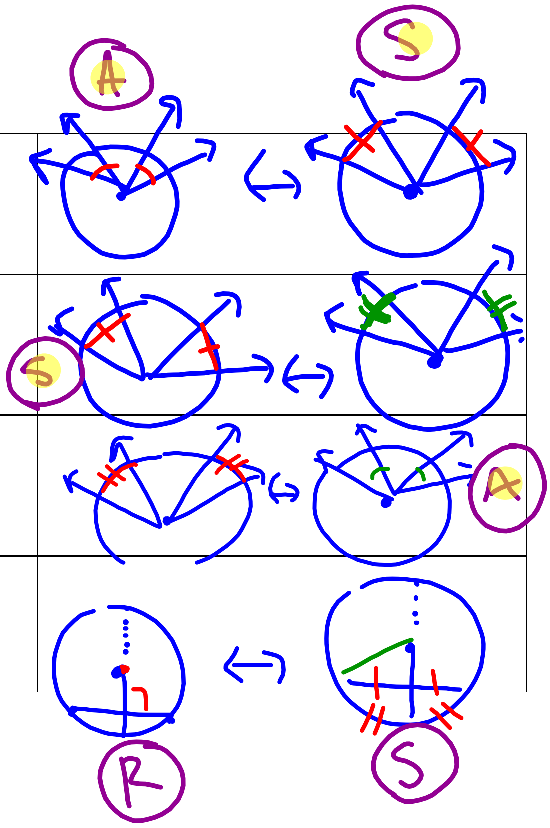
Lesson 11-13: Using Circle Properties in Proofs

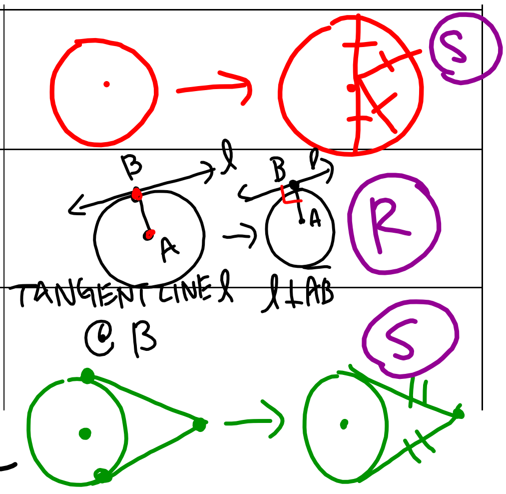
AGENDA:
- Check & Review Homework 11-12
- Lesson Notes & Guided Practice

HOMEWORK:
- Worksheet 11-13

R S A

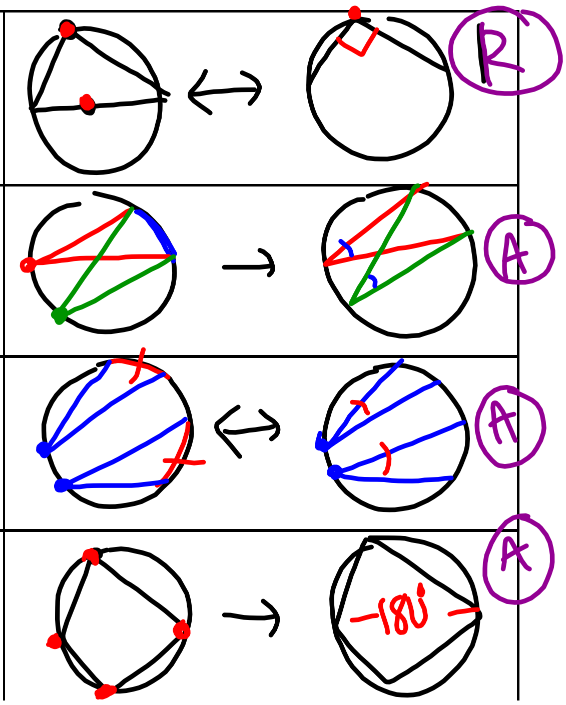
Arcs and Chords	• ≅ central angles have ≅ chords
	• ≅ chords have ≅ arcs
	• ≅ arcs have ≅ central angles
	• If a radius (or diameter) is ⊥ to a chord →it bisects the chord and the arc



Radii and Tangents	• In a circle all radii are ≅	
	• A tangent is ⊥ to the radius at the point of tangency	
	• 2 segs. tangent to circle from the same external point → segs. ≅	

TANGENT SEGMENTS FROM SAME EXTERNAL POINT ARE ≅

Graphic Organizer (back)

Inscribed Angles	• If an ∡ is inscribed in a semi circle → it is a right ∡ ✱DIAM	
	• If two inscribed ∡'s intercept the same arc → the ∡'s are ≅	
	• If two inscribed ∡'s intercept ≅ arcs → the ∡'s are ≅	
	• If a quadrilateral is inscribed in a circle → Opposite ∡'s are supplementary	

PUTTING IT ALL TOGETHER

Graphic Organizer

Circle Equation → Graph: $(x-h)^2+(y-k)^2=r^2$

1. Determine the center and Plot
2. Determine the radius, r
3. Count r- 4 times and plot 4 points
4. Connect points with ARCs and label

Given Diameter Endpoints → Equation or Graph

1. Calculate center using midpoint formula
2. Calculate radius using distance formula w/center & pt
3. Equation: Plug in center and radius

   Graph: Plot center, count r to plot 4 pts, connect pts w/arcs

Geometry LAB
Worksheet 11-12

Name ____________________ Section ________ Date _______

1) Determine the area and circumference for a circle with the equation $x^2-4x+y^2-6y=0$

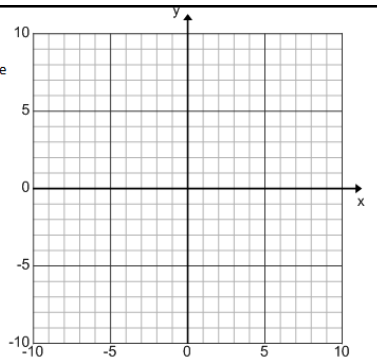
2) Is the point (-2,3) on the circle centered at (0,5) with a radius of 3? Explain your reasoning.

3) The diameter of a circle has endpoints at $(-2,3)$ and $(6,3)$. What is an equation of the circle?

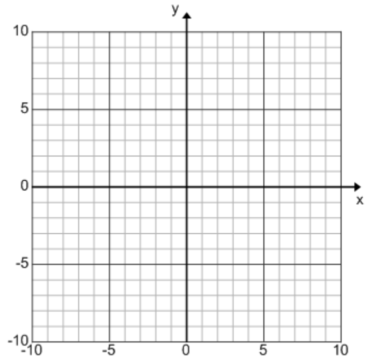
[1] $(x-2)^2+(y-3)^2=16$ [3] $(x+2)^2+(y+3)^2=16$

[2] $(x-2)^2+(y-3)^2=4$ [4] $(x+2)^2+(y+3)^2=4$

4) Write an equation of the line h that is tangent to circle A whose equation is $(x+3)^2 + y^2 = 53$ at the point B (4,2) in point-slope form. Use of the grid is optional.



5) Determine and state the points of intersection of the equations:
   a. $(x-4)^2 + (y+3)^2 = 9$
   b. $y = x - 4$



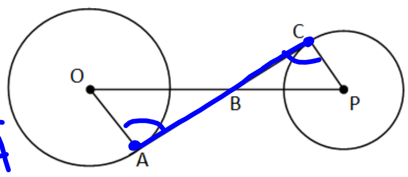
Use circle vocabulary to describe the line as it relates to the circle: ______________________________

11-13 Notes: Using Circle Properties in Proofs

****Be sure your graphic organizer is completed!**

Example 1:

If $\overline{AC}$ tangent to $\odot O$ at A and $\overline{AC}$ tangent to $\odot P$ at C, explain why $\measuredangle OAC \cong \measuredangle PCA$



① $\overline{OA} \perp \overline{AC}$; $\overline{PC} \perp \overline{CA}$

TANGENT ⊥ RADIUS @ PT OF TANGENCY

② $\measuredangle OAC$ & $\measuredangle PCA$ ARE RT $\measuredangle$'S

⊥ LINES → RT $\measuredangle$'S

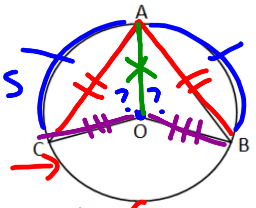
③ $\measuredangle OAC \cong \measuredangle PCA$

ALL RIGHT $\measuredangle$'S ARE $\cong$

Example 2:

1) If $\overset{\frown}{AC} \cong \overset{\frown}{AB}$ in $\odot O$, explain why $\measuredangle AOC \cong \measuredangle AOB$ and $\overline{AC} \cong \overline{AB}$.

A C



A IN A ⊙, ≅ ARCS ↔ ≅ CENTRAL ∡'S
SO ∡AOC ≅ ∡AOB.

2) Explain why $\overline{OC} \cong \overline{OB}$ in $\odot O$.

C IN A ⊙, ≅ ARCS → ≅ CHORDS.

R $\overline{OC} \cong \overline{OB}$ B/C ALL RADII IN A ⊙ ARE ≅

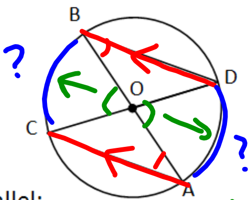
IF ASKED TO PROVE ≅ Δ'S THEN USE SSS ≅ OR SAS ≅

Example 3:

1) If $\overline{AC} \parallel \overline{BD}$ in $\odot O$, explain why $\overset{\frown}{CB} \cong \overset{\frown}{AD}$.

$\overset{\frown}{CB} \cong \overset{\frown}{AD}$ B/C ?

PARALLEL CHORDS → ≅ ARCS IN A ⊙



2) If you didn't know that $\overline{AC} \parallel \overline{BD}$, describe two ways you could prove the chords are parallel:

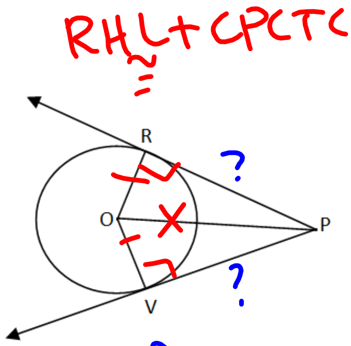
1. ≅ CENTRAL ∡'S BY VERTICAL ∡'S ≅ → ≅ ARCS → ∥ CHORDS

2.

3) If you didn't know that $\overline{AC} \parallel \overline{BD}$, explain why you would still be able to say $\overset{\frown}{CB} \cong \overset{\frown}{AD}$. → ≅ INSCRB ∡'S THAT ARE ALT INT

Example 4:

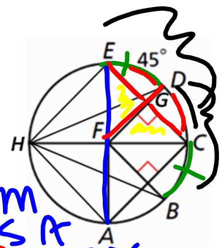
If $\odot O$ with tangents $\overline{PR}$ and $\overline{PV}$, explain why $\overline{PR} \cong \overline{PV}$.



$\overline{PR} \cong \overline{PV}$ B/C TANGENT SEGMENTS FROM THE SAME EXTERNAL POINT ARE $\cong$.

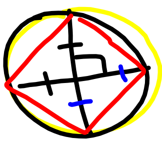
Example 5:

Given $\odot F$ with $\overline{EFA}$, $\widehat{ED} \cong \widehat{CB}$, $\overline{FD} \perp \overline{EC}$,



1) Fill in each reason:
- $\measuredangle EHD \cong \measuredangle CHB$: INSCRIBED $\measuredangle$'S FROM THE SAME ARC ARE $\cong$
- $\measuredangle ECA$ is a right angle: INSCRIBED $\measuredangle$ FROM A DIAM → RT $\measuredangle$
- $\overline{EG} \cong \overline{CG}$ and $\widehat{ED} \cong \widehat{CD}$: RADIUS ⊥ CHORD → BISECTS A CHORD & ARC
- $\widehat{DC} \cong \widehat{DC}$: REFLEXIVE then $\widehat{EC} \cong \widehat{DB}$: OVERLAPPING ARC THM or ARC ADDITION POSTULATE

2) If $\overline{EFA} \perp \overline{HFC}$, explain why ACEH is a square.



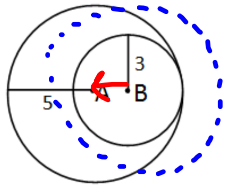
GIVEN ⊥ → RHOMBUS

IN A $\odot$, DIAMETERS $\cong$ → RECT

IN A $\odot$, RADII $\cong$ → F IS A MIDPOINT BY DEFN → DIAGONALS BISECT EACH OTHER

▱ + RECT + RHOMBUS = SQUARE ▱

<u>Example 6</u>



Explain why the two circles with given centers and radii are similar using a transformational approach. Be specific.

① DILATE ⊙B BY SCALE FACTOR OF $\frac{r_A}{r_B} = \frac{5}{3}$ CENTERED AT B

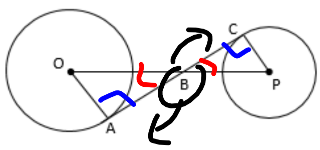
② TRANSLATE ⊙B BY VECTOR $\overrightarrow{BA}$

Practice Proof 1:

GIVEN: $\overline{AC}$ tangent to ⊙O at A
$\overline{AC}$ tangent to ⊙P at C

PROVE: $\Delta BAO \sim \Delta BCP$

Ask yourself...
1) *Do I have a radius?*
2) *What do I know about tangent lines and radii in a circle?*
3) *How do I prove Similar Triangles?*



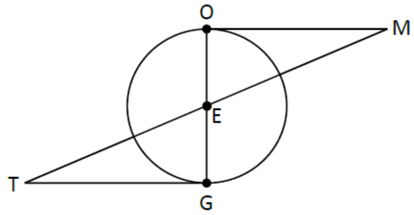
~~SSS~~
~~SAS~~
AA ~

TAN → ⊥ → RT ∡'s ≅ VERT

STATEMENTS	REASONS
1. $\overline{AC}$ tangent to ⊙O at A $\overline{AC}$ tangent to ⊙P at C	1. Given
2. $\overline{OA}$ and $\overline{PC}$ are RADII	2. Definition of radius
3. $\overline{OA}$ ⊥ $\overline{AB}$ and $\overline{PC}$ ⊥ $\overline{CB}$	3. A tangent line is ⊥ to a radius at the point of tangency
4) ∡BAO and ∡ BCP are RIGHT ∡'s	4. ⊥ Lines form RT ∡'s
5) ∡BAO ≅ ∡ BCP	5. All RT∡'S are ≅
6) ∡ABO ≅ ∡CBP	6. VERTICAL ∡'s are ≅
7) $\Delta BAO \sim \Delta BCP$	7. AA ~ AA

Geometry LAB Name: ____________________________ Section: __________ Date: _________

11-13 HW Worksheet Using Circle Properties in Proofs



Problem A

If $\odot E$ has tangents $\overline{OM}$ and $\overline{TG}$ at O and G, explain why

1) $\overline{OM} \perp \overline{OE}$ and $\overline{TG} \perp \overline{GE}$ ____

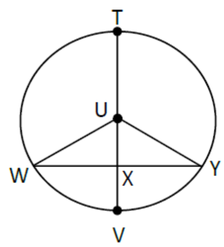
2) $\measuredangle EOM$ & $\measuredangle EGT$ are right angles ____________________________________

3) $\measuredangle EOM \cong \measuredangle EGT$ ____

4) $\overline{EO} \cong \overline{EG}$ ____

5) $\measuredangle OEM \cong \measuredangle GET$ ____

6) $\Delta OEM \cong \Delta GET$ by (check all which apply): ❐SSS≅ ❐SAS≅ ❐ASA≅ ❐AAS≅ ❐RHL≅



Problem B

If $\odot U$ has diameter $\overline{TUXV}$ that bisects $\overline{WY}$, explain why

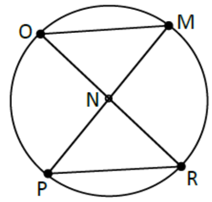
1) $\overline{WX} \cong \overline{YX}$ ____

2) $\overline{UW} \cong \overline{UY}$ ____

3) $\Delta WUX \cong \Delta YUX$ by (check all which apply): ❐SSS≅ ❐SAS≅ ❐ASA≅ ❐AAS≅ ❐RHL≅

4) $\measuredangle WUX \cong \measuredangle YUX$ ____

5) Triangle WUY is isosceles ____



Problem C

If $\odot N$ with diameters $\overline{OR}$ & $\overline{NP}$ intersecting at N, explain why

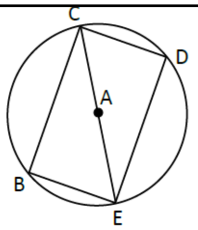
1) $\overline{ON} \cong \overline{NR}$ ______________________

2) $\measuredangle ONM \cong \measuredangle RNP$ ______________________

3) $\overline{OM} \cong \overline{RP}$ ______________________

4) $\widehat{OMR} \cong \widehat{RPO}$ ______________________

5) $\Delta NMO \cong \Delta NPR$ by (check all which apply): ❐SSS ≅ ❐SAS ≅ ❐ASA ≅ ❐AAS ≅ ❐RHL ≅



Problem D

If diameter $\overline{CE}$ in $\odot A$; $CD \cong BE$, explain why

1) $\overline{CD} \cong \overline{BE}$ ______________________

2) $\overline{CB} \parallel \overline{DE}$ ______________________

3) $\measuredangle BCE \cong \measuredangle DEC$ ______________________

4) $\overline{CAE}$ is a diameter ______________________

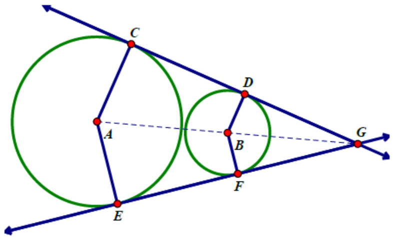
4) $\measuredangle BCE$ *and* $\measuredangle DEC$ are right angles ______________________

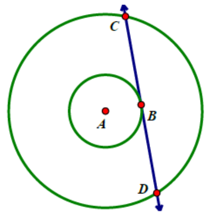
____

5) $\measuredangle CBE \cong \measuredangle EDC$ ______________________

6) ΔBCE is a right triangle ______________________

Problem E - Fill in the following congruency or similarity statements and specify which criteria is met:





$\Delta ACG \cong \Delta$______ by (check all which apply):
❒SSS ≅ ❒SAS ≅ ❒ASA ≅ ❒AAS ≅ ❒RHL ≅

$\Delta ACB \cong \Delta$______ by (check all which apply):
❒SSS ≅ ❒SAS ≅ ❒ASA ≅ ❒AAS ≅ ❒RHL ≅

$\Delta ACG \sim \Delta$______ by (check all which apply):
❒SSS ~ ❒SAS ~ ❒AA ~

Problem F:

Given Parallelogram BCDE with diagonal $\overline{CE}$ in $\odot A$, it could also be proven that BCDE is a ________________ because ____________________________.