

Lesson 11-12: Circles Equations and Completing the Square

AGENDA:

- Check & Review Homework 11-11
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- Lesson Notes & Guided Practice
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HOMWORK:

- Worksheet 11-12

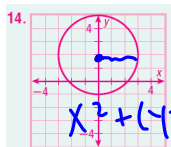
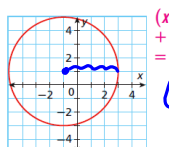
p. 802-804
 #10, 12, 14, 20, 28, 30, 31, 34, 35, 39

Write the equation of each circle.

10. $\odot R$ with center $R(-12, -10)$ and radius 8 $(x + 12)^2 + (y + 10)^2 = 64$

11. $\odot S$ with center $S(1.5, -2.5)$ and radius $\sqrt{3}$ $(x - 1.5)^2 + (y + 2.5)^2 = 3$

12. $\odot C$ that passes through $(2, 2)$ and that has center $C(1, 1)$ $(x - 1)^2 + (y - 1)^2 = 2$

14. $x^2 + (y - 2)^2 = 9$  20.  $(x + 1)^2 + (y - 1)^2 = 16$

16. $x^2 + y^2 = 100$

28. Consider the circle whose equation is $(x - 4)^2 + (y + 6)^2 = 25$. Write, in point-slope form, the equation of the line tangent to the circle at $(1, -10)$. $y + 10 = -\frac{3}{4}(x - 1)$

Find the center and radius of each circle.

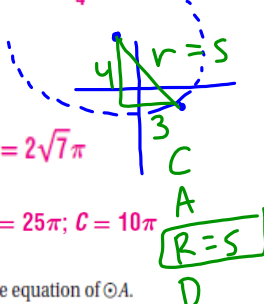
30. $(x - 2)^2 + (y + 3)^2 = 81$ 31. $x^2 + (y + 15)^2 = 25$
 center: $(2, -3)$; $r = 9$ center: $(0, -15)$; $r = 5$

34. circle with equation $(x - 8)^2 + (y + 5)^2 = 7$ $A = 7\pi$; $C = 2\sqrt{7}\pi$

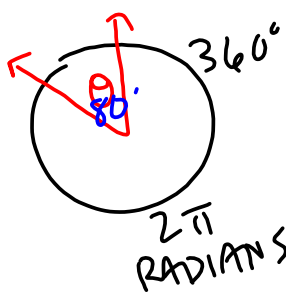
35. circle with center $(-1, 3)$ that passes through $(2, -1)$ $A = 25\pi$; $C = 10\pi$

39. $\odot A$ has a diameter with endpoints $(-3, -2)$ and $(5, -2)$. Write the equation of $\odot A$.

$39. (x - 1)^2 + (y + 2)^2 = 16$



$$\frac{\theta^\circ}{360^\circ} = \frac{\theta_{\text{RAD}}}{2\pi \text{ RADIANS}}$$



~~$$\frac{80^\circ}{360^\circ} = \frac{x}{2\pi \text{ RADIANS}}$$~~

$$\frac{(80)(2\pi)}{360} = \frac{360x}{360}$$

$$\frac{160\pi}{360} = x$$

$$\frac{4}{9} \pi \text{ RADIANS}$$

$(x \pm \quad)^2 = \square$ $y = ax^2 + bx + c$

COMPLETING THE SQUARE FOR CENTER-RADIUS FORM FOR A CIRCLE In order to get an equation like $x^2 + 10x + y^2 - 8y - 9 = 0$ into center-radius form, we need to complete the square for both x and y .

| | | |
|---|--|--|
| <p>1. Be sure that the coefficient of the highest power is one. If it is not, divide each term by that value to create a leading coefficient of one.</p> | $x^2 + 8x - 4 = 0$ | Recall: $ax^2 + bx$ |
| <p>2. Move the constant term to the right hand side.</p> | $x^2 + 8x = 4$ | O Find half the value of b , i.e. $\frac{b}{2}$ |
| <p>3. Prepare to add the needed value to create the perfect square trinomial. Be sure to balance the equation. The boxes may help you remember to balance.</p> | $x^2 + 8x + \square = 4 + \square$ | O Square it: $\left(\frac{b}{2}\right)^2$ |
| <p>4. To find the needed value for the perfect square trinomial, take half of the coefficient of the <i>middle term</i> (x-term), square it, and add that value to both sides of the equation.</p> <p style="text-align: center; color: red;">Take half and square</p> <p style="text-align: center;">↓</p> $x^2 + 8x + \square = 4 + \square$ | $x^2 + 8x + \boxed{16} = 4 + \boxed{16}$ | O Add $\left(\frac{b}{2}\right)^2$ to both sides of the equation |

$$(x \pm \quad)^2 + (y \pm \quad)^2 = r^2$$

Practice: put each of the following equations into center-radius form for a circle. Then determine the center and radius.

A) $x^2 - 12x + y^2 = 64$

B) $x^2 + 10x + y^2 - 2y + 1 = 0$

$$x^2 - 12x + 36 + y^2 = 64 + 36$$

$$\left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$$

$$(x - 6)^2 + y^2 = 100$$

CENTER (6, 0) r = 10

$$(x-6)(x-6)$$

$$x^2 - 12x + 36$$

Equations into center-radius form for a circle. Then determine the

B) $x^2 + 10x + y^2 - 2y + 1 = 0$

$$x^2 + 10x + 25 + y^2 - 2y + 1 = -1 + 25 + 1$$

$$\left(\frac{10}{2}\right)^2 = (5)^2 = 25 \quad \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

$$(x + 5)^2 + (y - 1)^2 = 25$$

CENTER (-5, 1)
r = 5

$$(x+3)(x+3)$$

$$x^2 + 6x + 9$$

$$(x+5)(x+5) = x^2 + 10x + 25$$

ADDITIONAL APPLICATIONS OF CIRCLES IN THE COORDINATE PLANE**FINDING AREA AND CIRCUMFERENCE FROM AN EQUATION OF A CIRCLE**

Find the area and circumference of $\odot G$ with the equation $x^2 + 6x + y^2 - 8y = 10$. Express your answer in terms of π .

$$x^2 + 6x + y^2 - 8y = 10$$

$$x^2 + 6x + \boxed{9} + y^2 - 8y + \boxed{16} = 10 + \boxed{9} + \boxed{16}$$

$$\left(\frac{6}{2}\right)^2 = (3)^2 = 9 \quad \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$$

$$(x+3)^2 + (y-4)^2 = 35$$

$$C = 2\pi r = 2\pi\sqrt{35} \text{ units} \quad r^2 = 35$$

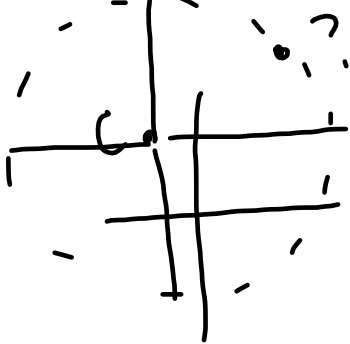
$$A = \pi r^2 = \pi(\sqrt{35})^2 = 35\pi \text{ units}^2 \quad r = \sqrt{35}$$

$$\boxed{R = \sqrt{35}}$$

D

DETERMINING IF A POINT IS ON A CIRCLE

Is the point $(7,8)$ on the circle with the equation $(x+2)^2 + (y-5)^2 = 100$?



$$(7+2)^2 + (8-5)^2 = 100$$

$$9^2 + 3^2 = 100$$

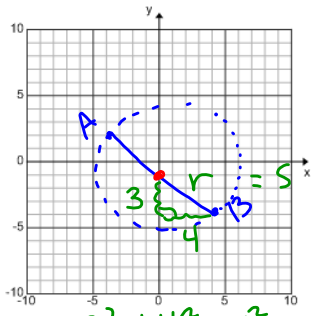
$$81 + 9 = 100$$

$$90 \neq 100$$

NO $(7, 8)$ DOES NOT SATISFY THE EQUATION OF THE CIRCLE.

WRITING AN EQUATION GIVEN THE ENDPOINTS OF A DIAMETER x+y

Write the equation for $\odot C$ that has endpoints of the diameter at $A(-4,2)$ and $B(4,-4)$. Use of the grid is optional.



$(x-h)^2 + (y-k)^2 = r^2$

CENTER MIDPOINT

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{-4+4}{2}, \frac{2+(-4)}{2}\right)$

$(0, -1)$

$3^2 + 4^2 = r^2$

$9 + 16 = r^2$

$25 = r^2$ $r \rightarrow 5$

$(x-h)^2 + (y-k)^2 = r^2$ (h, k)

$(x-0)^2 + (y-(-1))^2 = 5^2$ $(0, -1)$

$(x-0)^2 + (y+1)^2 = 25$

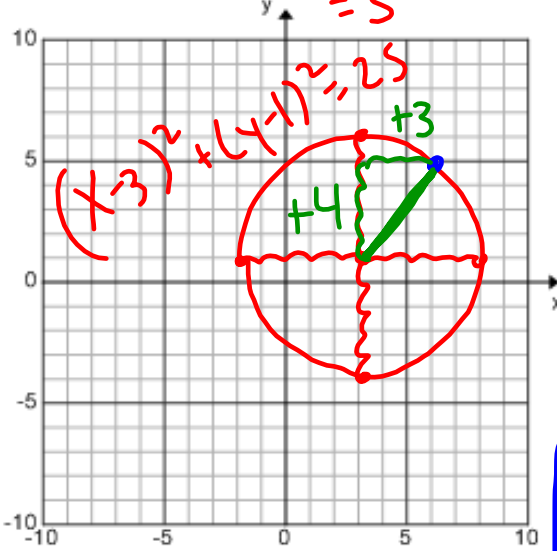
$x^2 + (y+1)^2 = 25$

FINDING THE EQUATION OF A TANGENT LINE

Determine the equation of the line tangent to the circle with equation $(x-3)^2 + (y-1)^2 = 25$ at the point $(6,5)$. Use of the grid is optional.

Recall: $y - y_1 = m(x - x_1)$

$(3, 1)$ $r = \sqrt{25} = 5$ (x_1, y_1)



$(x-3)^2 + (y-1)^2 = 25$

L LINE THRU GIVEN POINT

SLOPE OF RADIUS = $\frac{4}{3}$ \perp $-\frac{3}{4}$

$(y-5) = -\frac{3}{4}(x-6)$

FINDING POINTS OF INTERSECTION ON A GRAPH - Find the points of intersection of the equations:

⊙ CENTER (0,0)

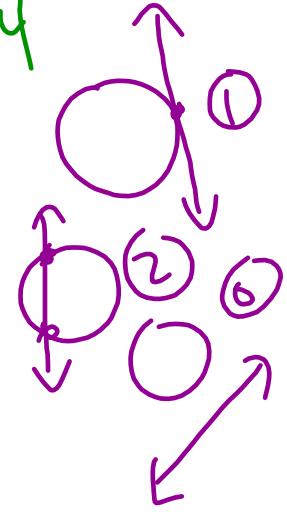
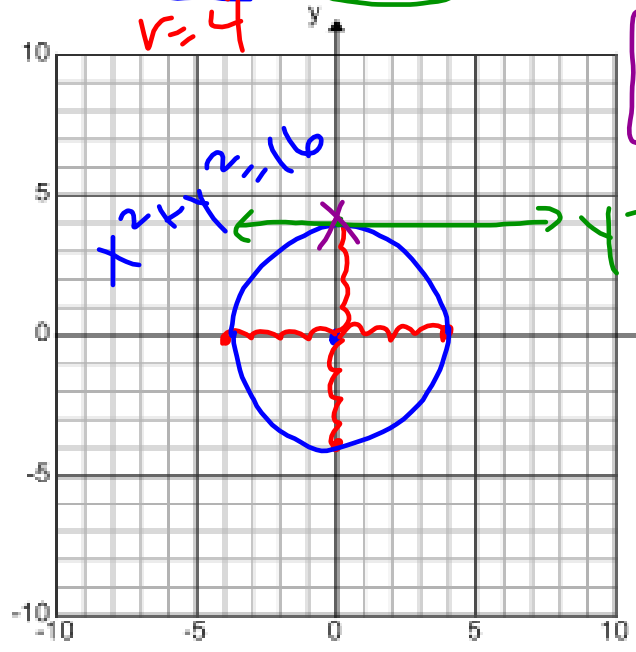
A) $x^2 + y^2 = 16$ & $y = 4$

$r = 4$

(0,4)

SOLN:

(0,4)



⊙ C(3,-2) r=2

$y - y_1 = m(x - x_1)$

B) $(x-3)^2 + (y+2)^2 = 4$ & $y+6 = 1(x-1)$

$m = \frac{1}{1}$

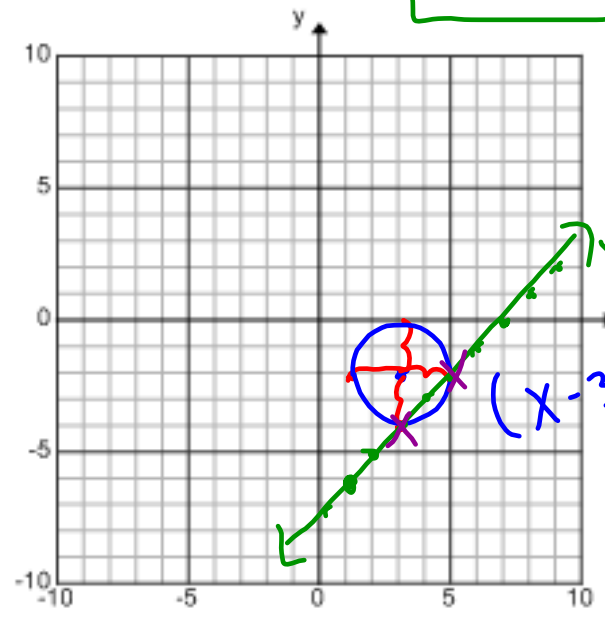
POINT (1,-6)

$y+6 = 1(x-1)$

$(x-3)^2 + (y+2)^2 = 4$

(5,-2)
(3,-4)

SOLUTIONS



Geometry LAB
Worksheet 11-12

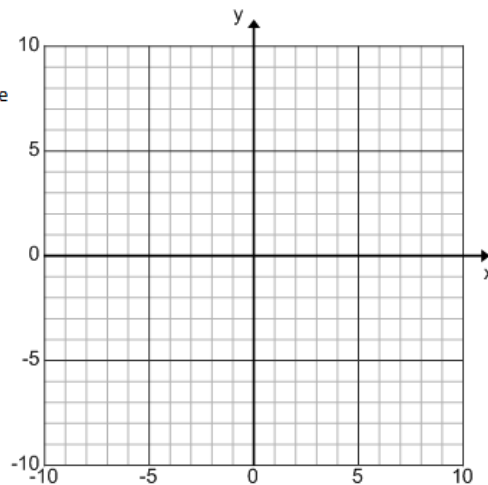
Name _____ Section _____ Date _____

- 1) Determine the area and circumference for a circle with the equation $x^2 - 4x + y^2 - 6y = 0$

- 2) Is the point $(-2,3)$ on the circle centered at $(0,5)$ with a radius of 3? Explain your reasoning.

- 3) The diameter of a circle has endpoints at $(-2,3)$ and $(6,3)$.
What is an equation of the circle?
 [1] $(x - 2)^2 + (y - 3)^2 = 16$ [3] $(x + 2)^2 + (y + 3)^2 = 16$
 [2] $(x - 2)^2 + (y - 3)^2 = 4$ [4] $(x + 2)^2 + (y + 3)^2 = 4$

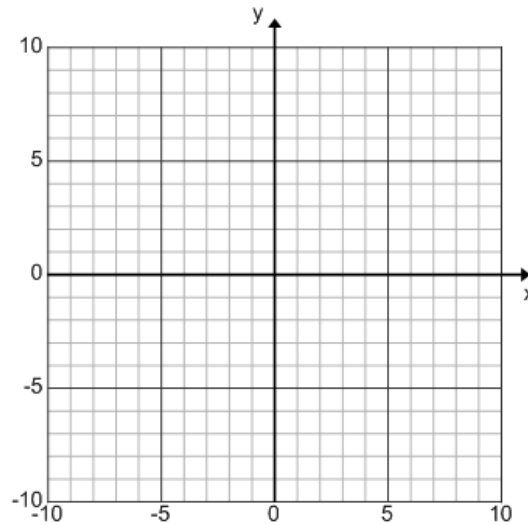
- 4) Write an equation of the line h that is tangent to circle A whose equation is $(x + 3)^2 + y^2 = 53$ at the point B $(4,2)$ in point-slope form. Use of the grid is optional.



5) Determine and state the points of intersection of the equations:

a. $(x - 4)^2 + (y + 3)^2 = 9$

b. $y = x - 4$



Use circle vocabulary to describe the line as it relates to the circle: _____

PUTTING IT ALL TOGETHER

Graphic Organizer

Circle Equation → Graph: $\sqrt{(x-h)^2 + (y-k)^2} = r^2$

1. **Determine the center and Plot** _____
2. **Determine the radius, r** _____
3. **Count r- 4 times and plot 4 points** _____
4. **Connect points with ARCs and label** _____

Given Diameter Endpoints → Equation or Graph

1. Calculate center using **midpoint** _____ formula
2. Calculate radius using **distance** _____ formula w/center & pt
3. Equation: Plug in center and radius

Graph: Plot center, count r to plot 4 pts, connect pts w/arcs