Lesson 11-8 Segment Relationships in Circles

AGENDA:
- Check & Review Homework 11-7
- Notes and Guided Practice

HOMEWORK:
- p. 796 # 12, 17- 22, 25, 27

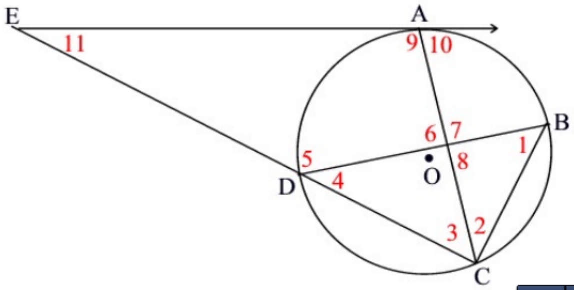
$3x+3x+4x+5x=360$

$15x=360$

$x=24$

Circle O with no diameters shown. $\overline{EA}$ is tangent at point A.

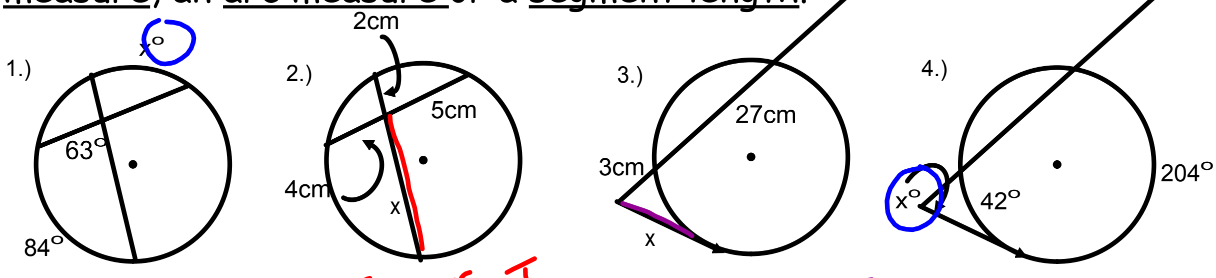
$m\overset{\frown}{AB} : m\overset{\frown}{BC} : m\overset{\frown}{CD} : m\overset{\frown}{DA} = 3:3:4:5$



$m\angle 1$ 48	$m\angle 6$ 96
$m\angle 2$ 36	$m\angle 7$ 84
$m\angle 3$ 60	$m\angle 8$ 96
$m\angle 4$ 36	$m\angle 9$ 108
$m\angle 5$ 144	$m\angle 10$ 72
	$m\angle 11$ 12

- Arc-angle relationships are measured in DEGREES.
- Segments and chords are measured in LINEAR UNITS, such as inches or centimeters.

In each problem, identify whether you are asked to find an angle measure, an arc measure or a segment length.



ARC MEASURE

SEGMENT LENGTH cm

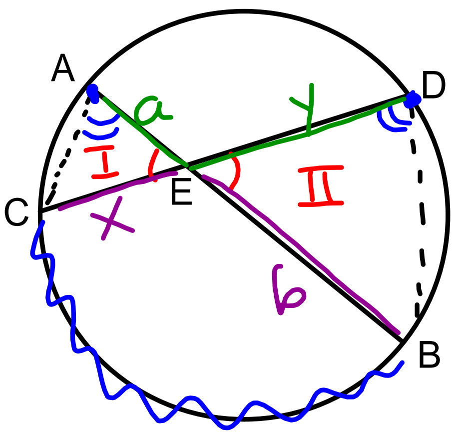
SEGMENT LENGTH cm

∡ MEASURE IN °

CHORD-CHORD PRODUCT THEOREM

Consider an interior or a central angle formed by two chords.
What relationship can you find among the parts of the chords?

Hint: Draw in other chords to form triangles AEC and DEB...
What could you prove about them?



$\triangle I \sim \triangle II$ BY AA ~

$\frac{\triangle I}{\triangle II}$ $\frac{a}{y} = \frac{x}{b}$

$a \cdot b = x \cdot y$

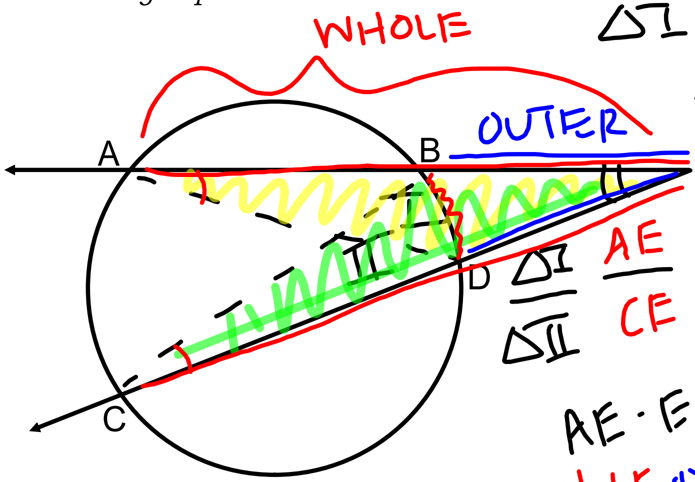
PART · PART = PART · PART

PT OF INTERSECTION

2. SECANT-SECANT PRODUCT THEOREM

Consider an exterior angle formed by two secants.
What relationship can you find among the chords and other segments?
Hint: Draw in other chords to form triangles ADE and CBE...
What could you prove about them?

$\Delta I \sim \Delta II$
AA ~



$\frac{\Delta I}{\Delta II}$ $\frac{AE}{CE} = \frac{ED}{EB}$

$AE \cdot EB = CE \cdot ED$

WHOLE · OUTER = WHOLE · OUTER

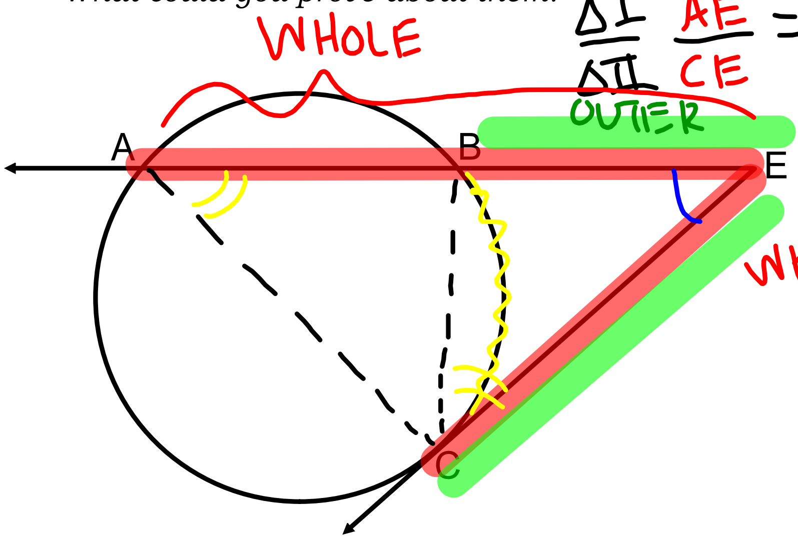
$W_1O_1 = W_2O_2$

3. SECANT-TANGENT PRODUCT THEOREM

$\Delta I \sim \Delta II$ By
AA ~

Consider an exterior angle formed by a secant and a tangent.
What relationship can you find among the chord and other segments?
Hint: Draw in other chords to form triangles ACE and CBE...
What could you prove about them?

$\frac{\Delta I}{\Delta II}$ $\frac{AE}{CE} = \frac{CE}{BE}$

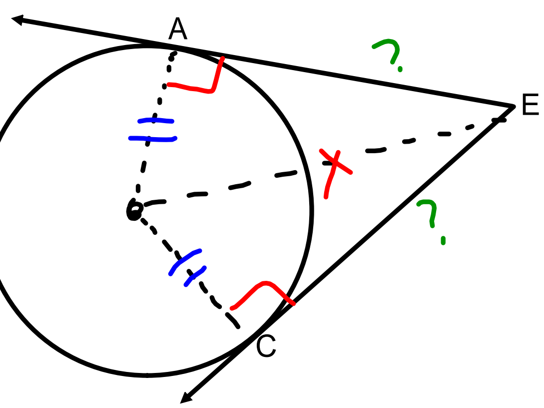


$AE \cdot BE = CE \cdot CE$

WHOLE · OUTER = WO

4. TANGENT-TANGENT SEGMENT CONGRUENCY

Consider an exterior angle formed by two tangents.
What relationship do we know already about the outer segments?



RHL $\cong$

+

CPCTC

$\overline{AE} \cong \overline{CE}$

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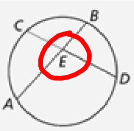
1) CHORD – CHORD PRODUCT THEOREM

If 2 chords intersect in a circle, then the products of the segments of the chords are Equal.



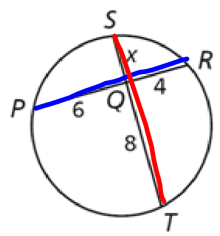
Chords $\overline{AB}$ and $\overline{CD}$ intersect at E.

$AE \cdot EB = CE \cdot ED$

DON'T JUMP CHORDS

PRACTICE: Find each chord length

A.



$P_1 \cdot P_1 = P_2 \cdot P_2$

$PQ \cdot QR = SQ \cdot QT$

$(6)(4) = (x)(8)$

$24 = 8x \rightarrow x = 3$

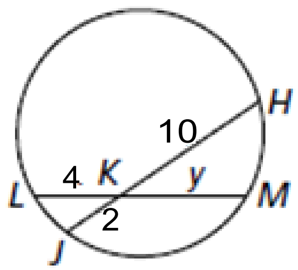
$PR = 10$ units

$ST = 3 + 8$

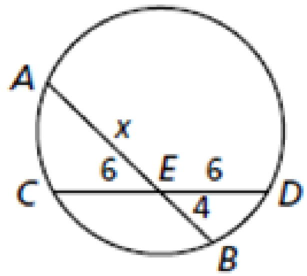
$ST = 11$ units

Try These:

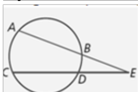
2.

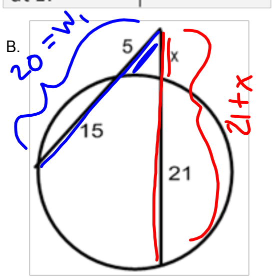


3.



2) SECANT-SECANT PRODUCT

 Secants $\overline{AE}$ and $\overline{CE}$ intersect at E.	$AE \cdot BE = CE \cdot DE$	If 2 secants intersect in the exterior of a circle, then product of the lengths of one secant and its external segment equals the product of the other secant and its external segment.



$W_1O_1 = W_2O_2$

$(20)(5) = (21+x)(x)$

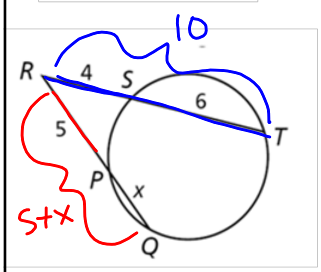
$100 = 21x + x^2$

$0 = x^2 + 21x - 100$

$0 = (x+25)(x-4)$

$x+25=0 \quad x-4=0$

$x = -25 \quad \boxed{x=4}$



$W_1O_1 = W_2O_2$

$(10)(4) = (5+x)(5)$

$40 = 25 + 5x$

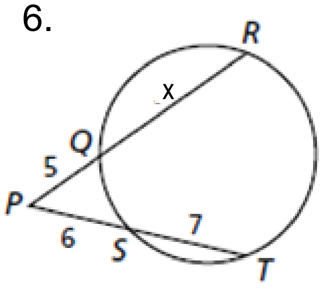
$-25 \quad -25$

$15 = 5x$

$\boxed{3 = x}$

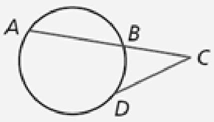
Extra Example

6.



3) SECANT – TANGENT PRODUCT THM:

If a secant & tangent intersect outside a circle, then the product of the lengths of the secant and its external segment = the length of tangent squared.



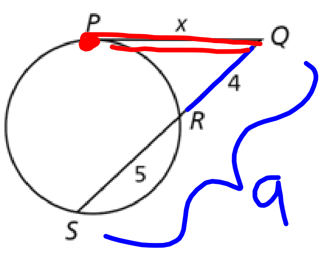
Secant $\overline{AC}$ and tangent $\overline{DC}$ intersect at C.

$AC \cdot BC = DC^2$

$W_1 O_1 = W_2 O_2$

PRACTICE: Find the Value of x:

C)



$(SQ)(RQ) = (PQ)(PQ)$

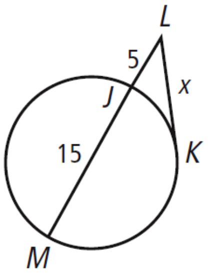
$W_1 O_1 = W_2 O_2$

$(9)(4) = (x)(x)$

$\pm\sqrt{36} = \sqrt{x^2}$

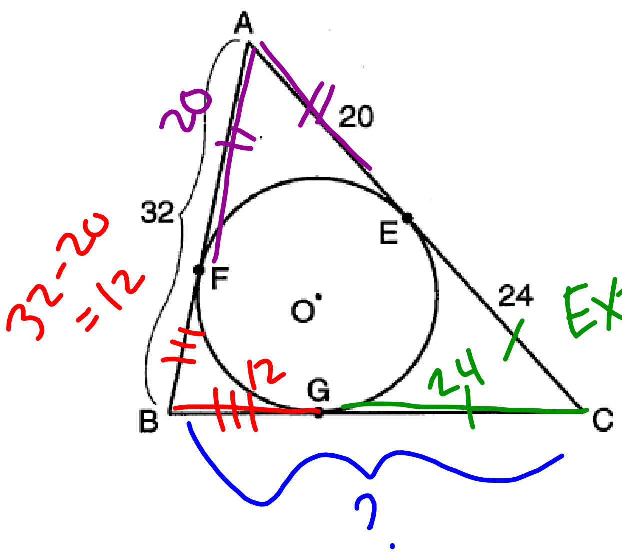
$\boxed{6 = x}$

Extra Example if needed.



MIXED PRACTICE

In the accompanying diagram, $\overline{AFB}$, $\overline{AEC}$, and $\overline{BGC}$ are tangent to circle O at F, E, and G, respectively. If $AB = 32$, $AE = 20$, and $EC = 24$, find BC.



O INCENTER

⊙O INSCRIBED IN △ABC

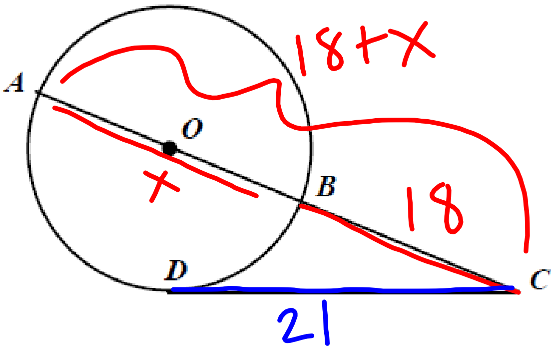
$BC = \underline{BG} + \overline{GC}$

$= 12 + 24$

EXTERNAL TAN SEGS FROM SAME PT → ≅

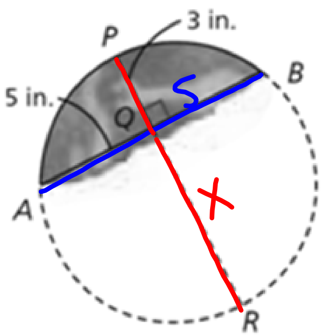
$\boxed{BC = 36}$ units

Find the diameter $=x$ of the circle (not drawn to scale). $BC = 18$, and $DC = 21$. Round your answer to the nearest tenth.



$$WO = WO$$
$$(AC)(BC) = (DC)(DC)$$
$$(18+x)(18) = (21)(21)$$
$$324 + 18x = 441$$
$$-324$$
$$18x = 117$$
$$x = \frac{117}{18}$$
$$\boxed{6.5 \text{ UNITS}}$$
DIAMETER

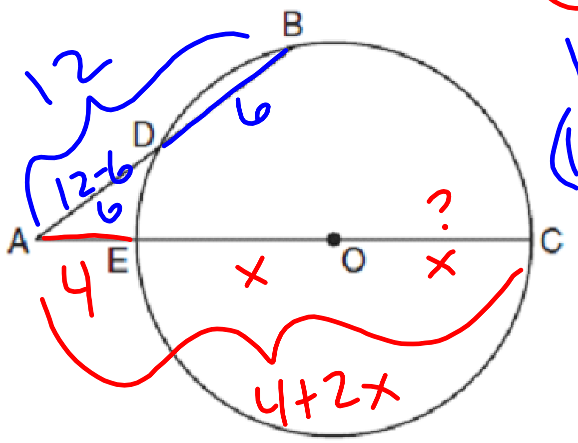
EXIT PASS Archaeologists discovered a fragment of an ancient disk. To calculate the original diameter, they drew the chord $\overline{AB}$ and the perpendicular bisector $\overline{PQ}$. Find the diameter of the disk.



$$P \cdot P = P \cdot P$$
$$(PQ)(QR) = (AQ)(QB)$$
$$(3)(x) = (5)(5)$$
$$3x = 25$$
$$x = \frac{25}{3}$$
$$PR = 3 + \frac{25}{3} = \boxed{11.3 \text{ IN}}$$

EXIT PASS

REGENTS QUESTION - In the diagram below of $\odot O$, $\overline{AB}$ intersects $\odot O$ at D, secant $\overline{AOC}$ intersects $\odot O$ at E. If AE=4, AB=12, and DB=6, find OC. $= x$



(Not drawn to scale)

$W \cdot O = W \cdot O$

$(12)(6) = (4+2x)(4)$

$72 = 16 + 8x$

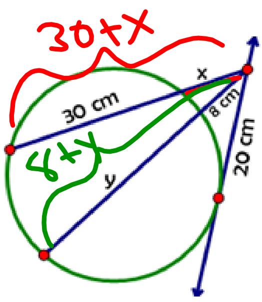
$56 = 8x$

$7 = x$

$\boxed{OC = 7}$ units

Want more? Try these:

1) Solve for x & y:



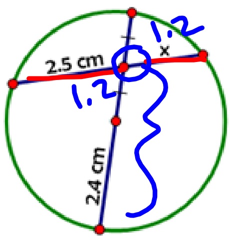
$W_2 = O_2$

$WO = WO$

$(30+x)(x) = (20)(20)$

$(8+y)(8) = (20)(20)$

2) Solve for x:

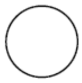
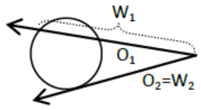
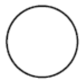
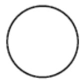


NOT RT △

$P \cdot P = P \cdot P$

$(2.5)(x) = (3.6)(1.2)$

Fill in your graphic organizer before beginning your homework:

Segment Length Relationships	Same External Point:			Same Internal Pt:
	2 Tangents	Secant – Tangent	Secant - Secant	Chord-Chord
Example				
Algebraic Equation	Tangent segment$_1$ = Tangent segment$_2$			
Derived from			Similar Triangles (AA~)	