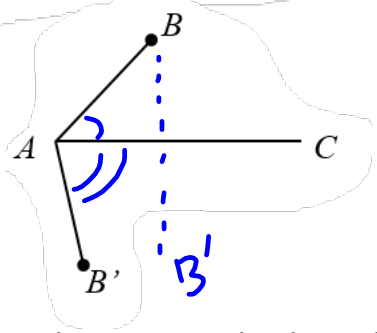
Since two rays intersect in exactly 1 *point and C is on both* $\overrightarrow{BC'}$ *and* $\overrightarrow{AC'}$, *then C' is the same as* C''', *meaning C maps to* C'''

Therefore, ΔABC *maps to* $\Delta ABC'''$ *proving that the triangles are congruent.*

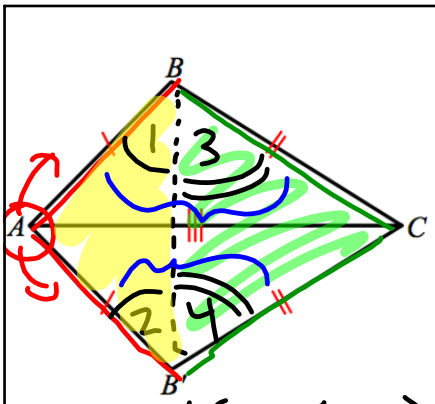
The gist: the angle measures (given as congruent) direct the rays; the congruent side locks in the points of intersection.

Side-Side-Side Triangle Congruence Criteria (SSS≅SSS): Given two triangles ABC and $A'B'C'$. If $AB = A'B'$ (Side), $AC = A'C'$ (Side), and $BC = B'C'$ (Side) then the triangles are congruent.





Since no angle measure is given, you cannot assume that B maps to B' under a reflection. Counterexample:



1) Draw in $\overline{BB'}$ since two points determine a line segment

2) Prove $\Delta ABC \cong \Delta AB'C$ by SAS≅SAS:

S	A	S
$\overline{AB} \cong \overline{A'B'}$	GIVEN	$\overline{BC} \cong \overline{B'C}$
$\Delta ABB'$ is isosceles	DEFN OF ISOS Δ	$\Delta CBB'$ is isosceles
$\angle 1 \cong \angle 2$	$m\angle 1 = m\angle 2$ $m\angle 3 = m\angle 4$	$\angle 3 \cong \angle 4$
	$m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	ADD PROP OF EQ
	$m\angle ABC = m\angle AB'C$	$\angle$ ADD POST
	$\angle ABC \cong \angle AB'C$	= MEAS ↔ ≅

ISOS Δ → BASE $\angle$'s ≅ = MEAS ↔ ≅

Therefore $\Delta ABC \cong \Delta AB'C$ by SAS≅SAS

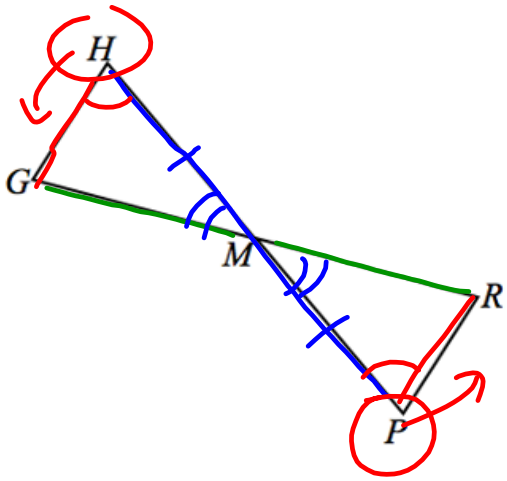
Exercises

→ 2 ≅ SEGS

1. Given: M is the midpoint of $\overline{HP}$, $\angle H \cong \angle P$.

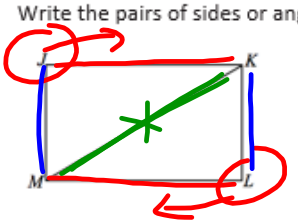
   Prove: $\Delta HGM \cong \Delta PRM$ by ASA≅ASA

~~SSS≅~~ ~~SAS≅~~ ASA≅

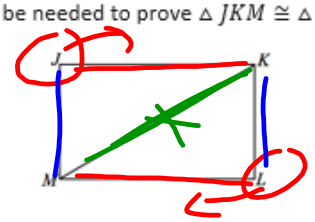


2. Given: $JKLM$ with $\overline{JK} \parallel \overline{LM}$, $\overline{JK} \cong \overline{LM}$, $\overline{MJ} \cong \overline{KL}, \overline{JK} \perp \overline{JM}, \overline{KL} \perp \overline{LM}$ (note: these are properties of a rectangle)

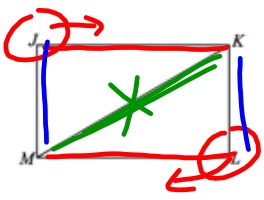
Write the pairs of sides or angles that would be needed to prove $\triangle JKM \cong \triangle LMK$ using each set of criteria:



S S S≅



S A S≅

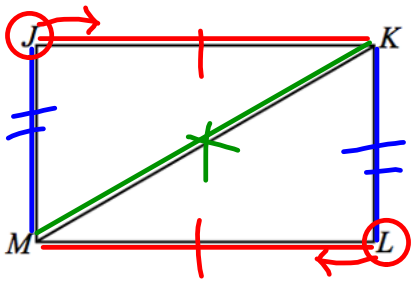


A S A≅

Color all the correspondence first

Given: $JKLM$ with $\overline{JK} \parallel \overline{LM}$, $\overline{JK} \cong \overline{LM}$, $\overline{MJ} \cong \overline{KL}, \overline{JK} \perp \overline{JM}, \overline{KL} \perp \overline{LM}$ (note: these are properties of a rectangle)

Write the pairs of sides or angles that would be needed to prove $\triangle JKM \cong \triangle LMK$ using each set of criteria:

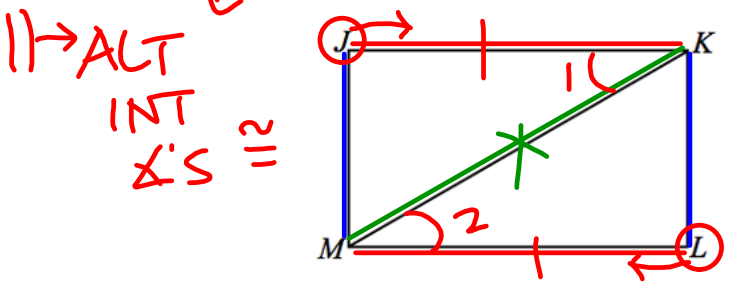


S S S≅

$\overline{JK} \cong \overline{LM}$ $\overline{LM} \cong \overline{MK}$ $\overline{MJ} \cong \overline{KL}$

Given: $JKLM$ with $\overline{JK} \parallel \overline{LM}$, $\overline{JK} \cong \overline{LM}$, $\overline{MJ} \cong \overline{KL}$, $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{LM}$ (note: these are properties of a rectangle)

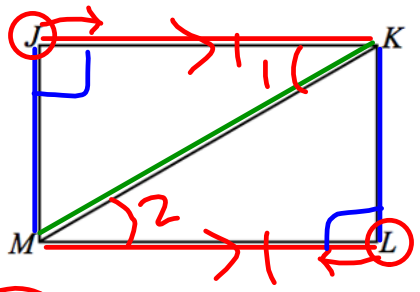
Write the pairs of sides or angles that would be needed to prove $\Delta JKM \cong \Delta LMK$ using each set of criteria:



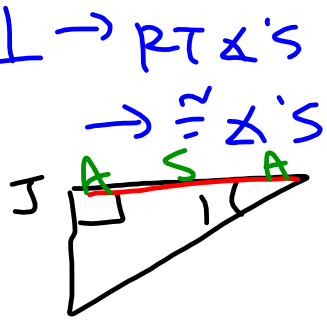
S $\overline{JK} \cong \overline{LM}$ A $\angle 1 \cong \angle 2$ S≅ $\overline{KM} \cong \overline{MK}$

Given: $JKLM$ with $\overline{JK} \parallel \overline{LM}$, $\overline{JK} \cong \overline{LM}$, $\overline{MJ} \cong \overline{KL}$, $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{LM}$ (note: these are properties of a rectangle)

Write the pairs of sides or angles that would be needed to prove $\Delta JKM \cong \Delta LMK$ using each set of criteria:



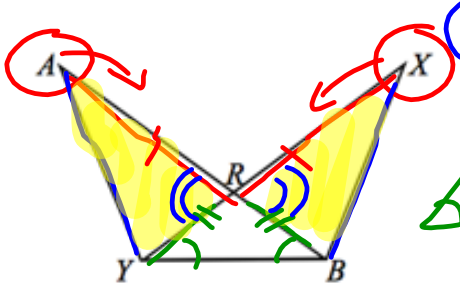
A $\angle 1 \cong \angle 2$ S $\overline{JK} \cong \overline{LM}$ A≅ $\angle J \cong \angle L$



Identify whether each of the following can be proven by SSS≅SSS, SAS≅SAS, or ASA≅ASA:

3. Given: $\measuredangle RYB \cong \measuredangle RBY, \overline{AR} \cong \overline{XR}$

Prove: $\Delta ARY \cong \Delta XRB$

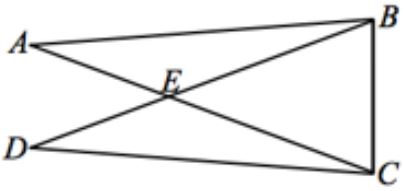


~~SSS~~ SAS ASA

Criteria: SAS ≅ SAS

4. Given: $\angle A \cong \angle D, \overline{AE} \cong \overline{DE}.$

Prove: $\triangle AEB \cong \triangle DEC$

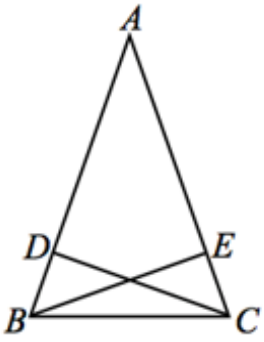


Criteria: _____

Identify whether each of the following can be proven by SSS≅SSS, SAS≅SAS, or ASA≅ASA:

5. Given: $\overline{AB} \cong \overline{AC}, \overline{BD} \cong \overline{CE}$

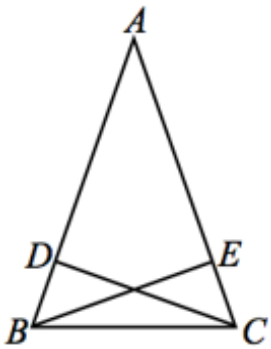
Prove: $\Delta BDC \cong CEB$



Criteria: _____

6. Given: $\overline{AB} \cong \overline{AC}, \overline{BD} \cong \overline{CE}$

Prove: $\Delta ABE \cong ACD$

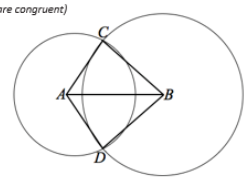


Criteria: _____

Problem Set 4-5R/4-7L **Use separate paper if you need more room.**

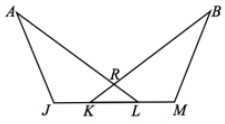
Use your knowledge of triangle congruence criteria to write proofs for each of the following problems.

1. Given: Circles with centers A and B intersect at C and D. *(Hint: radii of a circle are congruent)*
   Prove: $\Delta CAB \cong \Delta DAB$.



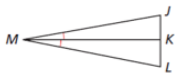
What specific rigid motion would map ΔCAB onto ΔDAB? _____

2. Given: $\angle J \cong \angle M$, $\overline{KR} \cong \overline{LR}$; $\overline{JK} \cong \overline{ML}$
   Prove: $\triangle AJL \cong \triangle BMK$

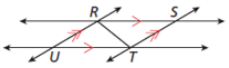


For 3&4 determine whether you can use ASA to prove the triangles are congruent. If possible, write out the proof. If not, explain why it is not possible.

3. $\triangle MKJ$ and $\triangle MKL$



4. $\triangle RST$ and $\triangle TUR$



Extra Credit:
Given: $\angle w \cong \angle x$ and $\angle y \cong \angle z$
Prove: $\triangle ABE \cong \triangle ACE$