

Lesson 2-4R : Geometric Proofs

Agenda

- Check and Review Homework 2-3 Proofs
- Turn in Cumulative Review #1
- Mini Quiz #2
- Guided Notes - need pouches, notes, and your unit outline/lesson summaries/axioms pages

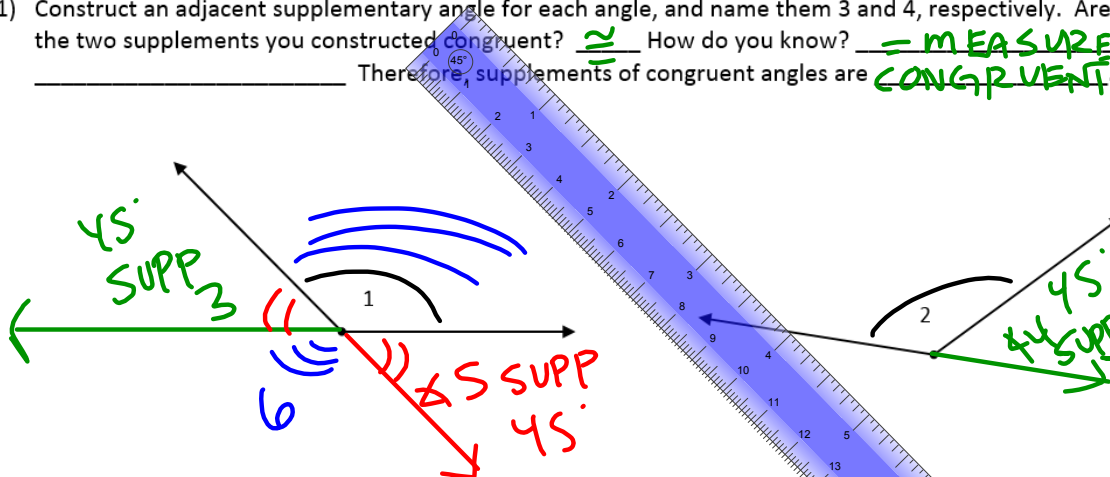
HW

- 2-4 problems in your packet (separate paper)

Congruent Supplements Theorem & Vertical Angles Theorem Exploration

Given $\angle 1 \cong \angle 2$,

- 1) Construct an adjacent supplementary angle for each angle, and name them 3 and 4, respectively. Are the two supplements you constructed congruent? \cong How do you know? MEASURE CONGRUENT.
 _____ Therefore, supplements of congruent angles are _____.



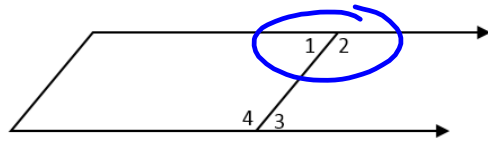
- 2) Go back to angle 1 and construct the other adjacent supplementary angle, naming it angle 5. How do angles 3 and 5 compare? \cong . Therefore, supplements to the same angle are CONGRUENT. Both of these cases are called the **Congruent Supplements Theorem**. Note that you need to have 2 pairs of supplementary angles.

- 3) Now we know that $\angle 3 \cong \angle 5$. What is another name for this angle pair? VERTICAL X'S. Label the last angle 6 that was created at the same point as angles 1, 3 & 5. Compare $\angle 1$ & $\angle 6$. The congruent supplements theorem therefore directly leads to the **Vertical Angles Theorem** which states that PAIRS OF VERTICAL X'S ARE \cong .

LINEAR PAIR THEOREM (review)

Anytime you have a LINEAR PAIR, you can *deduce* that the angles are SUPPLEMENTARY.
 Here is an example of how you would use the Linear Pair Theorem in a future proof:

Ex 1: Given: the diagram at the right
 Prove: $\angle 1$ and $\angle 2$ are supplementary \angle 's



Statements	Reasons
1) $\angle 1$ & $\angle 2$ LINEAR PAIR	1) DEFN OF LINEAR PAIR
2) $\angle 1$ SUPP $\angle 2$	2) LINEAR PAIR \rightarrow SUPP \angle 'S

How do we know if two angles are supplementary?

- 1) LINEAR PAIR \rightarrow SUPP \angle 'S (2 STEPS)
- 2) IT IS ^{DEFN} STATED (GIVEN)
- 3) SUM TO $180^\circ \rightarrow$ SUPP (DEFN SUPP \angle 'S)

CONGRUENT SUPPLEMENTS THEOREM (new)

Theorem

THEOREM

2-6-2 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.

Identify the hypothesis:

- SUPP TO SAME \sphericalangle $2x$'s
- SUPP TO $\cong \sphericalangle$'s

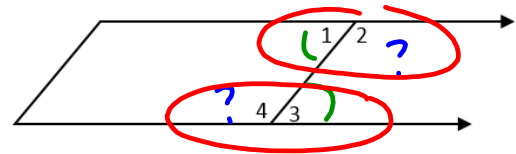
Identify the conclusion:

THOSE \sphericalangle 's \cong

Supplements to the same \sphericalangle are congruent OR Supplements to $\cong \sphericalangle$ s are congruent

Here is an example of how you would use the Congruent Supplements Theorem in a future proof:

Ex 2: Given: $\sphericalangle 1 \cong \sphericalangle 3$
 Prove: $\sphericalangle 2 \cong \sphericalangle 4$



Statements	Reasons
1. $\sphericalangle 1 \cong \sphericalangle 3$	1. GIVEN
2. $\sphericalangle 1$ and $\sphericalangle 2$ form a linear pair $\sphericalangle 3$ and $\sphericalangle 4$ form a linear pair	2. Defn: 2 adjacent angles whose non-common sides are opposite rays form a linear pair
3. $\sphericalangle 1$ and $\sphericalangle 2$ are supplementary $\sphericalangle 3$ and $\sphericalangle 4$ are supplementary	3. LINEAR PAIR \rightarrow SUPP
4. $\sphericalangle 2 \cong \sphericalangle 4$	4. CONGRUENT \sphericalangle 'S SUPPLEMENTS THM

Think back to the bridge...

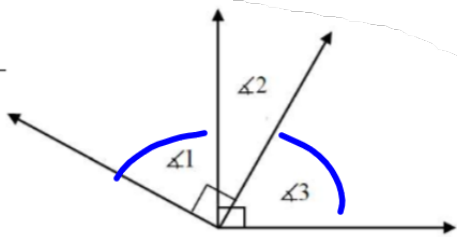
7. Look at the drawing at right.

a. What conclusion can you make about $\angle 1$ & $\angle 3$? 112

b. Here is the proof; see if you can match the reasons:

- 1) B $m\angle 1 + m\angle 2 = 90^\circ$ $m\angle 3 + m\angle 2 = 90^\circ$
- 2) A $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$
- 3) E $m\angle 2 = m\angle 2$
- 4) D $m\angle 1 = m\angle 3$
- 5) C $\angle 1 \cong \angle 3$

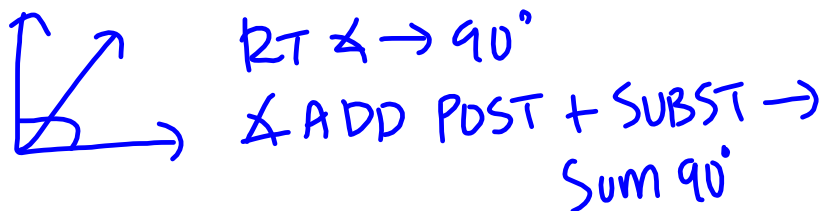
- | | |
|---------------------------------------|---|
| <input checked="" type="checkbox"/> A | Substitution |
| <input checked="" type="checkbox"/> B | Definition of Complementary Angles |
| <input checked="" type="checkbox"/> C | Definition of Congruency or = measure $\leftrightarrow \cong$ figures |
| <input checked="" type="checkbox"/> D | Subtraction Property of Equality |
| <input checked="" type="checkbox"/> E | Reflexive Property of Equality |



\cong COMPLEMENTS THEOREM

How do we know if two angles are complementary?

- 1) SUM TO 90° → COMP (DEFN OF COMP-LS)
- 2) IT IS STATED (GIVEN)



CONGRUENT COMPLEMENTS THEOREM

2-6-4 Congruent Complements Theorem
 If two angles are complementary to the same angle (or to two congruent angles), then the two angles are congruent.

Identify the hypothesis: **2 \angle 's**

- COMP TO SAME \angle
- COMP TO $\cong \angle$'s

Identify the conclusion: **THOSE \angle 's ARE \cong**

Complements to the same \angle are congruent OR Complements to $\cong \angle$'s are congruent

How Congruent Complements Theorem is used in a Proof:

Ex 3: Given: $m\angle 1 + m\angle 2 = 90^\circ$
 $m\angle 1 + m\angle 3 = 90^\circ$ \rightarrow COMP

Prove: $\angle 2 \cong \angle 3$

Statements	Reasons
1. $m\angle 1 + m\angle 2 = 90^\circ$ $m\angle 1 + m\angle 3 = 90^\circ$	1. GIVEN
2. $\angle 1$ and $\angle 2$ are complementary $\angle 1$ and $\angle 3$ are complementary	2. DEFN OF COMP \angle 's
3. $\angle 2 \cong \angle 3$	3. CONGRUENT COMPLEMENTS THM

RIGHT ANGLE CONGRUENCE THEOREM

Theorems

THEOREM

2-6-3 Right Angle Congruence Theorem
 All right angles are congruent.

So, anytime you have **RIGHT \angle 's**, you can **deduce** that they are **CONGRUENT**.

**Note: you can go right from right angle to congruency without going through equal measures of 90° .*

How Right Angle Congruence Theorem is used in a Proof:

Ex 4: Given: $m\angle 1 = 90^\circ$; $\angle 2$ is a right angle

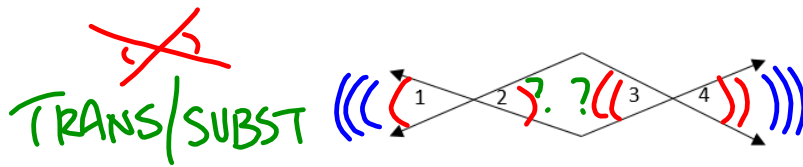
Prove: $\angle 1 \cong \angle 2$

Statements	Reasons
1. $m\angle 1 = 90^\circ$	1. GIVEN
2. $\angle 1$ is a right angle	2. DEFN OF RIGHT \angle
3. $\angle 2$ is a right angle	3. GIVEN
4. $\angle 1 \cong \angle 2$	4. RIGHT \angle 'S ARE \cong

VERTICAL ANGLE PAIR CONGRUENCE THEOREM

Anytime you have A PAIR OF VERTICAL \angle 'S, you can *deduce* that they are CONGRUENT.

Ex 5: Given: $\angle 1 \cong \angle 4$
 Prove: $\angle 3 \cong \angle 2$



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are vertical angles $\angle 3$ and $\angle 4$ are vertical angles	1. DEFN OF VERTICAL \angle'S VERTICAL \angle PAIRS ARE \cong
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	2. GIVEN
3. $\angle 1 \cong \angle 4$	3. SUBSTITUTION (STEP 3 INTO 2)
4. $\angle 3 \cong \angle 1$	4. SUBSTITUTION OR (STEP 2 INTO 4)
5. $\angle 3 \cong \angle 2$	5. TRANSITIVE (STEP 4 \rightarrow 2)

****Fill in axioms pages:**

- **Angle Pairs:** Congruent Supplements Theorem, Congruent Complements Theorem (*Linear Pair & Vertical Angles should already be filled in*)
- **Right Angles:** Definition (add in), Right Angle Congruence

Angle Pairs		
Conditional Statement	Diagram / Example	Stated as a Reason in a Proof
The sum of two angles = 90° if and only if the angles are _____		
The sum of two angles = 180° if and only if the angles are _____		
If two angles are adjacent and their noncommon sides form opposite rays, then the angles are a <u>LINEAR PAIR</u>		Defn. of a Linear Pair
If two \sphericalangle 's form a linear pair, then they are <u>SUPPLEMENTARY</u>		Linear pairs of \sphericalangle 's are supplementary. Linear Pair \rightarrow Supp \sphericalangle 's

Angle pairs (Con't)		
If 2 non-adjacent \sphericalangle 's are formed by intersecting lines then they are _____		Defn of Vertical Angles
If angles are vertical \sphericalangle 's , then the angles are _____		Vertical \sphericalangle pairs are equal in measure Vertical \sphericalangle pairs are \cong
If 2 angles are supplementary to the same angle, then they are <u>CONGRUENT</u>		Congruent Supplements Theorem Or Supplements of the same angle are congruent
If 2 angles are supplementary to congruent angles, then they are <u>CONGRUENT</u>		Congruent Supplements Theorem Or Supplements of congruent angles are congruent

Angle pairs (Con't)		
<p>If 2 angles are complementary to the same angle, then they are</p> <p><u>CONGRUENT</u></p>		<p>Congruent Supplements Theorem ^{Comp} Supplements of the same angle are congruent</p>
<p>If 2 angles are complementary to congruent angles, then they are</p> <p><u>CONGRUENT</u></p>		<p>Congruent Supplements Theorem ^{Comp} Supplements of congruent angles are congruent</p>

Add: Definition of a right angle		
<p>Right Angles $m \angle = 90^\circ \longleftrightarrow RT \angle$</p>		
<p>If 2 \angle's are right \angle's, then _____</p> <p><u>THEY ARE \cong</u></p>		<p>Right \angle's are \cong.</p>
<p>If 2 \cong \angle's are supplementary, then _____</p> <p><i>Note conjunction!</i></p>		<p>Congruent & supplementary angles are right \angle's</p>