Lesson 2-2/ 2-3L : Algebraic Proofs

Agenda
- Check and Review Homework 2-1
- Guided Notes - need notes and your unit outline/ lesson summaries/axioms pages

HW
- Worksheet 2-2R/2-3L is Problem Set in Notes

P. 77: #19 P. 85-86: #32,34;

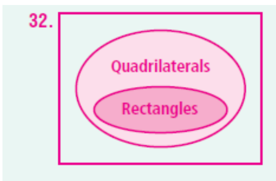
P.100-101:#11,15,18,23,24,25,29,33,36,40,53,61

Show that each conjecture is false by finding a counterexample.

19. Every pair of supplementary angles includes one obtuse angle. $m\angle 1 = m\angle 2 = 90°$

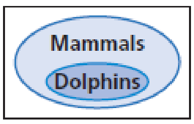
Draw a Venn diagram.

32. All rectangles are quadrilaterals.



Write a conditional statement from each Venn diagram.

34.



34. If an animal is a dolphin, then it is a mammal.

P.100-101:#11,15,18,23,24,25,29,33,36,40,53,61

11. A parallelogram is a rectangle if and only if it has four right angles.

11. Conditional: If a ▱ is a rect., then it has 4 rt. ∠s. Converse: If a ▱ has 4 rt. ∠s, then it is a rect.

15. If a triangle contains a right angle, then it is a right triangle.

Converse: If a triangle is a right triangle, then it contains a right angle.

Biconditional: A triangle is a right triangle ifand only if it contains a right angl

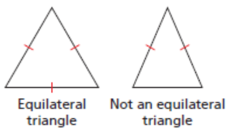
23. If $x > 0$, then $x^2 > 0$.

no; possible answer: $x = -2$

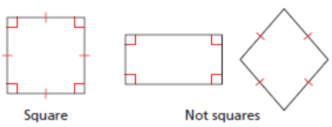
18. A circle is the set of all points in a plane that are a fixed distance from a given point.

Biconditional: A circle is a set of all points in a plane if and only if the points are at a fixed distance from a given point.

24.



25.



24. An equil. △ is a △ with 3 ≅ sides.

25. A square is a quad. with 4 ≅ sides and 4 rt. ∠s.

29. An angle is a geometric object formed by two rays.

29. Possible answer: The definition does not say that the rays have a common endpoint.

Complete each statement to form a true biconditional.

33. The circumference of a circle is 10π if and only if its radius is _?_ . 5

36. **Write About It** Use the definition of an angle bisector to explain what is meant by the statement "A good definition is reversible."

36. Possible answer: If you write the def. as a biconditional, "A ray is an ∠ bisector iff it divides the ∠ into 2 ≅ ∠s," then you can use it either forward or backward. If you know the ray is an ∠ bisector, then you can conclude that the 2 ∠s formed are ≅. If you know that 2 adj. ∠s formed by a ray are ≅, then you can conclude that the ray is an ∠ bisector.

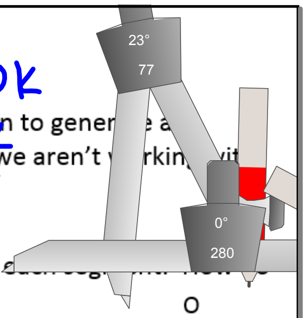
40. Which conditional statement can be used to write a true biconditional?

- (A) If a number is divisible by 4, then it is even.
- (B) If a ratio compares two quantities measured in different units, the ratio is a rate.
- (C) If two angles are supplementary, then they are adjacent.
- (D) If an angle is right, then it is not acute.

53. If x is prime, then $x + 2$ is also prime.

53. F; possible answer: $x = 2$

Warm Up

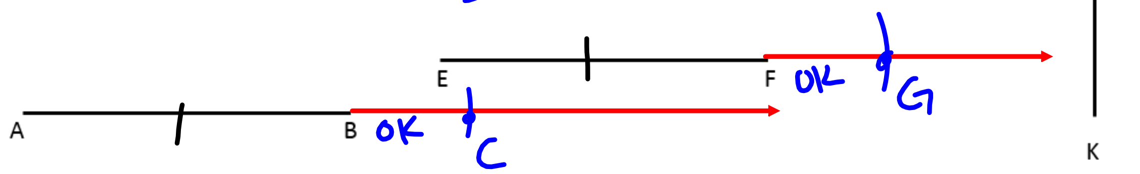


Addition/Subtraction Property of Equality

We know that you can add or subtract the same value from both sides of an equation to generate a equivalent equation using the addition or subtraction property of equality. What if we aren't working with strictly values but rather equal measures like OK or $m\angle 5$?

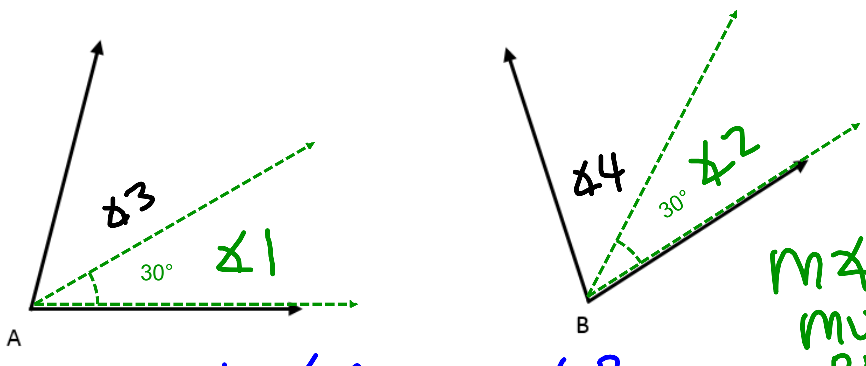
AB = EF OK = OK
AB + OK = EF + OK
AC = EG

Addition: Given $\overline{AB} \cong \overline{EF}$, construct $\overline{ABC}$ and $\overline{EFG}$ by adding the length of $\overline{OK}$ to each segment. How do the lengths of $\overline{ABC}$ and $\overline{EFG}$ compare? ≅



OK = OK
AB + OK = EF + OK

Subtraction: Given $\measuredangle A \cong \measuredangle B$, use your universal angle maker to draw in a ray in the interior with an endpoint at A or B such that a 30° angle is formed with one of the large angle's rays.
How much does the remaining little angle measure in each of larger angles? 45°



m∡1 = m∡2
MUST BE STATED

m∡A = m∡B
− 30°   − 30°
− m∡1   − m∡2

Geometry + LAB Name:______________________________ Date:_______ Section: _________

2-2R/2-3L Notes Algebraic Proofs

In order to prove a conjecture true, we must use **deductive reasoning**, which is using known facts and sound logic with precise definitions, postulates, and theorems to draw conclusions.

Deductive = USE rules vs. Inductive = MAKE rules.

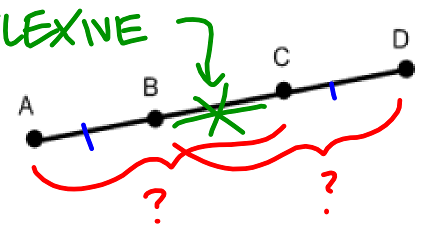
Formal 2-Column Geometric Proof:

Statements	Reasons
• Detailed steps to solve or prove the problem. $\overline{AB} \cong \overline{BC}$ • Names of angles, segments, lines, rays, etc. • Last statement is always what you are asked to prove SPECIFICALLY.	DEFN OF MIDPOINT • The definitions, postulates, properties, and theorems that allow you to make the corresponding statement and link your thinking between steps. **Look at what changes between steps. • Does not include names of figures – be general.

Refer to your lesson summaries and axiom pages for examples of & additional reasons used in proofs.

- Angle Addition Postulate/Segment Addition Postulate
- Reflexive & Transitive Properties
- Algebraic Properties & Substitution

EX 1: Segment Addition Postulate & Addition Property of Equality



Given: $\overline{AB} \cong \overline{CD}$; $\overline{ABCD}$

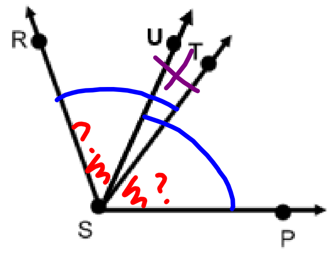
Prove: $\overline{AC} \cong \overline{BD}$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$, $\overline{ABCD}$	1. GIVEN
2. $AB = CD$	2. = MEASURE ⟷ ≅
3. $BC = BC$	3. REFLEXIVE PROPERTY OF EQ
4. $AB + BC = CD + BC$	4. ADDITION PROP OF EQUALITY
5. $AC = BD$	5. SEGMENT ADDITION POSTULATE
6. $\overline{AC} \cong \overline{BD}$	6. = MEASURE ⟷ ≅

EX 2: Angle Addition Postulate & Subtraction Property of Equality

Given: $m\measuredangle RST = m\measuredangle USP$

Prove: $m\measuredangle RSU = m\measuredangle TSP$



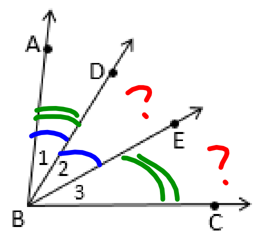
m∡RST =

Statements	Reasons
1. m∡RSU + m∡UST = m∡RST m∡TSP + m∡UST = m∡USP	1. ∡ ADDITION POSTULATE
2. m∡USP	2. GIVEN
3. m∡RSU + m∡UST = m∡TSP + m∡UST	3. SUBSTITUTION (STEP 1 INTO 2)
4. m∡UST = m∡UST	4. REFLEXIVE PROPERTY OF EQ.
5. m∡RSU = m∡TSP	5. SUBTRACTION PROP OF EQ.

EX 3: Application of Properties

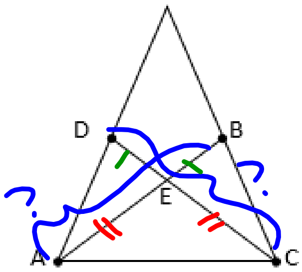
Given: $\measuredangle 2 \cong \measuredangle 1$, $\measuredangle 1 \cong \measuredangle 3$

Prove: $\measuredangle 2 \cong \measuredangle 3$



Statements	Reasons
1. ∡2 ≅ ∡1 , ∡1 ≅ ∡3	1. GIVEN
2. ∡2 ≅ ∡3	2. TRANSITIVE PROP OF ≅ (STEP 1 → 1)

EX 4: Segment Addition Postulate & Addition Property of Equality



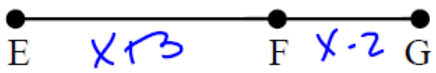
Given: $\overline{AE} \cong \overline{CE}$; $\overline{EB} \cong \overline{ED}$; $\overline{AEB}$; $\overline{CED}$

Prove: $\overline{AB} \cong \overline{CD}$

Statements	Reasons
1. $\overline{AE} \cong \overline{CE}$ $\overline{EB} \cong \overline{ED}$ $\overline{AEB}$; $\overline{CED}$	1. GIVEN
2. AE = CE , EB = ED	2. = MEASURE ⟷ ≅
3. AE + EB = CE + ED	3. ADDITION PROPERTY OF EQ.
4. AB = CD	4. SEGMENT ADDITION POSTULATE
5. $\overline{AB} \cong \overline{CD}$	5. = MEASURE ⟷ ≅

- A two-column proofs has statements on the ______________ side and reasons/justifications on the ______________ side.
- To write an algebraic proof, start with a GENERAL EQUATION using LETTER NAMES. Then use algebraic properties to justify each statement towards the solution.

Ex:



1) Statement #1 is EF + FG = EG
   Reason: SEGMENT ADDITION POSTULATE
2) Statement #2 uses GIVEN (EF = x+3, . . .)
3) Statement #3 uses SUBSTITUTION

Problem Set 2-2R/2-3L

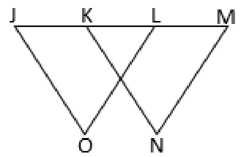
For questions 1-4, identify the property being used:

1. If AB = CD and CD = EF, then AB=EF. ____________________
2. If AB = CD and EF = CD, then AB=EF. ____________________
3. $\measuredangle TRY \cong \measuredangle TRY$ ____________________
4. If $m\measuredangle 1 = m\measuredangle 2$ and $m\measuredangle 3 = m\measuredangle 3$ then $m\measuredangle 1 + m\measuredangle 3 = m\measuredangle 2 + m\measuredangle 3$. ____________________

For questions 5-8, complete the following proofs:

5. Given: $\measuredangle J \cong \measuredangle OLJ; \measuredangle OLJ \cong \measuredangle M$

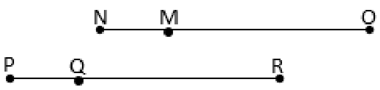
   Prove: $\measuredangle J \cong \measuredangle M$



Statements	Reasons
1. $\measuredangle J \cong \measuredangle OLJ$	1.
2. $\measuredangle OLJ \cong \measuredangle M$	2.
3. $\measuredangle J \cong \measuredangle M$	3.

6. Given: $\overline{NMQ}$ NMO, $\overline{PQR}, NO = PR, NM = PQ$

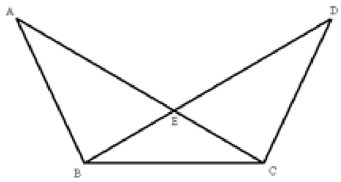
   Prove: $MO = QR$



Statements	Reasons
1. $\overline{NMQ}, \overline{PQR}, NO = PR$	1.
2. $NM + MO = NO; PQ + QR = PR$	2.
3. $NM + MO = PQ + QR$	3.
4. $NM = PQ$	4.
5. $MO = QR$	5.

7. **Given:** $m\measuredangle ABD = m\measuredangle DCA$; $m\measuredangle DBC = m\measuredangle ACB$

Prove: $\measuredangle ABC \cong \measuredangle DCB$

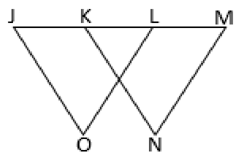


Statements	Reasons
1.	1. Given
2. $m\measuredangle ABD + m\measuredangle DBC = m\measuredangle DCA + m\measuredangle ACB$	2.
3.	3. Angle Addition Postulate
4. $m\measuredangle ABC = m\measuredangle DCB$	4.
5.	5.

8. **Given:** $\overline{JK} \cong \overline{ML}$

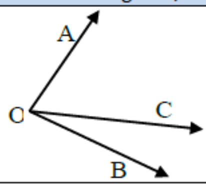
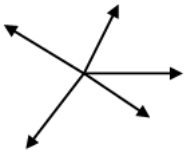
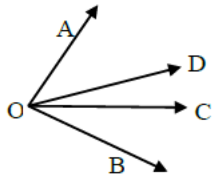
Prove: $\overline{JL} \cong \overline{MK}$

(Hint: how is this really just a segment problem?)

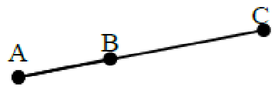


Statements	Reasons

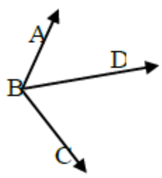
Sum of Parts

Conditional Statement	Diagram / Example	Stated as a Reason in a Proof
If point C is in the interior of $\measuredangle AOB$, then______________________		
Given a sequence of ***n*** consecutive adjacent angles whose interiors are all disjoint such that the angle formed by the first ***n* – 1** angles and the last angle are a linear pair, then the angle measures ______________	($\measuredangle$'s on a line)	Consecutive adjacent angles on a line sum to $180°$
If the sum of the measures of all angles formed by three or more rays with the same vertex and whose interiors do not overlap, then the angle measures ______________	 ($\measuredangle$'s at a point)	Angles at a point sum to $360°$
If points C and D are in the interior of $\measuredangle AOB$ AND $\measuredangle AOD \cong \measuredangle BOC$, then ______________________ *See lesson summaries for 3 step process (can also go ≅ bigs → ≅ littles)*		Common Angle Theorem *Or* Overlapping Angles Theorem

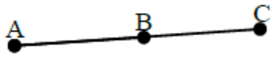
Sum of parts (con't)

If A, B, and C are collinear, then ______________________		
If points $A, B, C,$ AND D are collinear and $\overline{AB} \cong \overline{CD}$, then______________ *See lesson summaries for 3 step process (can also go ≅ bigs → ≅ littles)*		Common Segment Thm *Or* Overlapping Segments Thm

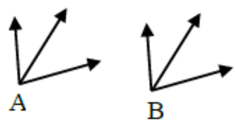
Bisectors

Conditional Statement	Diagram / Example	Stated as a Reason in a Proof
If $\overrightarrow{BD}$ bisects $\measuredangle ABC$, then ______________________	↔ 	Definition of Angle Bisector *Or* Angle Bisector ↔ two congruent adjacent angles
If $\measuredangle ABD \cong \measuredangle CBD$ and they are adjacent, then ______________________ *Note conjunction!*		

Bisectors (con't)

If B is the midpoint of $\overline{AC}$, then ______________________	↔ 	Definition of a Midpoint *or* Midpoint ↔ two congruent collinear segments
If $\overline{AB} \cong \overline{BC}$ and A, B, and C are collinear, then ______________________ *Note conjunction!*		
$\overline{CD}$ bisects $\overline{EF}$ at G and $\overline{EFG}$ ↔ G is the midpoint.	↔ 	
$\overline{CD}$ bisects $\overline{EF}$ at G and $\overline{EFG}$ ↔ $\overline{EG} \cong \overline{GF}$.	↔ 	Segment bisector ↔ 2 ≅ collinear segments. *Or* Definition of a Segment Bisector

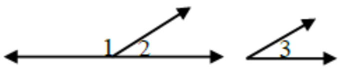
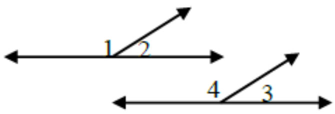
Bisectors (con't)

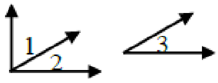
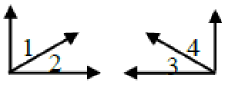
If 2 ≅ angles are bisected, then their ______________________		Halves of Congruent Angles are Congruent.
If 2 ≅ segments are bisected, then their ______________________		

Angle Pairs

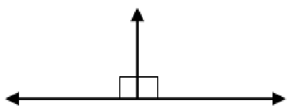
Conditional Statement	Diagram / Example	Stated as a Reason in a Proof
The sum of two angles $= 90°$ if and only if the angles are ______________		
The sum of two angles $= 180°$ if and only if the angles are ______________		
If two angles are adjacent and their noncommon sides form opposite rays, then the angles are a ____________________		Defn. of a Linear Pair
If two ∢'s form a linear pair, then they are ____________________		Linear pairs of ∢'s are supplementary. Linear Pair → Supp ∢'s

Angle pairs (Con't)

If 2 non-adjacent ∢'s are formed by intersecting lines then they are ______________________.		Defn of Vertical Angles
If angles are vertical ∢'s , then the angles are ________________		Vertical ∢ pairs are equal in measure Vertical ∢ pairs are ≅
If 2 angles are supplementary to the same angle, then they are ______________________		Congruent Supplements Theorem *Or* Supplements of the same angle are congruent
If 2 angles are supplementary to congruent angles, then they are ______________________		Congruent Supplements Theorem *Or* Supplements of congruent angles are congruent

Angle pairs (Con't)		
If 2 angles are complementary to the same angle, then they are ______________________		Congruent Supplements Theorem *Or* Supplements of the same angle are congruent
If 2 angles are complementary to congruent angles, then they are ______________________		Congruent Supplements Theorem *Or* Supplements of congruent angles are congruent

Right Angles

If 2 ∢'s are right ∢'s, then ______ ______________________		Right ∢'s are ≅.
If 2 ≅ ∢'s are supplementary, then ______________________ *Note conjunction!*		Congruent & supplementary angles are right ∢'s