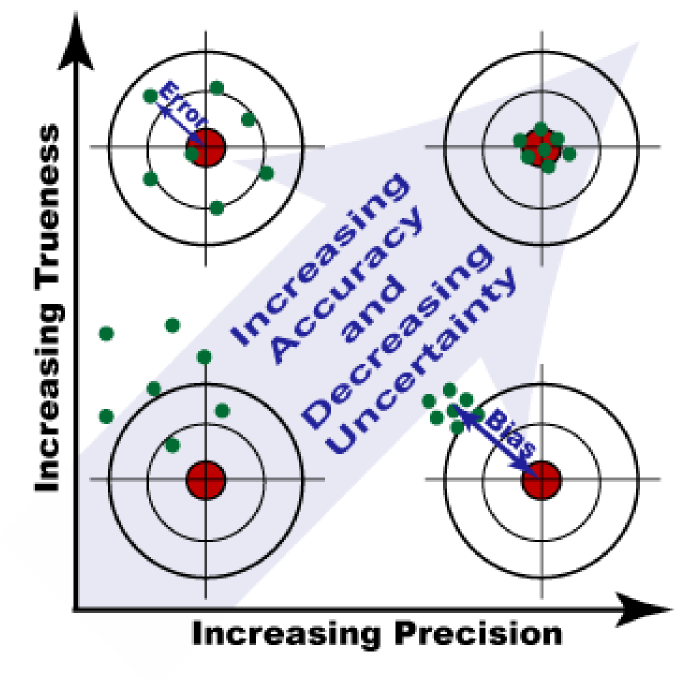
IB Physics (HL)

Student Guide for Measurement Error and Uncertainty Analysis

Ballston Spa High School



Error & Uncertainty

No measurement is ever perfectly exact or perfectly correct; every measurement has a degree of uncertainty associated with it. Thus, in order to reliably interpret the experimental data that you will collect, you need to have some idea as to the nature of the "errors" associated with the measurements.

In general, there are three types of errors: systematic, random, and measurement.

1) Systematic Error:

A systematic error is when measurements are incorrect in some consistent fashion.

The ***consistency*** of the error is the key point. There has to be a ***system*** to what has gone wrong. Systematic error is the opposite of a random error.

Systematic errors are caused by some consistent fault. This is going to be either

- a fault with the apparatus
- a faulty technique (but one that is consistently applied)

An example would be a balance that has not been zeroed and reads 5 g when there is no mass on it. All measurements with that balance would be 5 g too big as a result.

2) Random Uncertainty:

Random error leads to *unpredictable* inaccuracies when an experiment is repeated.

The classic example of a random uncertainty is timing the oscillations of a pendulum with a stopwatch.

- Your reaction times mean that you cannot start and stop the watch at exactly the right instants, so your measured time value will be inaccurate. It will have ***uncertainty***.
- However, you will sometimes measure a time that it too small (stopping the watch early) and sometimes too large (stopping it late). Your results may be a little bit wrong (you started and stopped almost perfectly), or more wrong. The error is ***unpredictable***.

It is the second part that makes this ***random*** uncertainty. There is an element of chance in your results. (The opposite of this is a systematic error.)

Random uncertainty is usually due to

- human limitations
- not being able to control all variables properly

3) Measurement Uncertainty (Device Reading Errors):

When recording data in IB Physics, it is important to record both the measured value and the associated uncertainty (± value). Here are some general rules:

a) Use your best judgment in the amount of uncertainty in the measurement.
b) Uncertainty should have only **one significant figure.**
c) Uncertainty should match the measurement in the number of **decimal places**.

For example, if the measurement of a distance with a meterstick was 78.42 cm, the number should be recorded as 78.42 cm ± .02 cm.

When using an **analog (non-digital) device**, one should record as many significant figures as the calibration of the instrument allows, plus one estimated digit. When using the scale on a standard analog device, a good rule-of-thumb is that the uncertainty in the reading is 20% of the smallest division. For example, on a centimeter ruler where the smallest division is 1 mm, the uncertainty in a given reading may be ± .2 mm (or ± .02 cm).

When using a **digital device**, the uncertainty may be obtained by placing a 1 in the furthest digit to the right that the device can display. An electronic balance that reports a reading of 142.6 g would have an uncertainty of ± 0.1 g. A stopwatch that reads 5.34 s would have an uncertainty of ± 0.01 s. **Please note** that the calibration of the digital gauge is extremely important in uncertainty determination.

The above guidelines help to establish the *minimum uncertainty* in a given measurement situation. The measurer also needs to evaluate how the measuring instrument is being used in order to determine if the estimated uncertainty should be higher than the above analog and digital guidelines. For example, if you are using a ruler to measure the height of a bouncing ball, the uncertainty may be ± .2 cm or ± .5 cm, rather than the ideal ± .02 cm due to the estimation that takes place in determining the bounce height. As stated above in rule "a", it is important to use your best judgment when dealing with uncertainty; you will get better at this skill as the course progresses.

Good laboratory technique typically dictates that multiple readings (trials) of a given measurement should be taken (rather than a single measurement) in an effort to reduce the effect of random errors. Technically, the correct treatment of uncertainty in average values is done using the standard deviation. If you are recording a measurement using **multiple trials** (to obtain an average), the uncertainty should be recorded as ½ of the range of the raw data measurements. Please note that the range of the data is the difference between the highest value and the lowest values. As an example if the following readings were taken for lengths (± .02 cm):

1.74 cm, 1.73 cm, 1.75 cm, 1.74 cm, 1.75 cm

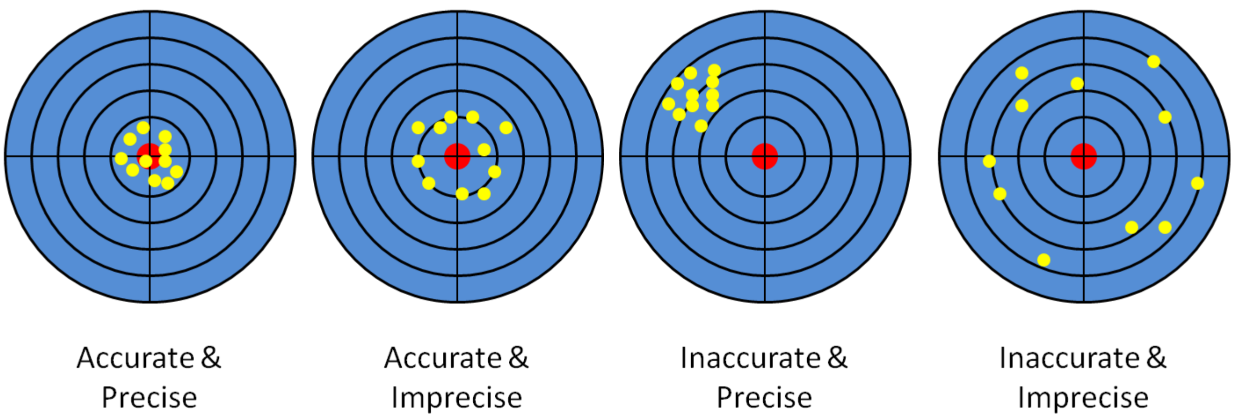
the uncertainty in the average value (1.742) would be ½ [(1.75-1.73)] = .01 cm
Thus the average value would be reported as 1.74 cm ± .01 cm

If **<u>several cycles</u>** of the same event are measured, the uncertainty can be recorded as the uncertainty of the entire measurement divided by the number of cycles. For example, the time for a child to swing back and forth on a swing 10 is recorded as 36.27s ± 0.01s on a digital stopwatch. The time for one swing may be taken as 3.627s ± 0.001s when both the time and uncertainty are divided by 10. Notice that timing multiple cycles reduces the uncertainty for a single cycle.

Measurements are considered **accurate** if they are close to the expected (or actual) value. In general, measurements are accurate if the systematic error is small.

If the same thing is measured many times and the same value is obtained, then the measurements are said to be **precise**. In general, measurements are precise if the random uncertainty is small.

The following target analogy provides a nice visual illustration of accuracy and precision.



Data Processing:

The IB refers to the "propagation of measurement uncertainty" as data processing. The general idea is that the uncertainty in measured quantities will lead to an uncertainty in a number obtained by using the measured quantities in a calculation.

In this course, we will express uncertainty in the following two ways:

Absolute Uncertainty: uses the units of the expressed quantity

Example: L = 1.27m ± 0.02m

Important Note: it is sufficient to express uncertainties with 1 significant figure

Relative or Percent Uncertainty: uses a fraction or a percentage:

Example: L = 1.27m ± 1.57480315% before rounding

L = 1.27m ± 2% after rounding (1 sig fig in uncertainty)

The following rules summarize some accepted ways to determine the uncertainty in calculated quantities.

Handling Uncertainties in Addition and Subtraction:

Technique: When adding or subtracting quantities, the uncertainty in the sum or difference is the sum of the absolute uncertainties in the measured quantities.

This is stated in the IB Data booklet (reference table) as . . .

If y = a ± b, then Δy = Δa + Δb

Example: Adding Lengths

Data: L_1 = 3.91m ± 0.02m L_2 = 0.57m ± 0.02m

L_{total} = $L_1 + L_2$

= (3.91m ± 0.02m) + (0.57m ± 0.02m)

L_{total} = 4.48 ± 0.04 m

See Example 1 later in the packet for a worked sample involving subtraction.

An alternate approach is to find the highest and lowest results of the sum or difference using the estimated uncertainties, take the range of the results, and divide the range by two. This approach will yield the same result as the above rule for adding absolute uncertainties.

Example: Data: L_1 = 3.91m ± 0.02m L_2 = 0.57m ± 0.02m

$L_{total\ largest}$ = 3.93m + .59m = 4.52m

$L_{total\ smallest}$ = 3.89m + .55m = 4.44m

Range = 4.52m - 4.44m = .08m

½ Range = .04m

L_{total} = 4.48 ± 0.04 m

Handling Uncertainties in Multiplication and Division:

Technique:
a) Calculate the percent uncertainty for each quantity.
b) Add the percent uncertainties.
c) Convert back to absolute uncertainty.

This is stated in the IB Data booklet (reference table) as . . .

If $y = \frac{ab}{c}$ then $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$

Again, an alternate approach is to find the highest and lowest results of the product or quotient using the estimated uncertainties, take the range of the results, and divide the range by two. This approach will yield the same result as the above rule for adding percent uncertainties.

Example: Calculating Area

Data: L = 21.7m ± 0.2m W = 4.57m ± 0.01m

Area = (L x W) = 21.7m x 4.57m

= 99.169 m^2 (unrounded)

$Area_{largest}$ = 100.302 m^2 (unrounded)

$Area_{smallest}$ = 98.04 m^2 (unrounded)

Range = 2.262 m^2 (unrounded)

½ Range = 1.131 m^2 (unrounded)

Area = 99 ± 1m^2 (1 sig fig for uncertainty)

Please note that there are less significant figures in the answer than expected (could have 3) due to the requirement of 1 significant figure in the answer and the place value agreement between the number and the uncertainty. **See Example 2 for a worked sample involving division.**

Handling Uncertainties in Mathematical Operations (in general):
The use of powers, roots, and trigonometric functions are commonplace in IB physics. When dealing with any mathematical operation, the ½ range technique will provide the uncertainty in the calculated result. Please note that the uncertainties will not always be symmetrical, especially when dealing with trigonometric functions.

Technique:

a) Use the recorded uncertainty to find the highest and lowest possible values of the measured quantity.

b) Use these values to find the extremes of the calculated quantity.

c) Use the extremes to find the uncertainty of the calculated quantity.

d) The uncertainty is ½ of the range ($value_{high}$ - $value_{low}$)

Example: Uncertainty of a trig function

Given θ = 22º ± 1º, find the uncertainty in sinθ.

a) $\theta_{high} = 23^o$, $\theta_{low} = 21^o$

b) sinθ = .3746 $\sin\theta_{high} = .3907$ $\sin\theta_{low} = .3584$

c) ½ ($\sin\theta_{high} - \sin\theta_{low}$) = .01615

d) sin θ = .37 ± .02

Uncertainties in calculated results using factional or percentages uncertainties

When two or more quantities are *multiplied, divided or raised to a power*, one can often determine the fractional (or percentage) uncertainty in the final result simply by adding the uncertainties in the several quantities.

Example: Calculating Area using values from above

Data: L = 21.7m ± 0.2m W = 4.57m ± 0.01m

ΔL/L = ± (0.2m/ 21.7m) = 0.0092 =±0.9% (1 sig fig in uncertainty)

ΔW/W = ± (0.01m/ 4.57m) = 0.00218 = ±0.2 %

The total % uncertainty in the result = (0.9 +0.2)% = ±1.1% (1 sig fig in uncertainty)

1.1% of 99.2 = 1.0912 m^2 (unrounded)

So Area = 99.0 ± 1m^2 (rounded 1 sig fig in uncertainty)

Example Uncertainty Calculations:

Example 1: Determine the mass of the product.

Mass of crucible & product:	74.10 g ± 0.01 g
Mass of empty crucible:	72.35 g ± 0.01 g
Mass of product:	74.10 g - 72.35 g = 1.75 g
Uncertainty in mass of product:	± 0.01 g + ± 0.01 g = ± 0.02 g
Result:	1.75 g ± 0.02 g

The half-range technique can also be used to obtain the same result.
½ [(74.11-72.34) – (74.09-72.36)] = .02 g

Note that the largest possible result is obtained by subtracting the smallest value from the largest value, and the smallest result is obtained by subtracting the smallest possible value from the largest possible value.

Note that the random error introduced by a centigram balance is very tiny when you are weighing quantities of about 5 grams or more. However, the random error of the balance begins to contribute a large uncertainty if you try to weigh a very tiny quantity.

Example 2: A student did an experiment to measure the density of a liquid. She massed an empty graduated cylinder, placed a volume of liquid in the cylinder, and then massed it again. Her data is shown below:

Mass of empty graduated cylinder:	25.64 g ± 0.01 g
Mass of grad cylinder with liquid:	28.02 g ± 0.01 g
Volume of liquid:	3.00 mL ± 0.05 mL

The mass of the liquid (via subtraction) is 2.38 g ± 0.02 g

Since density = mass/volume, the student calculates the experimental density value:
2.38 g / 3.00 mL = 0.793 g/mL

The half-range technique can be used to obtain the uncertainty:
½ [(2.40/2.95) – (2.36/3.05)] = .02 g/mL

Note that the largest possible value is obtained by dividing the largest numerator by the smallest denominator, and the smallest result is obtained by dividing the smallest possible numerator by the largest possible denominator.

This makes the density 0.79 g/mL ± 0.02 g/mL

Note that the calculated result has been reduced to 2 significant figures to obtain place-value agreement with the uncertainty.

Suppose the literature value for this density is 0.809 g/mL. The literature value would then fall within the bounds of experimental uncertainty (.77 g/mL to .81 g/mL). This result indicates that random error alone can account for the difference between the student's value and the literature value. *We can say that the student got the same value as the literature value, within the limitations of random error. Systematic error, if present, did not appear to affect the result.*

<u>Example 3</u>: A student performs a calorimetry experiment to determine the amount of energy transferred in an experiment. He takes the following measurements:

Mass of water:	100.00 g ± 0.01 g	(negligible uncertainty)
Initial temperature of water:	23.6 ± 0.2 ^oC	
Final temperature of water:	27.4 ± 0.2 ^oC	
Change in temp (ΔT)	3.8 ± 0.4 ^oC	(added uncertainties)

His calculation would be: Q = (mass) (ΔT) (Cp of water)

Q = (100.00 g ± 0.01 g) (3.8 ^oC ± 0.4 ^oC) (4.184 J/g^oC) = 1589.92 J = 1.6 kJ

The half-range technique can be used to obtain the uncertainty:

½ [(1757.46) – (1422.42)] = 167.52 J = .16752 kJ = .2kJ (to 1 sig. fig.)

Thus the energy transferred would be 1.6 kJ ± 0.2 kJ

What if the literature value was 2.4 kJ? In this case, random error alone cannot account for the discrepancy. Therefore, some systematic error must have occurred. It is this error that the student should seek to identify and make some suggestions for eliminating it next time.

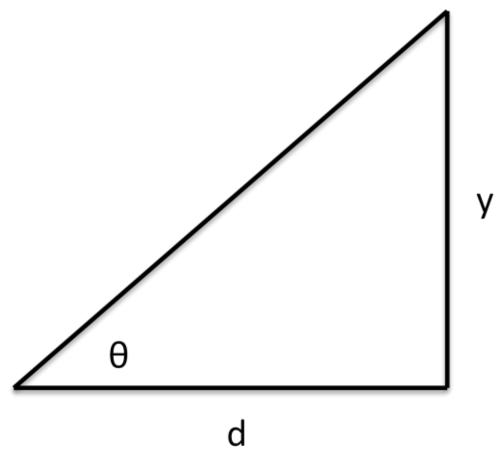
Independent Practice:

Exercise 1: Measure and record the length and width of this sheet of paper in centimeters (with uncertainty). Calculate the perimeter of this sheet of paper with associated uncertainty.

Exercise 2: Using the length and width measured above, calculate the area of this sheet of paper in cm^2. Report the absolute uncertainty as part of your answer.

Exercise 3: Calculate the maximum height (y) of a toy rocket (with uncertainty) given the following data.

Data: Max angle of elevation $\theta = 75^\circ \pm 3^\circ$.
Horizontal distance of vertex from launch pad: $d = 32.72\text{m} \pm 0.02$ m



Exercise 4: The resistance of a wire (R) can be calculated by dividing the potential difference (V) by the current (I) flowing in the wire. Calculate the resistance of the wire with associated uncertainty.

Data: Potential Difference (V) = 4.75 V ± .02 V
Current (I) = .0153 A ± .0002 A