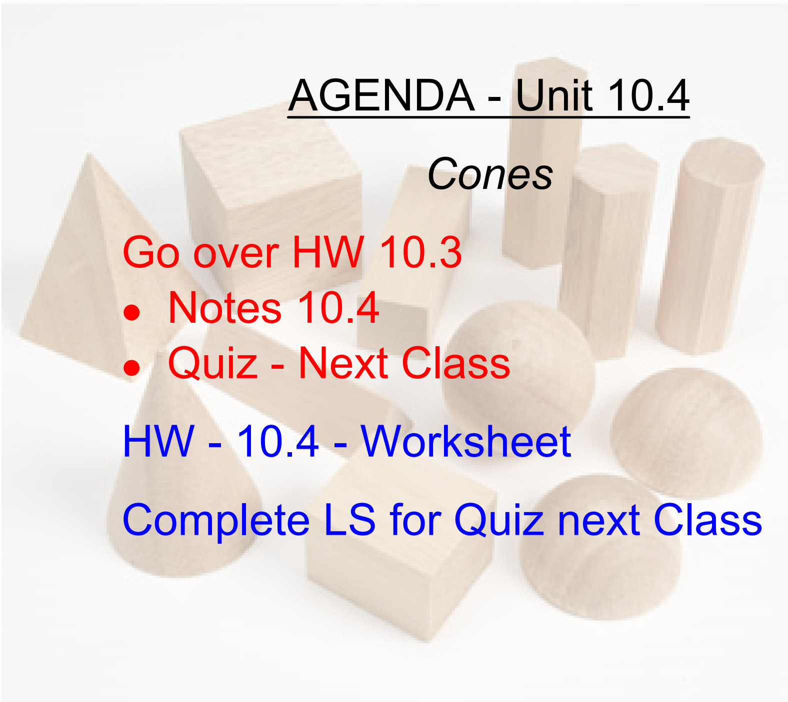


AGENDA - Unit 10.4

Cones

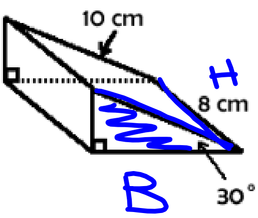
Go over HW 10.3
- Notes 10.4
- Quiz - Next Class

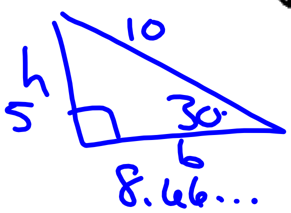
HW - 10.4 - Worksheet

Complete LS for Quiz next Class

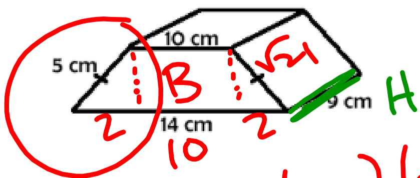
7. Determine the volume of each of the following prisms to the nearest tenth. First solve for the missing dimension.

a. Right Triangular Prism





b. Right Trapezoidal Prism

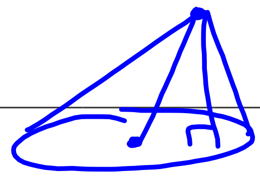


$$B = \frac{(b_1 + b_2)h}{2}$$

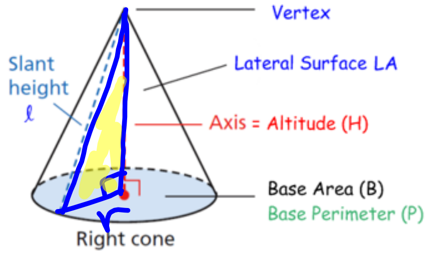
$$\frac{(10+14)(\sqrt{21})}{2}$$

Geometry LAB Name ____________ Section __________ Date________

10-4 Notes: Cones

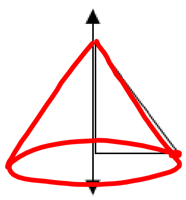


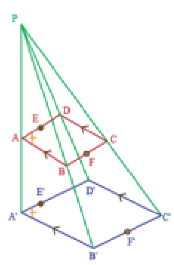
ADDITIONAL VOCABULARY

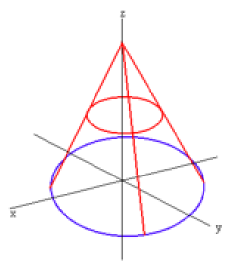


- **Axis:** The segment connecting the vertex of a cone to the center of its base. If the cone is right, the axis is COINCIDENT with the altitude.
- **Slant Height (ℓ):** The segment connecting the vertex of a cone to the edge of its base.
  - This segment is on the LATERAL face of the cone.
  - The segments forming the right triangle are the ALTITUDE (leg), a RADIUS (leg), and the SLANT HEIGHT (hypotenuse).

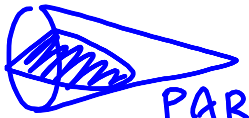
Recall: Rotating a planar figure around an axis/line and dilating a circle from a point







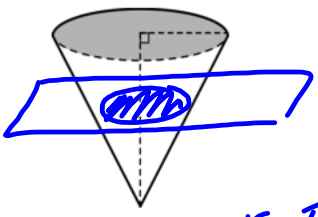
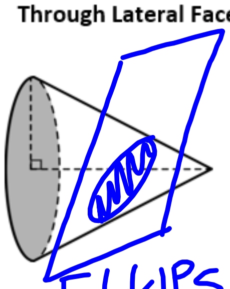
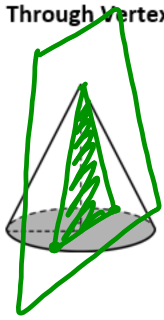
https://www.geogebra.org/m/UMHXAvhb



PARABOLA

CROSS SECTIONS AND SLICES

Draw and describe the common planar cross sections and slices made by the intersecting planes:

Parallel to Base	Through Lateral Face	Through Vertex
		
~ CIRCLE TO BASE	ELLIPSE	△

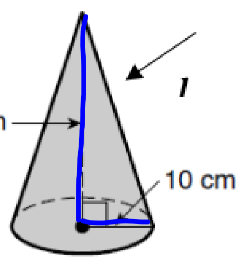
Summary: Common intersecting planes create slices that are CURVED **or** △.

⊙, ELLIPSE, PARABOLA

http://www.shodor.org/interactivate/activities/CrossSectionFlyer/

Finding missing dimensions: Find each dimension.

1. Slant Height



$a^2 + b^2 = c^2 \leftarrow \ell$

$10^2 + 30^2 = c^2$

$100 + 900 = c^2$

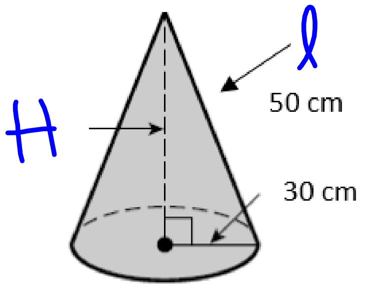
$1000 = c^2$

$\pm\sqrt{1000} = \sqrt{c^2}$

$10\cancel{\sqrt{100}}\sqrt{10} = c$

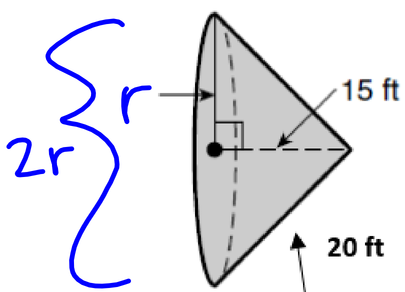
$10\sqrt{10} = \ell$ cm

2. Altitude



$$a^2 + b^2 = c^2$$
$$30^2 + H^2 = 50^2$$
$$900 + H^2 = 2500$$
$$H^2 = 1600$$
$$H = \pm\sqrt{1600}$$
$$H = 40 \text{ cm}$$

3. diameter



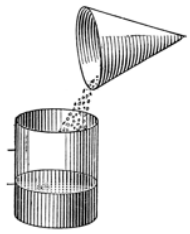
$$a^2 + b^2 = c^2$$
$$15^2 + r^2 = 20^2$$
$$225 + r^2 = 400$$
$$r^2 = 175$$
$$r = \pm\sqrt{175}$$
$$= \sqrt{25}\sqrt{7}$$
$$r = 5\sqrt{7}$$

$$d = 2(5\sqrt{7})$$
$$\boxed{= 10\sqrt{7} \text{ ft}}$$

VOLUME OF A CONE

How does the volume of a cone relate to the volume of a cylinder?

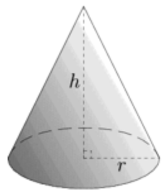
You can pour the sand or the water from the cone to the cylinder and you will find that the relationship is exactly one-third the volume of the cylinder.

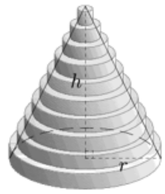


Therefore, the formula for the volume of a cone follows directly from the volume of a cylinder. The cone will always be 1/3 the volume of the cylinder with the same base radius and height.

$$\textbf{VOLUME}_{\text{CONE}} = \frac{1}{3}BH = \frac{1}{3}\pi r^2 H$$

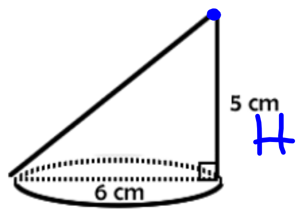
You can also use the same argument about approximately the volumes by stacking cylinders upon each other can compare those to the volume of the cylinder. This limit argument will again display the relationship when the number of layers becomes very large. Thus due to **Cavalieri's Principle,** the volume of a cone will be the same for both right and oblique cones.





PRACTICE:

1) Determine the capacity of the oblique cone, in terms of π.



$$V = \frac{BH}{3}$$

$$= \frac{(9\pi)(5)}{3}$$

$$\boxed{V = 15\pi \text{ cm}^3}$$

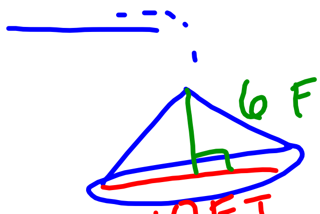
C

$A = \pi r^2 = \pi 3^2 = 9\pi$ B

$R = 3$

$\boxed{D = 6 \text{ cm}}$

2) Sand falls from a conveyor belt and forms a pile on a flat surface. The diameter of the pile is approximately 10 feet, and the height is approximately 6 feet. Estimate the volume of the pile of sand. State your assumptions used in the modeling. RT CONE



$V = \frac{BH}{3} = \frac{(25\pi)6}{3}$

$\boxed{V = 50\pi \ FT^3}$

C

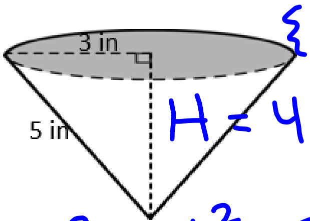
$A = \pi r^2 = \pi 5^2 = 25\pi \ FT^2$

$R = 5$

$\boxed{D = 10}$

3) How could this be harder? Solving for a missing dimension is a little more complicated, which usually involves a right triangle. Find the volume of the cone to the nearest tenth of a cubic centimeter.

a. Given the slant height



{3-4-5}

$H = 4 \ IN$

$3^2 + H^2 = 5^2$

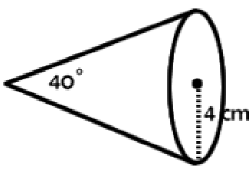
C

$A = \pi r^2 = \pi 3^2 = 9\pi \ IN^2$

$\boxed{R = 3 \ IN}$

D

b. Given the vertex angle



$V = \frac{BH}{3} = \frac{(9\pi)(4)}{3} \ IN^3$

$= \frac{36\pi}{3} \ IN^3$

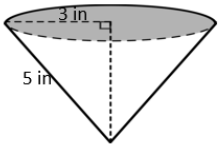
$= 113.0973 \div 3$

$= 37.6991\ldots$

$\boxed{V = 37.7 \ IN^3}$

3) How could this be harder? Solving for a missing dimension is a little more complicated, which usually involves a right triangle. Find the volume of the cone to the nearest tenth of a cubic centimeter.

a. Given the slant height



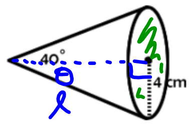
$V = \frac{BH}{3}$

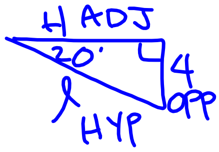
$V = \frac{16\pi(10.9899)}{3}$

$= 184.1375$

$\boxed{= 184.1 \text{ IN}^3}$

b. Given the vertex angle





$\sin 20° = \frac{4}{\ell}$

$\tan 20° = \frac{4}{H}$

$\frac{H(\tan 20°)}{\tan 20°} = \frac{4}{\tan 20°}$

$H = \frac{4}{\tan 20°}$

$10.9899\ldots$

C

$A = \pi r^2 = \pi 4^2 = 16\pi$ B

$\boxed{R = 4}$

D

4) Backsolving: If the volume of a right cone is 320π ft^3 with an altitude of 5 ft, what is the radius of the cone?

$V = \frac{BH}{3}$ $\quad V = \frac{\pi r^2 H}{3}$

$320\pi \text{ FT}^3 = \frac{\pi r^2 \, 5 \text{ FT}}{3}$

$320(3)\pi = 5\pi r^2$

$\frac{960\cancel{\pi}}{5\cancel{\pi}} = \frac{\cancel{5\pi} r^2}{\cancel{5\pi}}$

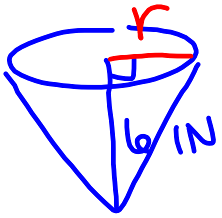
$192 = r^2$

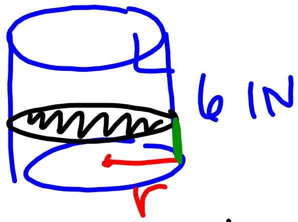
$\pm\sqrt{192} = r$

$\sqrt{64}\sqrt{3} = r$

$\boxed{r = 8\sqrt{3} \text{ FT}}$

5) Suppose you fill a conical paper cup with a height of 6" with water. If all the water is then poured into a cylindrical plastic cup with the same radius and same height as the conical paper cup, to what height will the water reach in the cylindrical plastic cup?





$V = \frac{BH}{3}$

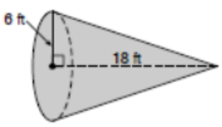
$V = BH$

VOL IN CYL WOULD $\frac{1}{3}$

$\frac{1}{3}(6 \text{ IN}) = \boxed{2 \text{ IN}}$

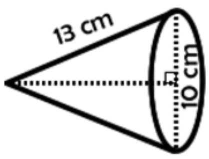
WORKSHEET 10-4 LAB Name ______________________ Due ________ Section________

1. **Find the slant height to the nearest tent of a ft. :**

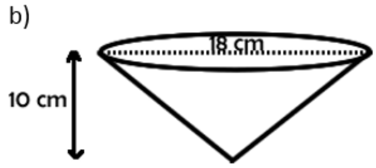


2. Determine the volume of the cone in terms of π.

a)



b)



c)