Lesson 2-2L : New Axioms

Agenda

- Check and Review Homework 2-1
- Discoveries - need pouches, notes, and your unit outline/lesson summaries/axioms pages

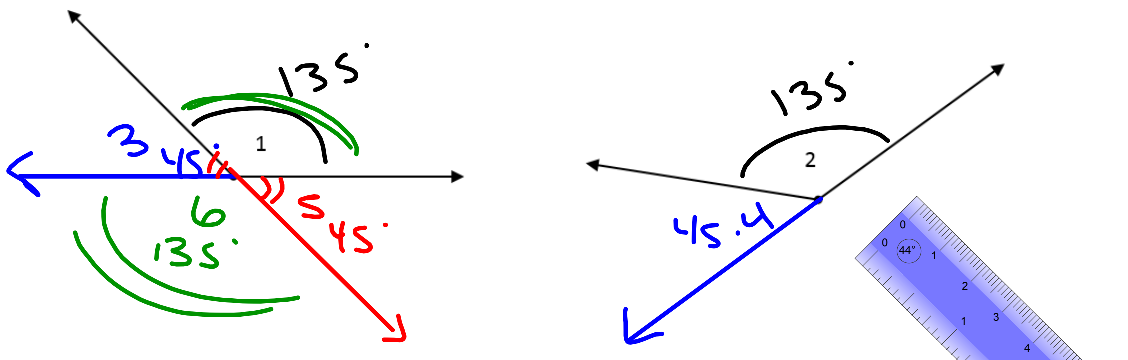
HW

- Worksheet 2-2L

Congruent Supplements Theorem & Vertical Angles Theorem

Given $\measuredangle 1 \cong \measuredangle 2$,

1) Construct an adjacent supplementary angle for each angle, and name them 3 and 4, respectively. Are the two supplements you constructed congruent? YES How do you know? MEASURES = 45° → ≅ Therefore, supplements of congruent angles are CONGRUENT.

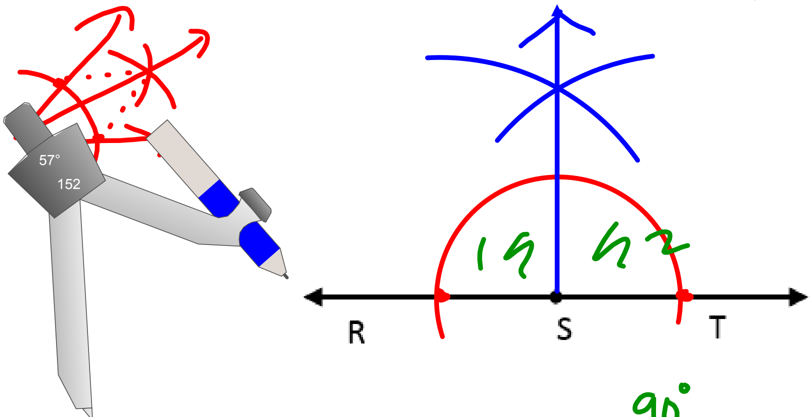


2) Go back to angle 1 and construct the other adjacent supplementary angle, naming it angle 5. How do angles 3 and 5 compare? ≅. Therefore, supplements to the same angle are CONGRUENT. Both of these cases are called the **Congruent Supplements Theorem**. *Note that you need to have 2 pairs of supplementary angles.*

3) Now we know that $\measuredangle 3 \cong \measuredangle 5$. What is another name for this angle pair? VERTICAL ∡'S. Label the last angle 6 that was created at the same point as angles 1, 3 & 5. Compare $\measuredangle 1 \& \measuredangle 6$. The congruent supplements theorem therefore directly leads to the **Vertical Angles Theorem** which states that VERTICAL ∡ PAIRS ARE CONGRUENT.

Right Angle Congruency and Congruent + Supplementary Angles

1) Construct ray $\overrightarrow{SU}$ such that it bisects the given the straight angle $\measuredangle RST$. Describe this specific linear pair of adjacent angles using two relationships: CONGRUENT, SUPPLEMENTARY



2) How much does each of the little angles measure? 90° By definition, each of these angles is a RIGHT angle and therefore **right angles are** CONGRUENT

3) Will supplementary and congruent angles always be right angles? Can you find a counterexample? NO
Therefore if 2 **angles are both** SUPPLEMENTARY ~~and~~ AND CONGRUENT**, then they are right angles.** *Note conjuction!*

CONJUNCTION!

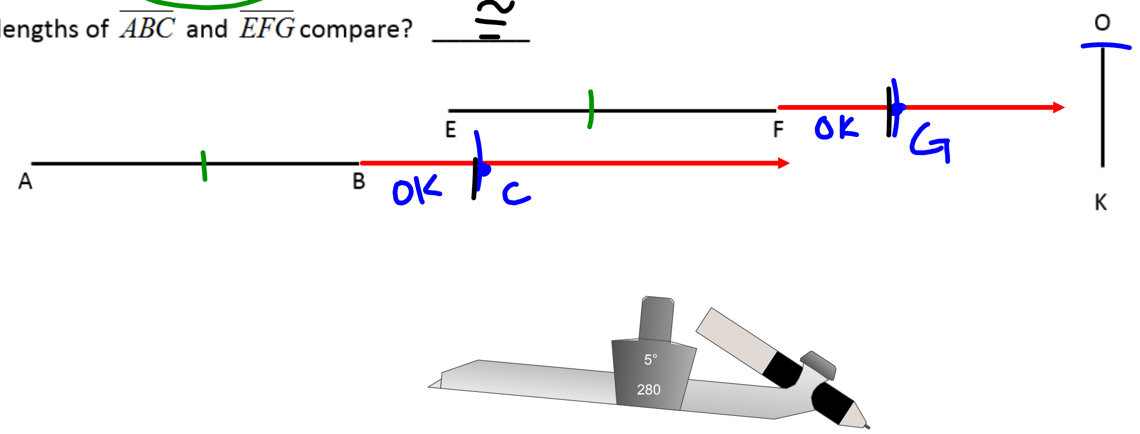
OK = OK

AB + OK = EF + OK

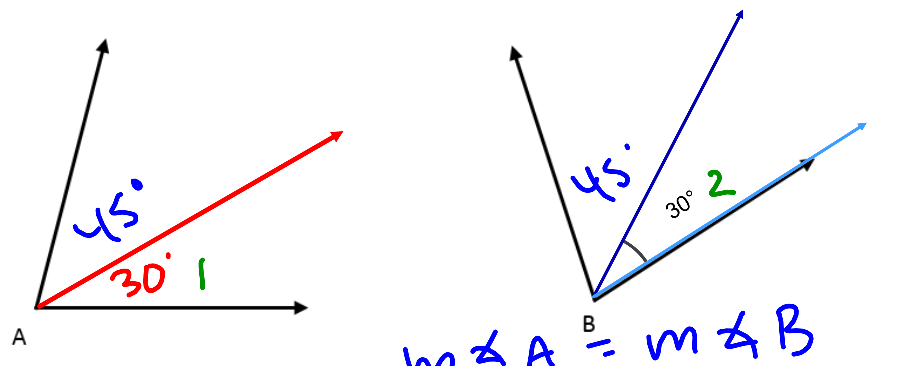
Addition/Subtraction Property of Equality

We know that you can add or subtract the same value from both sides of an equation to generate an equivalent equation using the addition or subtraction property of equality. What if we aren't working with strictly values but rather equal measures like OK or $m\measuredangle 5$?

Addition: Given $\overline{AB} \cong \overline{EF}$, construct $\overline{ABC}$ and $\overline{EFG}$ by adding the length of $\overline{OK}$ to each segment. How do the lengths of $\overline{ABC}$ and $\overline{EFG}$ compare? ≅



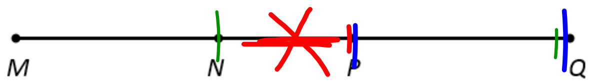
Subtraction: Given $\measuredangle A \cong \measuredangle B$, use your universal angle maker to draw in a ray in the interior with an endpoint at A or B such that a 30° angle is formed with one of the large angle's rays.
How much does the remaining little angle measure in each of larger angles? 45°

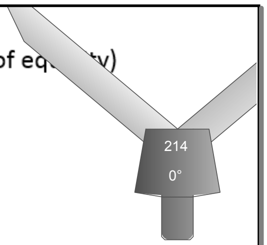


$m\measuredangle A \cong m\measuredangle B$
$-30° \qquad -30°$
$-m\measuredangle 1 \qquad -m\measuredangle 2$

$m\measuredangle 1 = m\measuredangle 2$

Common Segment Theorem (A shortcut to the addition postulates w/addition property of eq...ty)
Given the segment below, use your compass to investigate and fill in the relationships:

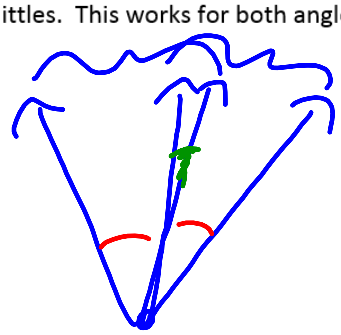




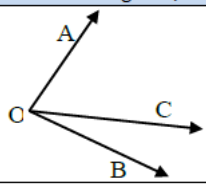
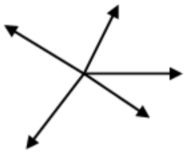
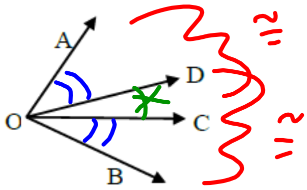
1. $\overline{MN}$ ≅ $\overline{PQ}$ GIVEN
2. $\overline{NP}$ ≅ $\overline{NP}$ (Property: REFLEXIVE PROP OF ≅)
3. $\overline{MP}$ ≅ $\overline{NQ}$ OVERLAPPING/COMMON SEGMENT THM

This is really the addition property of equality applied to segment addition postulate, with the special case that what we are adding is a figure that both the other figures have in common. This allows us to skip the algebraic approach by using the **Common/Overlapping Segment Theorem**.

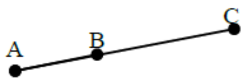
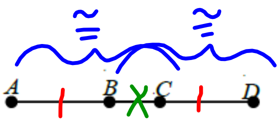
If instead you were given $\overline{MP} \cong \overline{NQ}$ with $\overline{NP} \cong \overline{NP}$, what could you conclude about $\overline{MN}$ & $\overline{PQ}$? ≅ So we can either add the overlapping piece to congruent littles to get congruent bigs *or* subtract the overlapping piece from congruent bigs to get congruent littles. This works for both angles and segments.



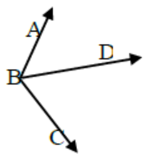
Sum of Parts

Conditional Statement	Diagram / Example	Stated as a Reason in a Proof
If point C is in the interior of $\measuredangle AOB$, then________________		
Given a sequence of ***n*** consecutive adjacent angles whose interiors are all disjoint such that the angle formed by the first ***n* – 1** angles and the last angle are a linear pair, then the angle measures ____________	($\measuredangle$'s on a line)	Consecutive adjacent angles on a line sum to $180°$
If the sum of the measures of all angles formed by three or more rays with the same vertex and whose interiors do not overlap, then the angle measures ____________	 ($\measuredangle$'s at a point)	Angles at a point sum to $360°$
If points C and D are in the interior of $\measuredangle AOB$ AND $\measuredangle AOD \cong \measuredangle BOC$, then $\measuredangle AOC \cong \measuredangle BOD$ *See lesson summaries for 3 step process (can also go ≅ bigs → ≅ littles)*		Common Angle Theorem *Or* Overlapping Angles Theorem

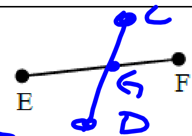
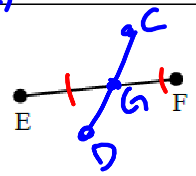
Sum of parts (con't)

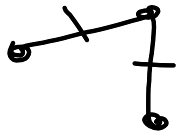
If A, B, and C are collinear, then ________________		
If points $A, B, C,$ AND D are collinear and $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$ *See lesson summaries for 3 step process (can also go ≅ bigs → ≅ littles)*		Common Segment Thm *Or* Overlapping Segments Thm

Bisectors

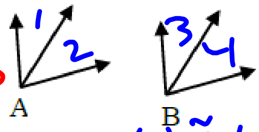
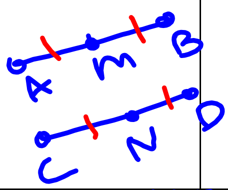
Conditional Statement	Diagram / Example	Stated as a Reason in a Proof
If $\overrightarrow{BD}$ bisects $\measuredangle ABC$, then ________________	↔ 	Definition of Angle Bisector *Or* Angle Bisector ↔ two congruent adjacent angles
If $\measuredangle ABD \cong \measuredangle CBD$ and they are adjacent, then ________________ *Note conjunction!*		

Bisectors (con't)

If B is the midpoint of $\overline{AC}$, then ________ If $\overline{AB} \cong \overline{BC}$ and A, B, and C are collinear, then ________ *Note conjunction!*	↔ 	Definition of a Midpoint *or* Midpoint ↔ two congruent collinear segments
$\overline{CD}$ bisects $\overline{EF}$ at G and EFG ↔ G is the midpoint.	$\overline{CD}$ BISECTS $\overline{EF}$ @ G ↔ 	SEGMENT BISECTOR ↔ MIDPOINT
$\overline{CD}$ bisects $\overline{EF}$ at G ~~and EFG~~ ↔ $\overline{EG} \cong \overline{GF}$. AND EFG	$\overline{CD}$ BISECTS $\overline{EF}$ @ G ↔  COLLINEAR	Segment bisector ↔ 2 ≅ collinear segments. *Or* Definition of a Segment Bisector



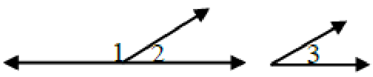
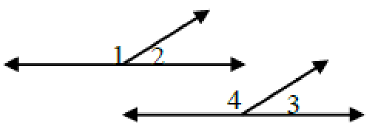
Bisectors (con't)

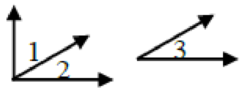
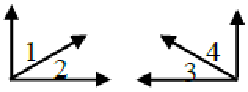
If 2 ≅ angles are bisected, then their HALVES ARE CONGRUENT	∡A & ∡B BISECTED + ∡A ≅ ∡B ↔  ∡1 ≅ ∡2 ≅ ∡3 ≅ ∡4 ADJACENT	Halves of Congruent Angles are Congruent.
If 2 ≅ segments are bisected, then their HALVES ARE CONGRUENT	$\overline{AB}$ & $\overline{CD}$ BISECTED + $\overline{AB} \cong \overline{CD}$ ↔  COLLINEAR	HALVES OF ≅ SEGMENTS ARE CONGRUENT

Angle Pairs

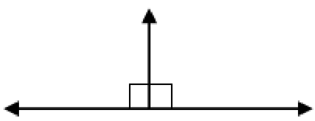
Conditional Statement	Diagram / Example	Stated as a Reason in a Proof
The sum of two angles = 90° if and only if the angles are COMPLEMENTARY	A B A 40° B 50° COMP ⟷ $m\angle A + m\angle B = 90°$	DEFN OF COMPLEMENTARY ∡'S
The sum of two angles = 180° if and only if the angles are SUPPLEMENTARY	SUPP ⟷ $m\angle 1 + m\angle 2 = 180°$ 100° 80°	DEFN OF SUPPLEMENTARY ∡'S
If two angles are adjacent and their noncommon sides form opposite rays, then the angles are a LINEAR PAIR	DIAGRAM	Defn. of a Linear Pair
* If two ∡'s form a linear pair, then they are SUPPLEMENTARY	1 2 → ∡1 SUPP ∡2	Linear pairs of ∡'s are supplementary. Linear Pair → Supp ∡'s

Angle pairs (Con't)

If 2 non-adjacent ∡'s are formed by intersecting lines then they are ______________.		Defn of Vertical Angles
If angles are vertical ∡'s, then the angles are ______________		Vertical ∡ pairs are equal in measure Vertical ∡ pairs are ≅
If 2 angles are supplementary to the same angle, then they are ______________		Congruent Supplements Theorem *Or* Supplements of the same angle are congruent
If 2 angles are supplementary to congruent angles, then they are ______________		Congruent Supplements Theorem *Or* Supplements of congruent angles are congruent

Angle pairs (Con't)		
If 2 angles are complementary to the same angle, then they are ____________________		Congruent Supplements Theorem *Or* Supplements of the same angle are congruent
If 2 angles are complementary to congruent angles, then they are ____________________		Congruent Supplements Theorem *Or* Supplements of congruent angles are congruent

Right Angles

If 2 $\measuredangle$'s are right $\measuredangle$'s, then ______ ____________________		Right $\measuredangle$'s are $\cong$.
If 2 $\cong$ $\measuredangle$'s are supplementary, then ____________________ *Note conjunction!*		Congruent & supplementary angles are right $\measuredangle$'s