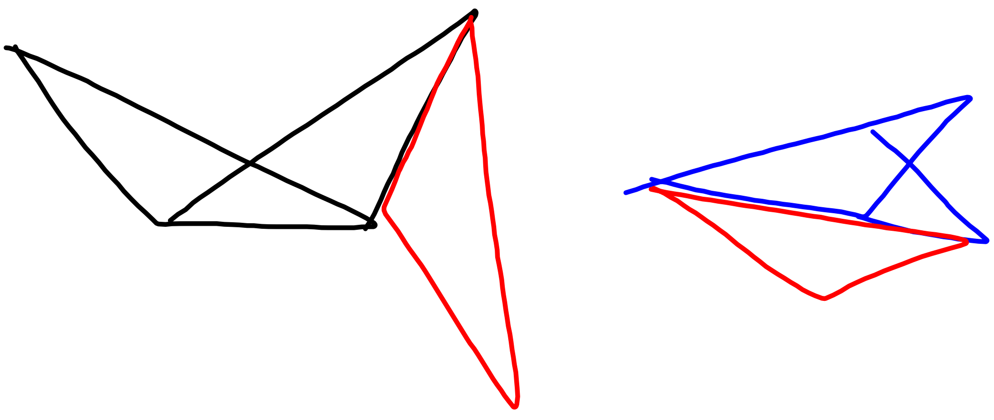
Lesson 4-14: Coordinate Proofs

AGENDA:

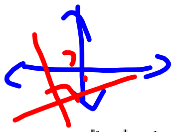
- Homework 4-13 Check & Review
- Lesson Notes with Guided Practice

Homework:

- Worksheet (Finish Packet) and Complete Lesson Summary
- Test is on Tuesday.



$y - y_1 = m(x - x_1)$



Review:

1. Given line $\overleftrightarrow{PQ}$ through P(9,3) and Q(-5,6). Write an equation of the line perpendicular to $\overleftrightarrow{PQ}$ through (-1,2).

$m_{\overleftrightarrow{PQ}} = \frac{\Delta y}{\Delta x} = \frac{6-3}{-5-9} = \frac{3}{-14}$ $\quad \perp + \frac{14}{3}$

$y = mx + b$

$y = \frac{14}{3}x + b$

2. Determine ~~the relationship that exists between the lines $y - 5 = \frac{3}{4}(x - 8)$ and $4y = 3x - 1$~~. Explain your reasoning.

$y - 2 = \frac{14}{3}(x - (-1))$

$y - 2 = \frac{14}{3}(x + 1)$

Review:

1. Given line $\overleftrightarrow{PQ}$ through P(9,3) and Q(-5,6). Write an equation of the line perpendicular to $\overleftrightarrow{PQ}$ through (-1,2).

2. Determine the relationship that exists between the lines $y - 5 = \frac{3}{4}(x - 8)$ and $4y = 3x - 1$. Explain your reasoning.

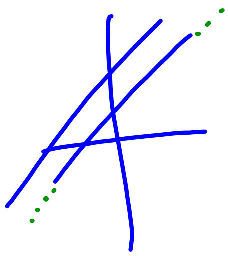
$\frac{3}{4} = m$

$\frac{4y}{4} = \frac{3x}{4} - \frac{1}{4}$

$y = \frac{3}{4}x - \frac{1}{4}$

$m = \frac{3}{4}$

PARALLEL B/C = SLOPES



When asked to prove something *in the coordinate plane*, we use a *coordinate proof* rather than a traditional 2 column proof. Coordinate proofs contain:

- A sketch — A GRAPH
- Algebraic work - CALCULATIONS
- Concluding statements - Geometric Relationships !

▶ **3 Algebraic Tools are used in Coordinate Proofs:**

1) **The slope formula** $m = \frac{\Delta y}{\Delta x}$

-- used to prove lines or segments parallel $m_1 = m_2$ → ALT INT ∡'S ≅ CORR ∡'S ≅

-- used to prove lines or segments perpendicular $m_1 \bullet m_2 = -1$ or opposite reciprocal slopes

→ RT ∡'S → ≅ ∡'S → RT △

2) **The distance formula** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ PYTHAG

$\sqrt{a^2 + b^2} = c$

-- used to show two <u>segments</u> have the <u>same length</u> and are therefore <u>congruent</u>

-- may also be used to prove a segment bisector

3) **The midpoint formula** $mdpt = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

-- used to find the coordinates of the midpoint of a segment or

-- may be used to prove a point is a midpoint or a segment/line is a segment bisector

Sometimes you need several statements!

Concluding Statements

Concluding statements are most often in the form "since numbers (from algebraic calculations), then concluded relationships (apply geometry)" and include re-writes of definitions, postulates, theorems and properties.

Slope: Since $m\overline{AB} = \frac{2}{3} = m\overline{CD}$, then $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

Since $m\overline{AB} = \frac{2}{3}$ and $\overline{BC} = -\frac{3}{2}$ are opposite reciprocals or have a product of -1, then $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$.

Distance: Since $AB = \sqrt{7} = BC$, then $\overline{AB} \cong \overline{BC}$.

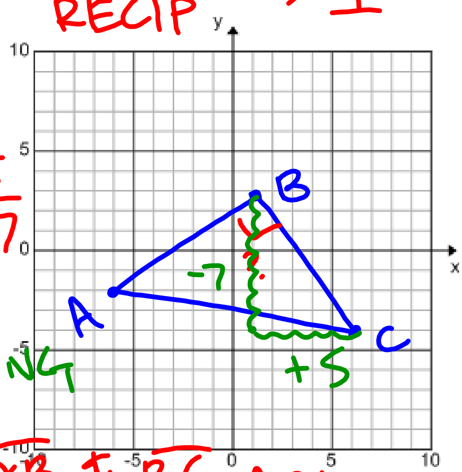
Midpoint: Since the midpoint of $\overline{PQ}$ is (5,0) and M is (5,0), then M IS THE MIDPOINT OF $\overline{PQ}$.

EXAMPLE 1

CALC SLOPES → OPP RECIP → $\perp$

GIVEN: A(-6,-2), B (1,3), C(6,-4)

PROVE: $\overline{AB} \perp \overline{BC}$



$$m_{\overline{AB}} = \frac{\Delta y}{\Delta x} = \frac{3-(-2)}{1-(-6)} = \frac{5}{7}$$

$$m_{\overline{BC}} = \frac{\Delta y}{\Delta x} = \frac{-7}{5}$$ BY COUNTING

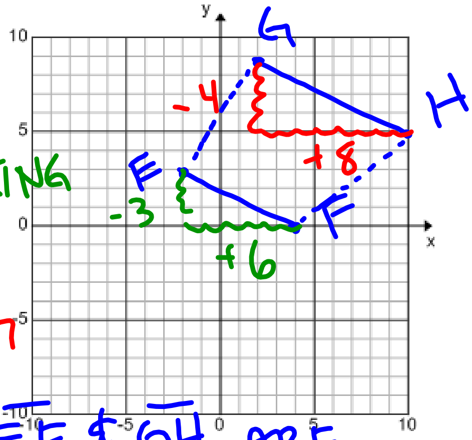
SINCE THE SLOPES OF $\overline{AB}$ & $\overline{BC}$ ARE OPPOSITE RECIPROCALS, THEN $\overline{AB} \perp \overline{BC}$.

SLOPES → = → ||

EXAMPLE 2

GIVEN: E(-2,3), F (4,0), G(2,9), H(10,5)

PROVE: $\overline{EF} \parallel \overline{GH}$



$m_{\overline{EF}} = -\frac{3}{6} = -\frac{1}{2}$ BY COUNTING

$m_{\overline{GH}} = \frac{-4}{8} = -\frac{1}{2}$ BY COUNTING

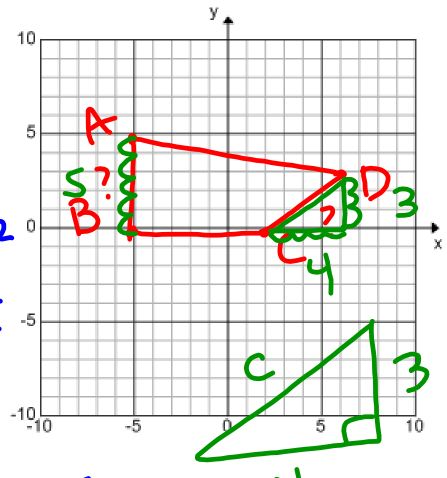
SINCE THE SLOPES OF $\overline{EF}$ & $\overline{GH}$ ARE EQUAL, THEN $\overline{EF} \parallel \overline{GH}$.

DISTANCE → = → ≅

EXAMPLE 3

GIVEN: A(-5,5), B(-5,0), C(2,0), D(6,3)

PROVE: $\overline{AB} \cong \overline{CD}$



$AB = 5$ BY COUNTING

$CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6-2)^2 + (3-0)^2}$

$= \sqrt{4^2 + 3^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

$a^2 + b^2 = c^2$

SINCE $AB = 5 = CD$,

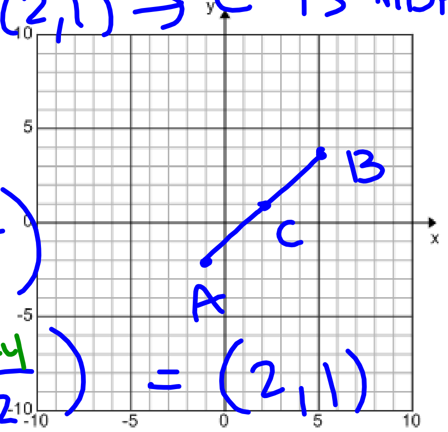
THEN $\overline{AB} \cong \overline{CD}$.

EXAMPLE 4

GIVEN: A(-1,-2), B (5,4), C(2,1)

PROVE: C is the midpoint of $\overline{AB}$

CALC MID → $=(2,1)$ → C IS MDPT



A (-1, -2)

$m(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

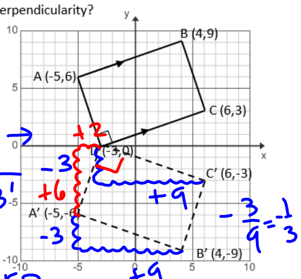
B (5, 4)

$= \left(\frac{-1+5}{2}, \frac{-2+4}{2}\right) = (2, 1)$

SINCE THE COORDINATES OF THE MIDPOINT OF $\overline{AB}$ ARE (2,1) AND C IS GIVEN AS (2,1), THEN C IS MIDPOINT OF $\overline{AB}$.

RIGID MOTION EXPLORATION - Do rigid motions preserve parallelism and perpendicularity?

Given rectangle ABCD with vertices as graphed, $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \perp \overline{DC}$. After a reflection in the x-axis,



A) Is $\overline{A'B'} \parallel \overline{C'D'}$? Prove it using coordinate geometry.

$m_{\overline{DC'}} = -\frac{1}{3}$

$m_{\overline{A'B'}} = -\frac{1}{3}$

= SLOPES → $\overline{DC'} \parallel \overline{A'B'}$

PARALLELISM PRESERVED

B) Is $\overline{A'B'} \perp \overline{B'C'}$? Prove it using

a. Coordinate geometry

$-\frac{1}{3}$

$m_{\overline{A'D}} = +\frac{3}{1}$

$m_{\overline{DC'}} = -\frac{1}{3}$

OPPOSITE RECIPROCALS SLOPES → $\overline{A'D} \perp \overline{DC'}$

b. Properties of rigid motions

PERPENDICULARITY PRESERVED

SINCE ⊥ → ∡D RT ∡ → 90° = m∡D

SINCE REFLECTIONS PRESERVE ∡ MEASURE,

THEN m∡D' = 90°.

→ RT ∡ → ⊥

PROBLEM SET 4-14 LAB

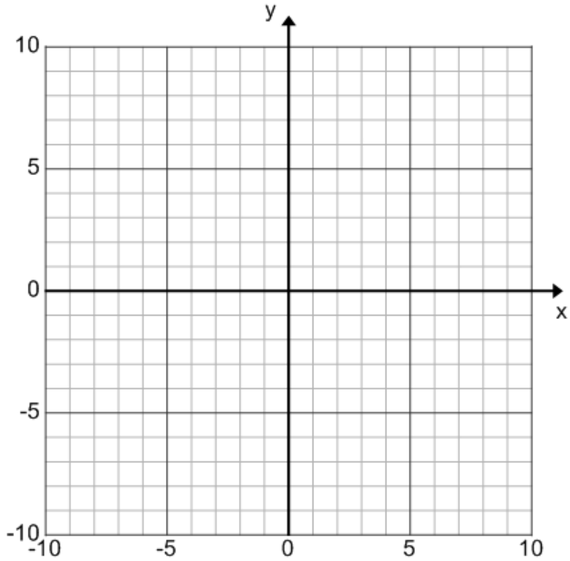
1) Fill in your lesson summaries from today's lesson.
2) Use the provided graph but do work on separate paper to prove each of the following. Given quadrilateral PQRS with P(2,4), Q(-5,2), R(-2,-1), S(5,1), M$\left(0, \frac{3}{2}\right)$

   a. Prove $\overline{PQ} \parallel \overline{RS}$

   b. Prove $\overline{PQ} \cong \overline{RS}$

   c. Prove $\overline{PQ}$ is not $\perp$ to $\overline{RS}$

   d. Prove M is the midpoint of the diagonal $\overline{PR}$



Attachments

4-9 & 4-12L Homework.pdf