

# Unit 2- Day 1 Conditional Statements and Axioms

## Agenda

- Go over Bridge
- Guided Notes

## HW

- Day2 - p. 77 #19,
- p. 85-86 #32, 34, 61
- p. 100-101 # 11, 15, 18, 23-25, 39, 33, 36, 40, 53, 61

**Bridge to Unit 2**

- Since we know that the statements "segments with equal measure are congruent" AND "congruent segments have equal measure", what do you think the symbol means when used like the following: "segments with equal measure  $\leftrightarrow$   $\cong$  segments"?
 

$\leftrightarrow$  **BICONDITIONAL**
- Inductive reasoning** means that we look at patterns and make a conjecture based on that pattern. I listed the following: A, E, I. What do you think comes next?  $\circ$  Defend your choice: **NEXT VOWEL EVERY 4TH**
- Given the algebraic equation, match the **algebraic property** which justifies each step that generates an equivalent equation for  $4(x+3)-2=6$ 

a. $\frac{4}{4}$ $4x+12-2=6$	1) Addition or Subtraction Property of Equality
b. $\frac{3}{3}$ $4x+10=6$	2) Multiplication or Division Property of Equality
c. $\frac{1}{1}$ $4x=-4$	3) Simplification
d. $\frac{2}{2}$ $x=-1$	4) Distribution
- If  $x=8$  and  $8=y$ , then what do you know about  $x$  &  $y$ ?  **$x=y$  TRANSITIVE**
- If I said that **ALL** Ballston Spa students clean their rooms on Friday night, would you believe me? **NO** Why or why not? **KIARA DOESN'T COUNTEREX**
- If I said, "Ballston Spa's mascot is the Scottie Dog, **and** our colors are red & black." was I being truthful? **NO** Why or why not? **FALSE CONJUNCTION**
- Look at the drawing at right.
 

a. What conclusion can you make about  $\angle 1$  &  $\angle 3$ ?  **$\cong$**

b. Here is the proof; see if you can match the reasons:

1) <b>B</b> $m\angle 1 + m\angle 2 = 90^\circ$ ; $m\angle 3 + m\angle 2 = 90^\circ$
2) <b>A</b> $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$
3) <b>E</b> $m\angle 2 = m\angle 2$
4) <b>D</b> $m\angle 1 = m\angle 3$
5) <b>C</b> $\angle 1 \cong \angle 3$

**COMPLEMENTS THEOREM**

<input checked="" type="checkbox"/> A	Substitution
<input checked="" type="checkbox"/> B	Definition of Complementary Angles
<input checked="" type="checkbox"/> C	Definition of Congruency or $\cong$ measure $\leftrightarrow$ figures
<input checked="" type="checkbox"/> D	Subtraction Property of Equality
<input checked="" type="checkbox"/> E	Reflexive Property of Equality

Regents Geometry & Lab Name \_\_\_\_\_ Date \_\_\_\_\_  
 Lesson 2 Day 1 Notes – Definitions and Axioms

CONDITIONAL STATEMENTS

- A conditional statement is an "if-then" ( $\rightarrow$ ) statement. It has a hypothesis and a conclusion.

Ex: If two angles are a linear pair, then the angles are supplementary. Is this a true statement? YES

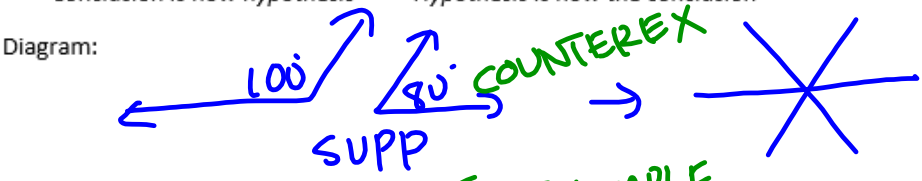
Hypothesis
Conclusion



- A conditional statement will not always be true if the hypothesis & conclusion are reversed. This is also called the CONVERSE statement to the original conditional statement.

Ex: If two angles are supplementary, then the angles are a linear pair. Is this a true statement? NO

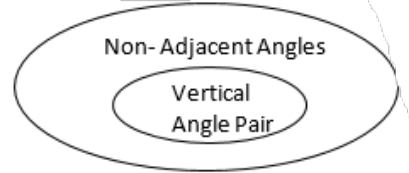
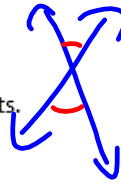
Conclusion is now hypothesis
Hypothesis is now the conclusion



- This diagram/example is called a COUNTEREXAMPLE, which disproves a statement (or shows the statement to be false).

- Venn Diagrams can also be used to write conditional statements.

EX:



Written as a conditional statement:

IF 2  $\sphericalangle$ 'S ARE A VERTICAL  $\sphericalangle$  PAIR,  
THEN THEY ARE NON-ADJACENT  
 $\sphericalangle$ 'S.

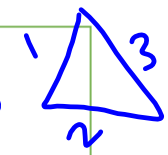
- A Biconditional Statement is an "if-and-only-if" statement.  $\leftrightarrow$ 
  - o These statements include both the CONDITIONAL statement AND the CONVERSE statement.
  - o A biconditional statement is true only when BOTH the conditional & converse statements are TRUE.
  - o Biconditional statements are used to write definitions.

Ex: A polygon is a triangle if and only if it has three sides.

Conditional: If a polygon is a triangle, then it has three sides.

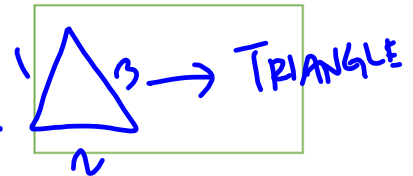
T

TRIANGLE  $\rightarrow$



Converse: If a polygon has three sides, then it is a triangle. T

Definition: A TRIANGLE IS A  
POLYGON WITH 3 SIDES.



Practice. Write the following definition in the appropriate forms.

An angle that measure  $90^\circ$  is a right angle.

iff

Conditional IF AN ANGLE MEASURES  $90^\circ$ , THEN IT IS A RIGHT  $\angle$ .

Converse IF AN ANGLE IS A RIGHT  $\angle$ , THEN IT MEASURES  $90^\circ$ .

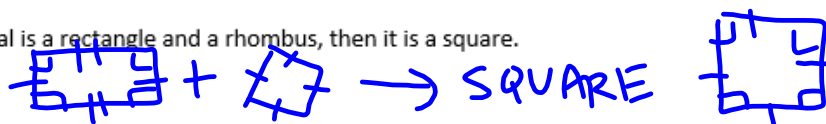
Biconditional AN ANGLE MEASURES  $90^\circ$  IF AND ONLY IF IT IS A RIGHT  $\angle$ .

CONJUNCTIONS

A conjunction is a compound sentence formed by combining two sentences (or facts) using the word "and." A conjunction is true only when BOTH sentences (or facts) are true.

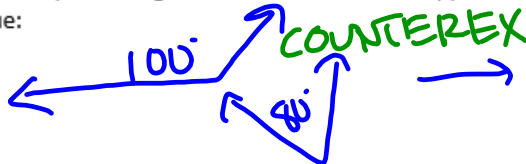
- Example 1: If a quadrilateral is a rectangle and a rhombus, then it is a square.

Truth value:

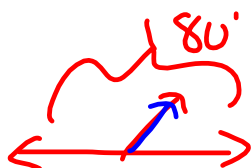


- Example 2: If a pair of angles sum to  $180^\circ$  and are supplementary, then they form a linear pair.

Truth value:



Often, to make a valid conclusion, you will need more than one condition/property. This is immensely important in your logical geometric reasoning. Rewrite example 2 such that it is a true/valid conclusion:



IF A PAIR OF  $\angle$ 'S ARE SUPP AND ADJACENT, THEN THEY ARE A LINEAR PAIR.

Sum of Parts		
Conditional Statement	Diagram / Example	Stated as a Reason in a Proof
If point C is in the interior of $\angle AOB$ , then $m\angle AOC + m\angle BOC = m\angle AOB$		ANGLE ADDITION POSTULATE
Given a sequence of $n$ consecutive adjacent angles whose interiors are all disjoint such that the angle formed by the first $n-1$ angles and the last angle are a linear pair, then the angle measures <u>sum to 180°</u>	$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180^\circ$ ( $\angle$ 's on a line)	Consecutive adjacent angles on a line sum to $180^\circ$
If the sum of the measures of all angles formed by three or more rays with the same vertex and whose interiors do not overlap, then the angle measures <u>sum to 360°</u>	$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360^\circ$ ( $\angle$ 's at a point)	Angles at a point sum to $360^\circ$
If points C and D are in the interior of $\angle AOB$ AND $\angle AOD \cong \angle BOC$ , then  See lesson summaries for 3 step process (can also go $\cong$ big $\rightarrow$ $\cong$ littles)		Common Angle Theorem Or Overlapping Angles Theorem

Sum of parts (con't)		
If A, B, and C are collinear, then $AB + BC = AC$		SEGMENT ADDITION POSTULATE
If points A, B, C, AND D are collinear and $\overline{AB} \cong \overline{CD}$ , then  See lesson summaries for 3 step process (can also go $\cong$ big $\rightarrow$ $\cong$ littles)		Common Segment Thm Or Overlapping Segments Thm

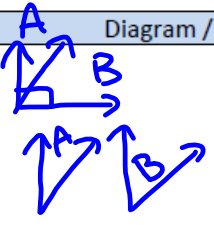
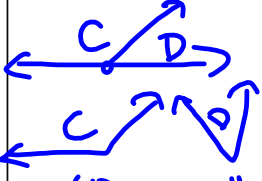
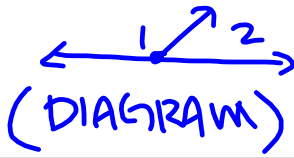
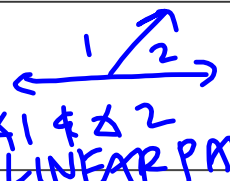
**Bisectors**

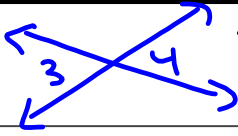
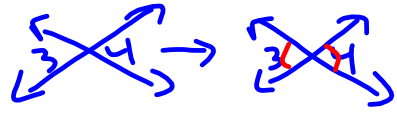
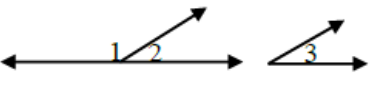
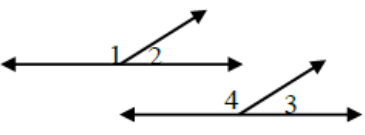
Conditional Statement	Diagram / Example	Stated as a Reason in a Proof
If $\overline{BD}$ bisects $\angle ABC$ , then $\angle ABD \cong \angle CBD$		Definition of Angle Bisector  Or
If $\angle ABD \cong \angle CBD$ and they are adjacent, then $\overline{BD}$ BISECTS $\angle ABC$	(Diagram from previous row)	Angle Bisector $\leftrightarrow$ two congruent adjacent angles

Note conjunction!

<u>Bisectors</u> (con't)		
<p>If B is the midpoint of <math>\overline{AC}</math>, then</p> <p style="text-align: center;"><math>\overline{AB} \cong \overline{BC}</math></p>	<p style="text-align: center;"><b>B MIDPOINT OF AC</b></p> <p style="text-align: center;"><math>\leftrightarrow</math></p> <p style="text-align: center;"><b>COLLINEAR</b></p>	<p>Definition of a Midpoint</p> <p style="text-align: center;">or</p> <p>Midpoint <math>\leftrightarrow</math> two congruent collinear segments</p>
<p>If <math>\overline{AB} \cong \overline{BC}</math> and A, B, and C are collinear, then</p> <p style="text-align: center;"><b>B IS MIDPOINT OF AC</b></p> <p><i>Note conjunction!</i></p>	<p style="text-align: center;"><b>CD BISECTS EF @ G</b></p> <p style="text-align: center;"><math>\leftrightarrow</math></p> <p style="text-align: center;"><b>G IS MIDPOINT OF EF</b></p>	<p style="text-align: center;"><b>SEGMENT BISECTOR <math>\leftrightarrow</math> MIDPOINT</b></p>
<p><math>\overline{CD}</math> bisects <math>\overline{EF}</math> at G and <math>\overline{EG} \cong \overline{GF}</math> <math>\leftrightarrow</math> G is the midpoint.</p>	<p style="text-align: center;"><b>CD BISECTS EF @ G</b></p> <p style="text-align: center;"><math>\leftrightarrow</math></p> <p style="text-align: center;"><b>COLLINEAR</b></p>	<p>Segment bisector <math>\leftrightarrow</math> 2 <math>\cong</math> collinear segments.</p> <p style="text-align: center;">Or</p> <p>Definition of a Segment Bisector</p>

<u>Bisectors</u> (con't)		
<p>If 2 <math>\cong</math> angles are bisected, then their</p> <hr/>		<p>Halves of Congruent Angles are Congruent.</p>
<p>If 2 <math>\cong</math> segments are bisected, then their</p> <hr/>		

Angle Pairs		
Conditional Statement	Diagram / Example	Stated as a Reason in a Proof
The sum of two angles = $90^\circ$ if and only if the angles are <b>COMPLEMENTARY</b> $m\angle A + m\angle B = 90^\circ$	 <p><math>\angle A</math> COMP <math>\angle B</math></p>	DEFN OF COMPLEMENTARY $\angle$ 'S
The sum of two angles = $180^\circ$ if and only if the angles are <b>SUPPLEMENTARY</b> $m\angle C + m\angle D = 180^\circ$	 <p><math>\angle C</math> SUPP <math>\angle D</math></p>	DEFN OF SUPPLEMENTARY $\angle$ 'S
If two angles are adjacent and their noncommon sides form opposite rays, then the angles are a <b>LINEAR PAIR</b>	 <p>(DIAGRAM)</p>	Defn. of a Linear Pair
If two $\angle$ 's form a linear pair, then they are <b>SUPPLEMENTARY <math>\angle</math>'S</b>	 <p><math>\angle 1</math> &amp; <math>\angle 2</math> LINEAR PAIR</p>	Linear pairs of $\angle$ 's are supplementary. Linear Pair $\rightarrow$ Supp $\angle$ 's

Angle pairs (Con't)		
If 2 non-adjacent $\angle$ 's are formed by intersecting lines then they are <b>VERTICAL <math>\angle</math>'S</b> .	 <p><math>\angle 3</math> &amp; <math>\angle 4</math> VERTICAL <math>\angle</math>'S</p>	Defn of Vertical Angles
If angles are vertical $\angle$ 's, then the angles are <b>CONGRUENT</b>		Vertical $\angle$ pairs are equal in measure Vertical $\angle$ pairs are $\cong$
If 2 angles are supplementary to the same angle, then they are _____		Congruent Supplements Theorem Or Supplements of the same angle are congruent
If 2 angles are supplementary to congruent angles, then they are _____		Congruent Supplements Theorem Or Supplements of congruent angles are congruent

Angle pairs (Con't)		
If 2 angles are complementary to the same angle, then they are _____ _____		Congruent Supplements Theorem Or Supplements of the same angle are congruent
If 2 angles are complementary to congruent angles, then they are _____ _____		Congruent Supplements Theorem Or Supplements of congruent angles are congruent

Right Angles <span style="margin-left: 50px;">A RIGHT <math>\sphericalangle</math> IS AN ANGLE MEASURING <math>90^\circ</math></span> <span style="float: right;">DEFN OF RIGHT <math>\sphericalangle</math></span>		
If 2 $\sphericalangle$ 's are right $\sphericalangle$ 's, then _____ <span style="color: blue; font-size: 1.2em;">THEY ARE <math>\cong</math></span>		Right $\sphericalangle$ 's are $\cong$ .
If 2 $\cong$ $\sphericalangle$ 's are supplementary, then _____ Note conjunction!		Congruent & supplementary angles are right $\sphericalangle$ 's