A Peer-Reviewed Article

Bits, Bytes, and Binary: *The Mathematics of Computers*

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THE purpose of the activities presented in this article is to help students understand the role of binary in technologies such as computers, smartphones, and the Internet. Students will learn how bits, bytes, and binary are used to make these technologies function. They will learn how to convert numbers from decimal format to binary and from binary to decimal.

Students will be shown how to access advanced features in a computer's calculator application to verify the accuracy of the number conversions. They will also be shown how to use a free online binary game to support mastery of the mathematics and conversion process.

Background

Binary is the foundation of almost all forms of modern communication (Bogomolny, n.d.). Whether you are talking about smartphones, computers, or mobile devices like iPads and other tablets, all rely on binary to operate and communicate with each other. Anthes (2007) noted that all data is built from binary. Everything from a cell phone to a supercomputer uses 0's and 1's to create, manipulate, and store information. Even a picture taken with a digital camera is created and stored using binary values.

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As Feaster, Ali, and Hallstrom (2012) stated, the binary number system is a very important part of building interest in the computer science field. It is the most basic level to which any code eventually is converted. The National Council of Teachers of Mathematics (2011) has

Table 1—Standards Met

ITEEA Standards for Technological Literacy Content Standards
(International Technology and Engineering Educators Association, 2012)

Standard 2	Students will develop an understanding of the core concepts of technology.	Students will discover how binary is the foundation of all computing technologies (e.g., computers, smart phones, Internet) work.
Standard 3	Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.	Students will integrate math with computing in a direct manner as well as understand the concept behind binary with reference to computers.
Standard 17	Students will develop an understanding of and be able to select and use information and communication technologies.	Students will use a variety of technologies to accomplish the activities.

NCTM Principles and Standards for School Mathematics
(National Council of Teachers of Mathematics, 2011)

4.6	Developing Estimation Strategies by Making Connections among Number, Geometry, Measurement, and Data Concepts	Students will develop connections between different numbering systems.
5.1	Communicating about Mathematics Using Games	Students will reinforce the math skills through the use of an online game. The game can be used individually or competitively between multiple students.
5.4	Accessing and Investigating Data Using the World Wide Web	Students will be engaged in online research.

NCTE Standards for the English Language
(National Council of Teachers of English, 2012)

Standard 8	Students use a variety of technological and information resources (e.g., libraries, databases, computer networks, video) to gather and synthesize information and to create and communicate knowledge.	Students will use a variety of information resources to develop an understanding of binary and its role in technology.

identified the need for students to learn about numbering systems like binary in their educational standards.

Forms of binary are used in many things. Data is transferred over telephone lines and radio lines by using high- and low-pitched tones to represent the ones and zeros (National Center for Women in Information Technology, 2008). On hard drives, binary is represented by magnetic fields.

Optical storage such as CDs, DVDs, and Blu-ray discs use reflected light to represent binary values. A reflective bit of the disc that reflects the laser in the drive represents a one and a nonreflective bit represents a zero (Alford, 1993). Bits are usually represented together to provide more information. These groups are called bytes.

The activities provided here will give students an opportunity to investigate the impact of binary on their lives, the binary number system, the conversion process between binary and decimal number systems, and strategies for reinforcing the initial learning.

Binary is also used to signify the computing capabilities of a computer's central processing unit (CPU). The CPU has evolved from 8-bit and 16-bit processing to the current 32-bit and 64-bit processing. These varying bit sizes refer to the amount of information that a CPU can process in a single clock cycle. This is then multiplied by the number of clock cycles per second a CPU can process. Current processors typically operate in the range of 3.0 to 4.0 MHz per second.

The bit size of the CPU is also an indication of the theoretical amount of RAM memory the computer is able to utilize. A 64-bit processor can use up to 2048 petabytes of RAM. Unfortunately, computer users will not see that level of performance. The actual RAM capacity is limited by other hardware components such as the motherboard.

Materials

- A computer with a Windows operating system, including the default calculator application
- A computer with Internet access
- Student worksheets

Learning Objectives

After conducting online research and completing the worksheets, students will be able to

1. Define binary.
2. Identify why computers use binary.
3. Identify three ways in which computers use binary.
4. Accurately convert decimal numbers to binary numbers.
5. Accurately convert binary numbers to decimal numbers.
6. Use the calculator on a Windows computer to verify decimal-to-binary and binary-to-decimal conversions.
7. Use the Cisco Binary Game to reinforce the learning of the binary conversion processes.

For a list of standards met, see Table 1.

Activity 1: What Is Binary and Why Should I Care?

In this activity, students will investigate the role of binary numbers in computing technologies. They will use an Internet search engine to answer the questions on the Binary Research Worksheet.

The teacher can use the items below as suggestions for guiding student research.

- Binary is a base 2 numbering system that uses a series of 1's and 0's.
- Binary is used by computers because it is relatively simple to use the presence of an electric charge to represent a 1 and the absence of an electrical charge to represent a 0.
- The transistors found in a microprocessor (found in everything from computers and tablets to smartphones) act as a switch to create binary values. Like a light switch, when the transistor is "turned on" it allows electricity to flow, which represents a 1. When the transistor is "turned off" it is registered as a 0.
- Electronic documents (e.g., word-processing documents, spreadsheets, pictures, music) are stored in a binary format.
- Electronic communications (e.g., email, Twitter, social networking, online gaming, Web surfing) use binary to route information from source to destination.
- The capacity of storage media (e.g., hard drives, flash drives, DVDs, Blu-ray discs, SD cards) is based on binary. Capacity is indicated by the number of bytes the storage media can hold. ▶

Binary Research Worksheet

The purpose of this activity is for you to learn more about binary numbers and their purpose in technology. You should use an Internet search engine to research each of the three questions below.

a. What is the binary numbering system?

b. Why is binary used by computers?

c. Describe ways that computers use binary.

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	128	64	32	16	8	4	2	1
Binary								

Fig. 1

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	128	64	32	16	8	4	2	1
Binary	1	1	1	0	1	0	1	0

Fig. 2

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	128	64	32	16	8	4	2	1
Binary	1	1	1	0	1	0	1	0
	128	64	32	0	8	0	1	0

Fig. 3

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	128	64	32	16	8	4	2	1
Binary	1	1	1	0	1	0	1	0

128 + 64 + 32 + 0 + 8 + 0 + 2 + 0 = 234

Fig. 4

• Each 0 or 1 represents 1 bit. Eight bits make a byte, 1024 bytes make a kilobyte (KB). 1024 KB make a megabyte (MB), 1024 MB make a gigabyte (GB).

After an appropriate period of time, the teacher will ask the students to share their findings.

Students should watch the video at http://www.youtube.com/user/MyWhyU?v=5sS7w-CMHkU&feature=pyv (Why U, 2011). Developed by the University of Central Florida and intended for students studying pre-algebra, the video covers decimal, binary, octal, and hexadecimal numbering systems.

Activity 2: Binary-to-Decimal Conversions

In this activity, the student works through binary-to-decimal and decimal-to-binary conversion processes without using a calculator.

Binary-to-Decimal Conversion Example:

Convert 11101010 to a decimal number.

1. Write out the conversion sheet (Fig. 1).

2. Write the binary values in the available boxes from right to left (Fig. 2).

3. Drop down the decimal value in any column that contains a 1 (Fig. 3).

4. Add the dropped decimal values to find that the binary value of 11101010 converts to the decimal value of 234 (Fig. 4).

Decimal-to-Binary Conversion Example

Convert 115 to binary format.

1. Write out the conversion sheet (Fig. 1).

2. Find the largest decimal value in the conversion sheet that does not exceed the number being converted. In this example, 64 is the largest number that is smaller than 115. Write a 1 in the binary box under the 64 (Fig. 5).

3. Subtract 64 from 115. This leaves a remainder of 51 (Fig. 5).

4. Repeat the process with the remainder from Step 3. Place a 1 in the binary box under 32 and then subtract that from 51. This will leave a remainder of 19 as seen in Fig. 6.

5. Repeat the process, placing a 1 in the binary box under 16 and then subtract 16 from 19 (Fig. 7).

6. Since 2 is the next number closest to the remainder of 3 from Step 5, a 1 should be placed in the binary box under the decimal value of 2. Subtracting 2 from 3 will leave a remainder of 1 (Fig. 8).

7. Place the last 1 in the binary box of the 1 column. Subtracting the 1 from 1 leaves a remainder of 0 (Fig. 9).

8. Place 0's in the remaining binary boxes (Fig. 10). The binary equivalent of the decimal

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	128	64	32	16	8	4	2	1
Binary		1						

115
-64
51

Fig. 5

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	128	64	32	16	8	4	2	1
Binary		1	1					

115
-64
51
-32
19

Fig. 6

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	128	64	32	16	8	4	2	1
Binary		1	1	1				

115
-64
51
-32
19
-16
3

Fig. 7

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	128	64	32	16	8	4	2	1
Binary		1	1	1			1	

115
-64
51
-32
19
-16
3
-2
1

Fig. 8

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	128	64	32	16	8	4	2	1
Binary		1	1	1			1	1

Fig. 9

115
-64
51
-32
19
-16
3
-2
1
-1
0

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	128	64	32	16	8	4	2	1
Binary	0	1	1	1	0	0	1	1

Fig. 10

value of 115 is 01110011. Any 0's to the left of the leftmost 1 in a binary value are typically dropped. Therefore, the conversion would typically be represented as 1110011.

The student should now complete the conversions on the Binary Conversions Worksheet independently.

Activity 3: Advanced Calculator

In this activity, students will learn how to use the advanced features in a computer's calculator to verify the conversions they completed on the Binary Conversions Worksheet. ►

Binary Conversions Worksheet

1. Convert the following numbers to binary
a. 255

b. 31

c. 27

d. 156

e. 243

2. Convert the following numbers to decimal format
a. 11011001

b. 00110101

c. 11001010

d. 10101010

e. 11100101

◄ Fig. 11

Fig. 12 ►

Fig. 13

Fig. 14

Fig. 15

Student Instructions

1. Open the calculator on the computer. This should open the calculator in its Standard Mode (Fig. 11).

2. Put the calculator into one of its advanced modes. To do this, click on the View menu and click on the Programmer option (Fig. 12). This will show a box with four options highlighted (Fig. 13). These represent the numbering systems that the calculator can use. The options include Hex (hexadecimal), Dec (decimal), Oct (octal), and Bin (binary). (The student should be familiar with each of these numbering systems from the video viewed in Activity 1.)

3. To verify the binary-to-decimal conversion from Activity 2, select the Bin option on the calculator and then enter the binary value of 11101010. (Fig. 13).

4. Select the Dec option. The output of the calculator should change to show the decimal equivalent of 234 (Fig. 14).

5. The calculator should still be set to the Dec option. To verify the decimal-to-binary conversion from Activity 2, enter 115 and choose the Bin option. The output should then display the binary value of 1110011. (Fig. 15).

As an alternative, the student may search for online binary/decimal converters. Students should use the calculator to check the conversions completed on the Binary Conversions Worksheet.

Activity 4: Online Binary Game

To reinforce the concepts introduced in the Binary Conversions Worksheet, students should play the Cisco Binary Game. In the game, students score points by successfully performing both types of conversions. If there are multiple students, the game can be used as a competition to see who can earn the highest score within a specified period of time.

Student Instructions

1. Go to http://forums.cisco.com/CertCom/game/binary_game_page.htm (Cisco Systems Inc., 2011). The game is also available as a free iPod/iPad app or an Android app.

2. Click "Start."

3. The game will present rows of 0's and 1's (Fig. 16). At the end of the row is a box that will either contain a number or be blank. If the box is blank, you must enter the decimal equivalent for the binary value of that row. For example, in the first row in Fig. 16, type 64 in the box and press the "Enter" key since there is a 1 in the 64 column.

4. If the row has a number in the

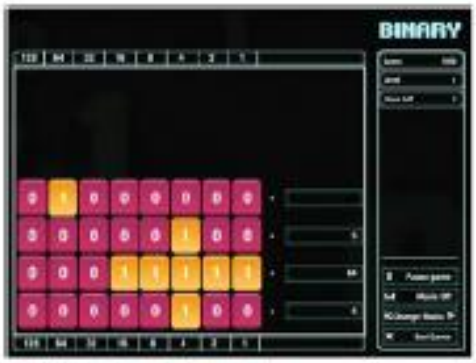

Fig. 16

Fig. 17

Worksheet Answer Key

1. Decimal to binary	2. Binary to decimal
a. 11111111	a. 217
b. 00011111	b. 53
c. 00011011	c. 202
d. 10011100	d. 170
e. 11110011	e. 229

box, you need to arrange that row's 1's and 0's to the binary equivalent of the value in that row's box. When you click on any of the binary values in the row, it will change a 1 to a 0 or a 0 to a 1. For example, the second row in Fig. 16 has a 5 in the box. The student would need to click on the 0 in the 1's column to create the binary equivalent of 101.

5. Once you have entered the correct decimal or binary value, the row will disappear. The game will add rows periodically. The game will add rows more frequently as the student successfully progresses through the game. The game ends when it can no longer add rows to the game screen. (Fig. 17).

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