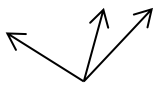
Geometry Main Concepts

Conditional Statements

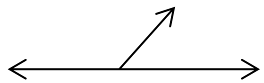
• Conditional	If hypothesis, then conclusion	if → then
• Converse	If conclusion, then hypothesis	"switch"
• Biconditional	Hypothesis if and only if conclusion	↔ (iff)

Angle Pairs

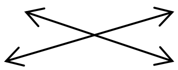
- Adjacent Angles



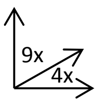
- Linear Pair (Thm → Supplementary)



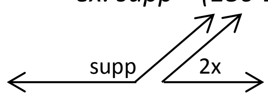
- Vertical Angles



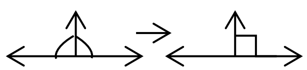
- Complementary Angles: $m\measuredangle$ + comp = 90° *ex: ratio 4:9*



- Supplementary Angles: $m\measuredangle$ + supp = 180° *ex: supp = (180-2x)°*

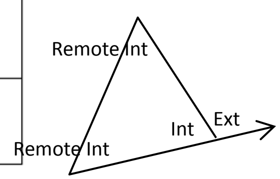


- Congruent Complements Theorem
- Congruent Supplements Theorem
- Congruent supplementary angles → right angles



Polygons

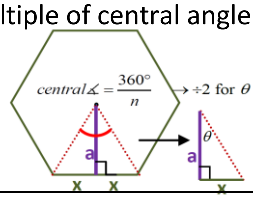
Angles	Sum	Each (regular)
Interior	$(n-2)180°$	$\frac{(n-2)180°}{n}$
Exterior	$360°$	$\frac{360°}{n}$



●Interior + Exterior = 180° (linear pair)

●Carry a regular polygon onto itself = multiple of central angle (360°/n)

●Area regular polygon: $A = n(area_{isos\Delta})$



$\tan\theta = \frac{x}{a}$

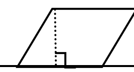
n = number of sides, isos triangles, vertices

Sum of Parts

- Angles at a point sum to 360°
- Consecutive adjacent angles on a line sum to 180°
- Angle Addition Postulate / Segment Addition Postulate
- Arc Addition Postulate (circle sum = 360° ; semicircle sum = 180°)
- Overlapping/Common Angle/Arc/Segment Theorem
- Halves of Congruent Angles/Segments/Arcs are Congruent
- Diameter → 2 semicircles in a circle

Quadrilaterals

Parallelogram:



●Both sets opposite sides || (defn)

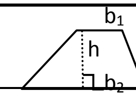
●Both sets opposite sides ≅

●Both sets opposite angles ≅

●Diagonals bisect each other

●Consecutive angles supplementary

Trapezoid:



●One pair || sides

●Same Side Int $\measuredangle$'s supplmnt

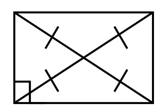
●Area = $\frac{(b_1+b_2)}{2}h$

● Midsegment - || to bases $= \frac{b_1+b_2}{2}$

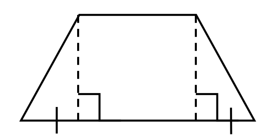
Rectangle:

●▱ with 4 right angles

●▱ with ≅ diagonals



Isosceles Trapezoid:

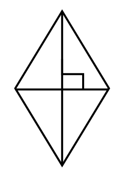


●Legs ≅

●Base angles ≅

●Diagonals ≅

Rhombus:



●All four sides congruent

●▱ w/consecutive ≅ sides

●▱ w/ ⊥ diagonals

●▱ w/diagonals bisect opposite angles

● Area = $\frac{1}{2}d_1d_2$ or treat as composite

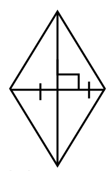
Note: use slope to find missing vertices of quadrilaterals in the coordinate plane

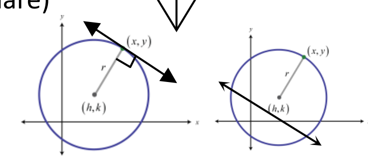
Square:

●All properties of ▱, rectangle,rhombus

Coordinate Geometry Tools

- Distance Formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
- Midpoint Formula: $Midpt\ (x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
- Slope Formula: $m=\frac{y_2-y_1}{x_2-x_1}$ or $m=\frac{\Delta y}{\Delta x}=\frac{rise}{run}=\pm\text{tangent}\,\theta$
    - Parallel: $m_1=m_2$
    - Perpendicular: $m_1 \bullet m_2=-1$ (opposite reciprocals)
    - Coincident lines: parallel lines w/same y-intercept
    - Skew lines: non-coplanar lines that never intersect
- Equations of Lines:
    - Vertical Line $x=c$ vs. Horizontal Line $y=c$
    - Slope-Intercept Equation of a Line: $y=mx+b$
    - Point-Slope Equation of a Line: $y-y_1=m(x-x_1)$
        - Perpendicular bisector equation – (x_1,y_1) is midpoint!
        - (Equation of other diagonal of a rhombus/square)
        - Tangent line: slope is perpendicular to radius & (x_1,y_1) is point of tangency





- Equations of Circles: $(x-h)^2+(y-k)^2=r^2$
- System of Equations: Graph each eq then label and state the points of intersection

Parallel and Perpendicular Lines

$\|$ l's $\leftrightarrow$ $\cong$ corresponding $\measuredangle$'s

$\|$ l's $\leftrightarrow$ $\cong$ alternate interior $\measuredangle$'s

$\|$ l's $\leftrightarrow$ $\cong$ alternate exterior $\measuredangle$'s

$\|$ l's $\leftrightarrow$ supplementary same side interior $\measuredangle$'s

Two l's $\perp$ same l $\rightarrow$ $\|$ l's

(Parallel Lines are Perpendicular to the Same Line)

Linear pair of $\cong$ $\measuredangle$'s $\rightarrow$ $\perp$ to other line

A line $\perp$ to one of 2 $\|$ l's $\rightarrow$ $\perp$ lines

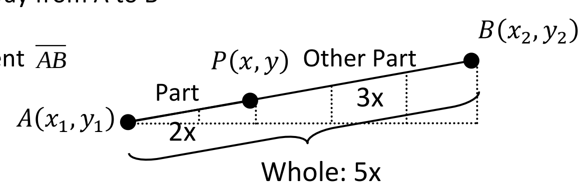
(Perpendicular Transversal Theorem)

Directed Line Segment

$$P(x,y)=\left(x_1+\frac{Part}{Whole}(x_2-x_1),y_1+\frac{Part}{Whole}(y_2-y_1)\right)$$

EX: P is $\frac{2}{5}$ of the way from A to B

or P divides segment $\overline{AB}$ into ratio 2:3



Transformations

- **Reflections (rigid)**

$r_{x-axis}(x,y)=(x,-y)$

$r_{y-axis}(x,y)=(-x,y)$

$r_{y=x}(x,y)=(y,x)$

$r_{origin}(x,y)=(-x,-y)=R_{180^\circ}$

Perpendicular in & out the same amount for reflection into a line! Same slope in & out same amount for reflection into point

Rotations (rigid)

* $R_{90^\circ}(x,y)=(-y,x)$

* $R_{180^\circ}(x,y)=(-x,-y)$

*If center rotation = origin; otherwise use slope from center of rotation

Translations (rigid)

$T_{a,b}(x,y)=(x+a,y+b)$

<a,b>

Dilations (non-rigid)

$D_k(x,y)=(kx,ky)$ if origin

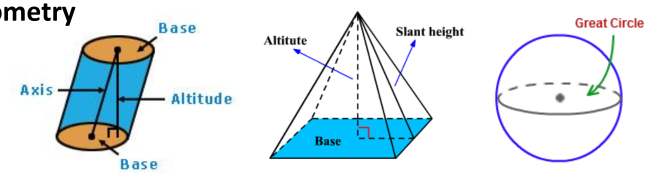
$Scale\ Factor\ k=\frac{image}{pre-image}$

$Sim\ Ratio=\frac{pre-image}{image}$

- **Compositions:** $2^{nd}(1^{st}$(pre-image)) → Image (be sure correspondence is correct)
    - Glide reflection = reflection + translation
    - 2 reflections into a line may be equivalent to a single rigid motion
- **Properties which can be preserved:** Parallelism, perpendicularity, orientation, distance (perimeter/ area), angle measure
    - Line reflections preserve all properties except orientation (indirect/opposite isometry)
    - Point reflections, Rotations and Translations preserve all properties (direct isometry)
    - Dilations preserve all properties except distance (not isometric)

Line Dilations: through center of dilation → coincident; otherwise parallel (SLOPE is preserved - SAME)

Solid Geometry



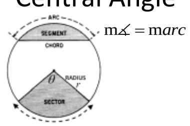
	Prism	Cylinder	Pyramid/Cone		Sphere
Volume	$V = BH$		$V = \frac{BH}{3}$		$V = \frac{4}{3}\pi r^3$
Lateral Face Shape	Parallelogram(s)		Triangles (Pyramid) Part of Circle (Cone)		
∥ Cross-Section	Congruent to base		Similar to base		Similar ⊙
Slice / Non-∥	Polygon	Conic; ▱	Polygon	Conic; Triangle	Similar ⊙

Formation: translate to parallel plane, rotate about an axis, dilate from pt, stack

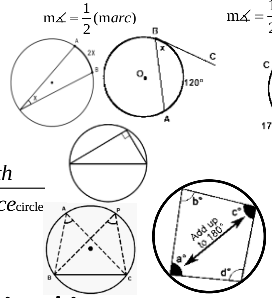
Circles

•Angle-Arc Relationships

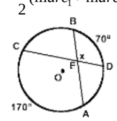
Central Angle — $m\measuredangle = m\text{arc}$



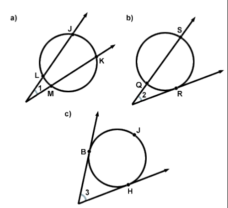
Inscribed Angle — $m\measuredangle = \frac{1}{2}(m\text{arc})$



Interior Angle — $m\measuredangle = \frac{1}{2}(m\text{arc}_1 + m\text{arc}_2)$



Exterior Angle — $m\measuredangle = \frac{1}{2}(m\text{large} - m\text{small})$



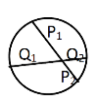
$$\frac{\theta°}{360°} = \frac{Area_{sector}}{Area_{circle}}$$

$$\frac{\theta°}{360°} = \frac{ArcLength}{Circumference_{circle}}$$

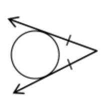
$$1\pi \text{ radian} = 180°$$

•Segment Relationships

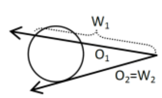
Chord-Chord: $P_1P_2 = Q_1Q_2$



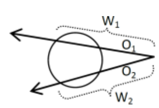
Two Tangents: $Tang_1 = Tang_2$



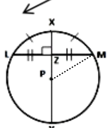
Secant-Tangent: $W_1O_1 = W_2O_2$



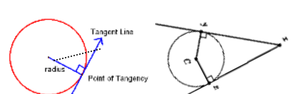
Secant-Secant: $W_1O_1 = W_2O_2$



Radius/Diameter ⊥ Chord

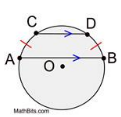


Tangent ⊥ Radius/Diameter

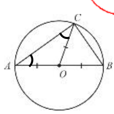


•Other

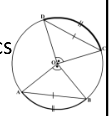
Parallel Chords



Congruent Radii in a ⊙



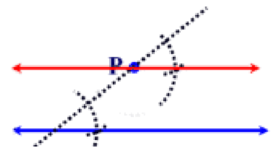
≅ Central∡ ↔ ≅Arcs ↔ ≅ Chords



Constructions – ≅ or ~Δ's criteria may use a combination of skills

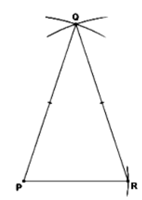
Copy Angle

Parallel line (≅ corr∡'s)

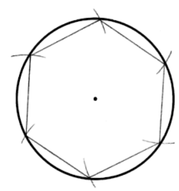


Copy Segment

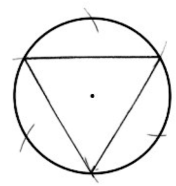
Isosceles/Equilateral Triangle



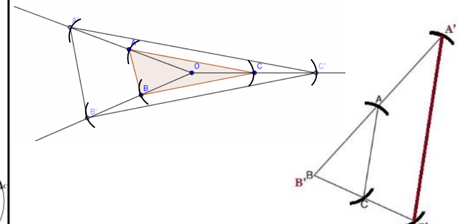
Inscribed Regular Hexagon in a Circle



Inscribed Equilateral Triangle in a Circle

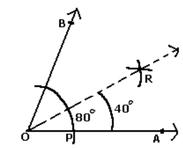


Dilation

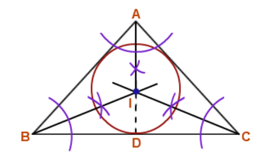


Bisect Angle

Bisect Angle



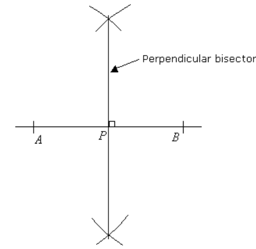
Incenter → Inscribed ⊙



Bisect Segment (→ Perpendicular Line) TAN

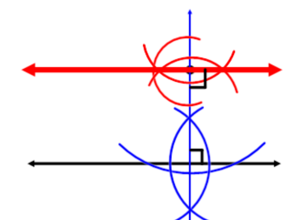
Line of Reflection

Mapping A to B

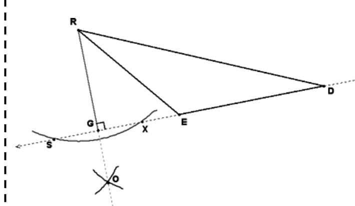


Parallel Line

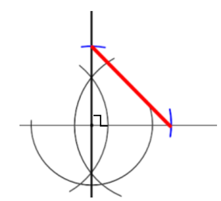
(2 perpendicular constructions)



Altitude (⊥ *from point off line*)

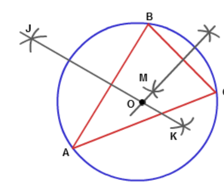


Right Triangle (⊥ *from pt on line*)



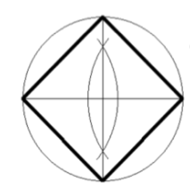
Circumcenter (⊥ *bisectors*)

→ Circumscribed ⊙



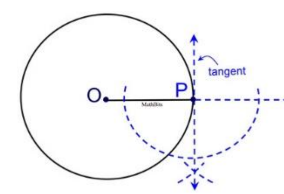
Inscribed Square

(⊥ *Diameters*)



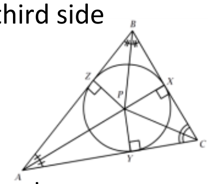
Tangent Line to a Circle

(⊥ to Radius)

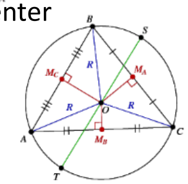


Triangle Properties

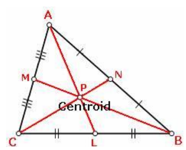
- Classification
  - o Sides:
    - Scalene, Isosceles, Equilateral
  - o Angles:
    - Acute, Right, Obtuse, Equiangular
- Equilateral ↔ Equiangular
- Exterior ∡ = sum 2 remote interior ∡'s
- Midsegment – || to and ½ of third side
- Points of Concurrency:
  - o Incenter (∡ Bisectors)
    - → Inscribed circle
    - → radii ⊥ to sides from incenter



  - o Circumcenter (⊥ Bisectors)
    - →Circumscribed circle
    - → radii to vertices of Δ



  - o Centroid (Medians)
    - → 1:2:3 ratio
    - → $(x,y)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$



  - o Orthocenter (Altitudes)
  - o Orthocenter and Circumcenter can be on/outside the triangle (right/obtuse Δ)
- Inequalities:
  - o Sum of two sides > 3rd alone
    Difference < 3rd Side < Sum
  - o Longest side is opposite largest angle
  - o Exterior angle > each remote interior ∡

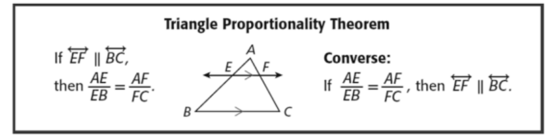
Congruent Triangles

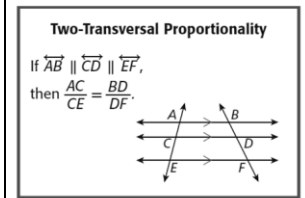
- Criteria First!
  - o SSS ≅
  - o ASA ≅ (included side)
  - o RtΔHL ≅
  - o AAS ≅
  - o SAS ≅ (no Donkey ASS or SSA; included angle)
- CPCTC (≅ Δ's → corresponding parts ≅)

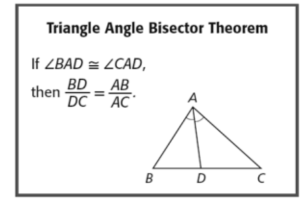
Similar Triangles

- Criteria First!
  - o SSS ~
  - o SAS ~ (no Donkey ASS or SSA)
  - o AA ~
- ~ Δ's →
  - o Corresponding sides proportional: $\frac{SideI}{SideII}=\frac{SideI}{SideII}$
  - o Corresponding angles ≅
  - o Perimeter Ratio = Similarity Ratio
  - o Area Ratio = (Sim Ratio)^2

Proportional Splitters







Right Triangle Similarity

- Special Right Triangles:

30°	60°	90°
n	$n\sqrt{3}$	2n

45°	45°	90°
n	n	$n\sqrt{2}$

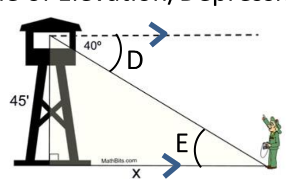
- Trigonometric Ratios: $\sin\theta=\frac{opp}{hyp};\cos\theta=\frac{adj}{hyp};\tan\theta=\frac{opp}{adj}$

$$\sin^{-1}\left(\frac{opp}{hyp}\right)=\theta;\cos^{-1}\left(\frac{adj}{hyp}\right)=\theta;\tan^{-1}\left(\frac{opp}{adj}\right)=\theta$$

- Complementary Acute Angles:

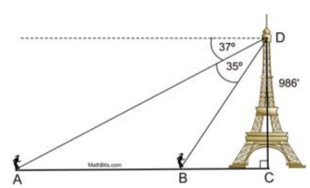
$$\sin\alpha=\cos\beta \text{ iff } \alpha+\beta=90° \ (\alpha \& \beta \text{ are complementary})$$

- Pythagorean Theorem: Right triangle $\leftrightarrow a^2+b^2=c^2$
- Angle of Elevation/Depression from Horizontal Line of Sight

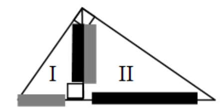


(alternate interior angles ≅)

- Double Trig: AB+BC=BC so BC=AC=AB

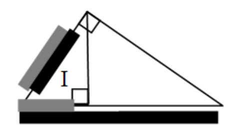


- Geometric Mean (Can you use inverse trig?)



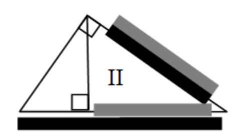
$$\frac{\Delta I \text{ short leg}}{\Delta II \text{ short leg}}=\frac{\Delta I \text{ long leg}}{\Delta II \text{ long leg}}$$

Geometric Mean: ΔIII Altitude



$$\frac{\Delta I \text{ short leg}}{\Delta III \text{ short leg}}=\frac{\Delta I \text{ hypotenuse}}{\Delta III \text{ hypotenuse}}$$

Geometric Mean: ΔIII Short Leg



$$\frac{\Delta II \text{ long leg}}{\Delta III \text{ long leg}}=\frac{\Delta II \text{ hypotenuse}}{\Delta III \text{ hypotenuse}}$$

Geometric Mean: ΔIII Long Leg