

## 4-6R/4-8L: AAS and $R_{\Delta}$ HL Congruency Criteria

### Agenda:

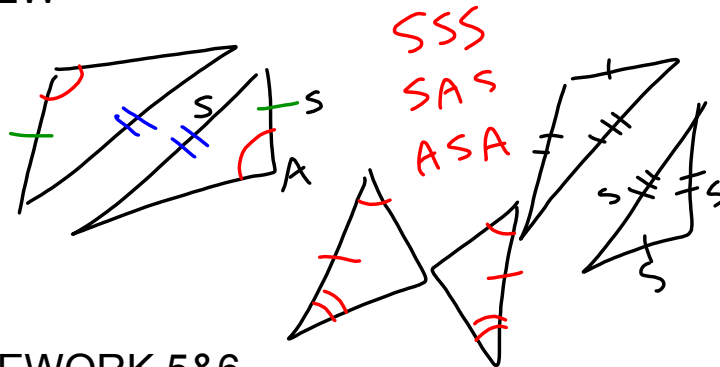
- Check & review homework
- Turn in CR#3
- Lesson Notes & Guided Practice

SSS  $\checkmark$   
 SAS  $\checkmark$   
 ASA  $\checkmark$   
 NOT POSS.

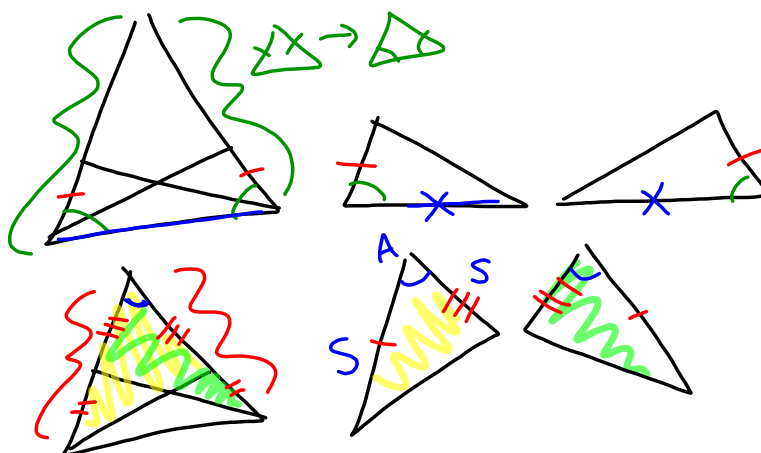
### Homework:

- Problem Set in Notes
- Unit 3 Test Remediation - Next Tu after school

### REVIEW



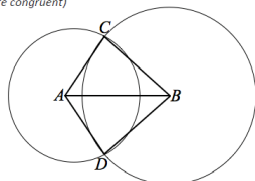
### HOMEWORK 5&6



**Problem Set 4-5R/4-7L** Use separate paper if you need more room.

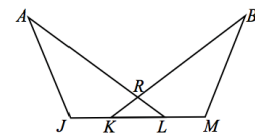
Use your knowledge of triangle congruence criteria to write proofs for each of the following problems.

1. Given: Circles with centers  $A$  and  $B$  intersect at  $C$  and  $D$ . (Hint: radii of a circle are congruent)  
 Prove:  $\triangle CAB \cong \triangle DAB$ .



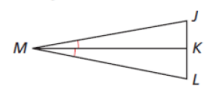
What specific rigid motion would map  $\triangle CAB$  onto  $\triangle DAB$ ?

2. Given:  $\angle J \cong \angle M$ ,  $\overline{KR} \cong \overline{LR}$ ;  $\overline{JK} \cong \overline{ML}$   
 Prove:  $\triangle AJL \cong \triangle BMK$

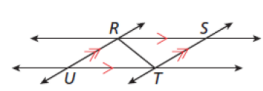


For 3&4 determine whether you can use ASA to prove the triangles are congruent. If possible, write out the proof. If not, explain why it is not possible.

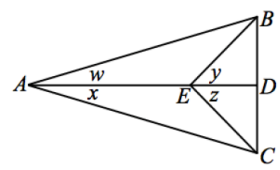
3.  $\triangle MKJ$  and  $\triangle MKL$



4.  $\triangle RST$  and  $\triangle TUR$



Extra Credit:  
 Given:  $\angle w \cong \angle x$  and  $\angle y \cong \angle z$   
 Prove:  $\triangle ABE \cong \triangle ACE$



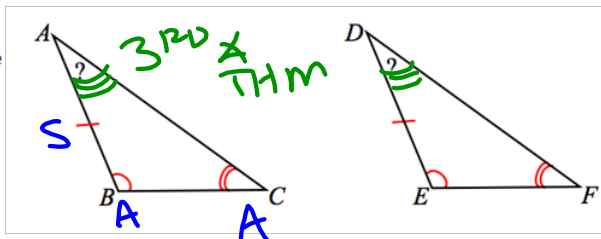
**Geometry 4-6 – AAS and R<sub>Δ</sub>HL Criteria**

**Angle-Angle-Side Triangle Congruence Criteria (AAS):**

Given two triangles  $ABC$  and  $A'B'C'$ . If  $AB = A'B'$  (Side),  $m\angle B = m\angle B'$  (Angle), and  $m\angle C = m\angle C'$  (Angle), then the triangles are congruent.

Proof:

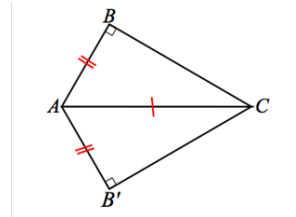
Since  $\angle A \cong \angle D$  by the Third Angle Theorem, this criteria is an extension of the ASA criteria and is therefore valid.



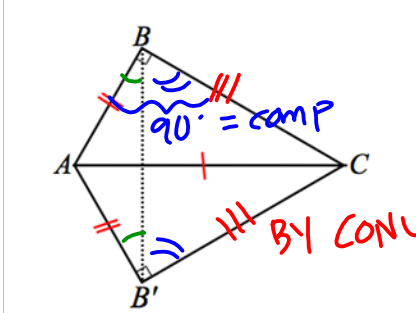
**Right Triangle Hypotenuse-Leg Triangle Congruence Criteria (R<sub>Δ</sub>HL):** Given two right triangles  $ABC$  and  $A'B'C'$  with right angles  $B$  and  $B'$ , if  $AB = A'B'$  (Leg) and  $AC = A'C'$  (Hypotenuse), then the triangles are congruent.

*Proof:*

Imagine that a congruence exists so that triangles have been brought together such that  $A = A'$  and  $C = C'$ ; the hypotenuse acts as a common side to the transformed triangles.



Similar to the proof for SSS, we add a construction and draw  $\overline{BB'}$ .



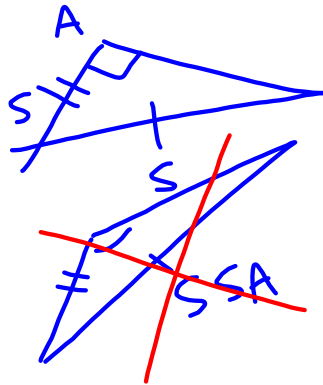
$\triangle ABB'$  is isosceles by definition so we can conclude that base angles  $m\angle ABB' = m\angle AB'B$  by the isosceles triangle theorem. Since  $\angle CBB'$  and  $\angle CB'B$  are both the complements of equal angle measures ( $\angle ABB'$  and  $\angle AB'B$ ), they too are equal in measure. (Note, this would be formally proven by perpendicular lines  $\rightarrow$  right angles  $\rightarrow$  substitution in the angle addition postulate  $\rightarrow$  definition of complementary angles).

Furthermore, since  $m\angle CBB' = m\angle CB'B$ , then  $\triangle CBB'$  is isosceles by the converse of the isosceles theorem. Therefore  $BC = B'C'$  by the definition of isosceles triangles.

Then, by  $SSS \cong SSS$  or by  $SAS \cong SAS$ , we can conclude  $\triangle ABC \cong \triangle A'B'C'$ . This proves R<sub>Δ</sub>HL as a valid criteria for triangle congruence.

Note: Regents uses "HL". We use R<sub>Δ</sub>HL because you must state the triangles are right triangles by |

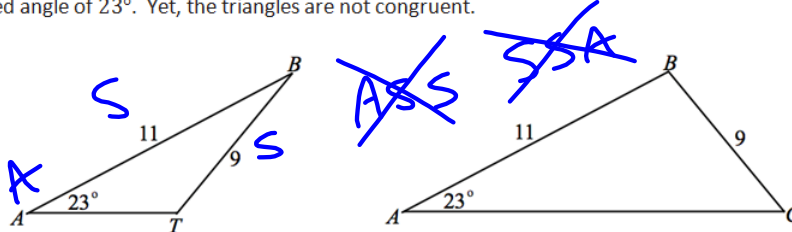
1) RIGHT  $\Delta \rightarrow$  RIGHT  $\Delta$  or 2) DEFN OF RIGHT  $\Delta$



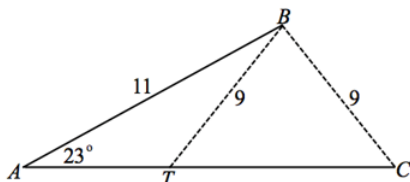
~~SSA~~  
LOOK TO SEE  
IF RT  $\Delta$   
FOR RHL  $\approx$

Criteria that do not determine two triangles as congruent: SSA and AAA

Side-Side-Angle (SSA): Observe the diagrams below. Each triangle has a set of adjacent sides of measures 11 and 9, as well as the non-included angle of  $23^\circ$ . Yet, the triangles are not congruent.



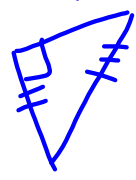
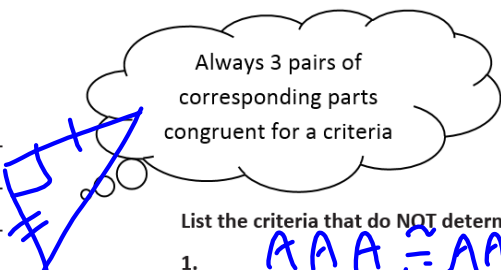
Examine the composite made of both triangles. The sides of lengths 9 each have been dashed to show their possible locations.



The triangles that satisfy the conditions of SSA cannot guarantee congruence criteria. In other words, two triangles under SSA criteria may or may not be congruent; therefore, we cannot categorize SSA as congruence criterion.

List all the triangle congruence criteria here:

1. SSS  $\cong$  SSS
2. SAS  $\cong$  SAS
3. ASA  $\cong$  ASA
4. AAS  $\cong$  AAS
5. RHL  $\cong$  RHL



Always 3 pairs of corresponding parts congruent for a criteria

List the criteria that do NOT determine congruence here:

1. AAA  $\cong$  AAA
2. SSA  $\cong$  SSA DONKEY

Examples

1. Given:  $\overline{BC} \perp \overline{CD}, \overline{AB} \perp \overline{AD}, \angle 1 \cong \angle 2$   
 Prove:  $\triangle BCD \cong \triangle BAD$

*Handwritten notes:*  $RT \Delta's \rightarrow \cong \Delta's$ ,  $RT \Delta$

*Handwritten notes on diagram:* RHL, SSS, SAS, ASA, AAS

*Flowchart:*

- Box 1:  $\overline{BC} \perp \overline{CD}$ ,  $\overline{AB} \perp \overline{AD}$  (GIVEN)
- Box 2:  $\angle C$  &  $\angle A$  (RIGHT  $\angle$ 'S)
- Text:  $\perp$  LINES  $\rightarrow$  RT  $\Delta$ 'S
- Box 3:  $\angle C \cong \angle A$  (ALL RIGHT  $\angle$ 'S ARE  $\cong$ )
- Box 4:  $\angle 1$  &  $\angle 3$  LINEAR PAIR,  $\angle 2$  &  $\angle 4$  PAIR
- Text: DEFN LINEAR PAIR
- Box 5:  $\angle 1$  SUPP  $\angle 3$ ,  $\angle 2$  SUPP  $\angle 4$
- Text: LINEAR PAIR  $\rightarrow$  SUPP  $\Delta$ 'S
- Box 6:  $\angle 1 \cong \angle 2$  (GIVEN)
- Text:  $\cong$  SUPPLEMENTS THM
- Box 7:  $\angle 3 \cong \angle 4$
- Text:  $\triangle BCD \cong \triangle BAD$  BY AAS  $\cong$  AAS

2. Given:  $\overline{AD} \perp \overline{BD}, \overline{BD} \perp \overline{BC}, \overline{AB} \cong \overline{CD}$   
 Prove:  $\triangle ABD \cong \triangle CDB$

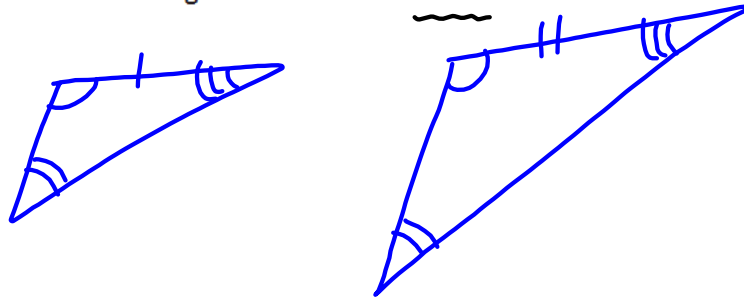
*Handwritten notes:*  $RT \Delta \rightarrow \cong RT \Delta's$

*Handwritten notes on diagram:* SSS, SAS, ASA, AAS, RHL

*Flowchart:*

- Box 1:  $\overline{AD} \perp \overline{BD}$ ,  $\overline{BD} \perp \overline{BC}$  (GIVEN)
- Text: DEFN  $\perp$  LINES
- Box 2:  $\angle 1$  &  $\angle 2$  (RIGHT  $\angle$ 'S)
- Text: DEFN RT  $\Delta$ 'S
- Box 3:  $\triangle ABD$  &  $\triangle CDB$  (ARE RT  $\Delta$ 'S)
- Text: DEFN RT  $\Delta$
- Box 4:  $\overline{AB} \cong \overline{CD}$  (GIVEN)
- Box 5:  $\overline{BD} \cong \overline{DB}$  (REFLEXIVE)
- Text:  $\triangle ABD \cong \triangle CDB$  BY RHL  $\cong$  RHL

3. Sketch an example of two triangles that meet the AAA criteria but are not congruent.



4. Sketch an example of two triangles that meet the SSA criteria that are not congruent.



5. Sketch an example of two right triangles that are not congruent.

