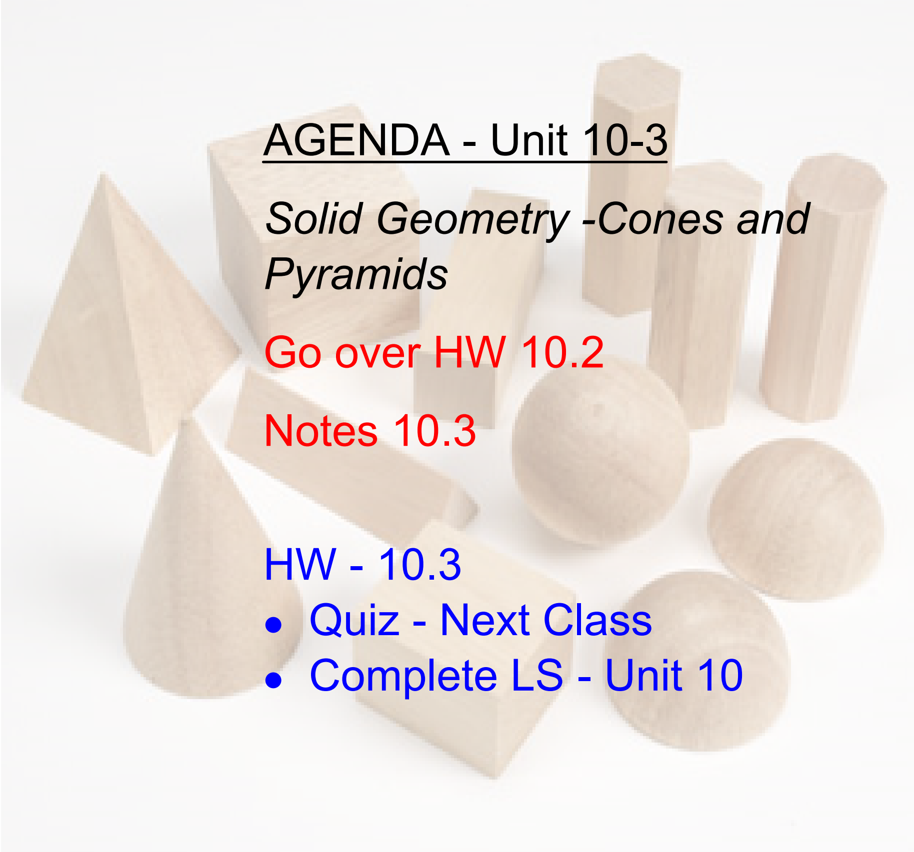


AGENDA - Unit 10-3

Solid Geometry -Cones and Pyramids

Go over HW 10.2

Notes 10.3

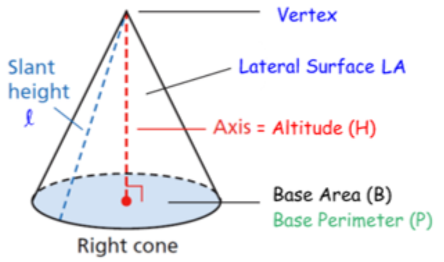
HW - 10.3

- Quiz - Next Class
- Complete LS - Unit 10

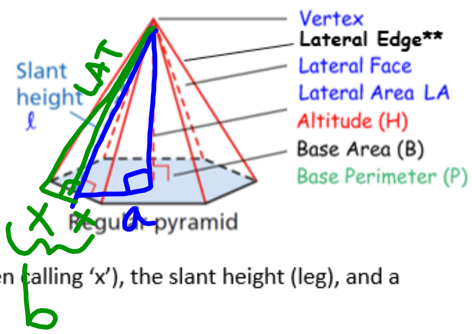
Geometry Name ______________________ Section __________ Date________

10-3R Notes: Cones & Pyramids

ADDITIONAL VOCABULARY



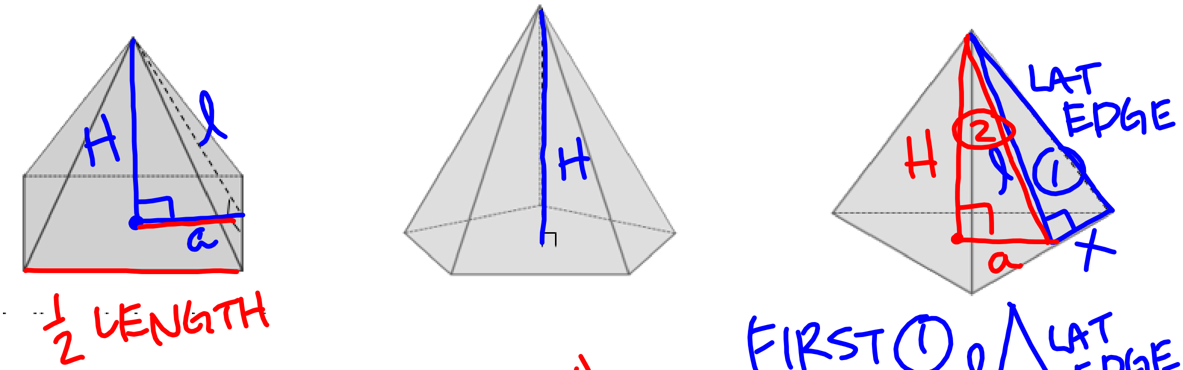
- **Axis:** The segment connecting the vertex of a cone to the center of its base. If the cone or pyramid is right, the axis is COINCIDENT with the altitude.
- **Apex:** The vertex of a pyramid where all the lateral edges concur.
- **Slant Height (ℓ):** The segment connecting the vertex of a cone or pyramid to the edge of its base. (For cone, it's the circumference).
  - This segment is on the LATERAL face of the cone.
  - The segments forming the right triangle for a cone are the ALTITUDE (leg), a RADIUS (leg), and the SLANT HEIGHT (hypotenuse).
  - The segments forming the right triangle for a right pyramid are either
    - (1) the altitude (leg), an APOTHEM (leg), and the slant height (hypotenuse) (INTERIOR Δ)
    - (2) half of the b base edge (leg – what we've been calling 'x'), the slant height (leg), and a LATERAL EDGE (hypotenuse).

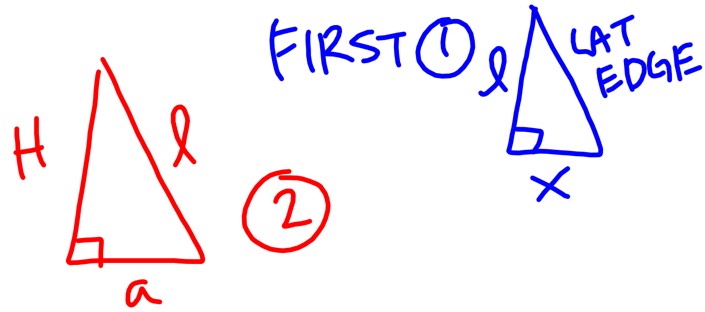


SLANT HEIGHT (Lateral Face) VS. ALTITUDE/AXIS (Solid)

Draw in the essential right triangle in each of the solid that would help you solve for a missing dimension. GOAL: H

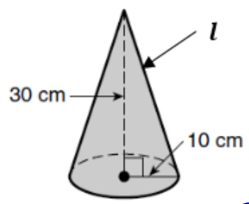
Given Slant Height | Given Altitude | Given Lateral Edge





Finding missing dimensions:

1. **Slant Height**



$a^2+b^2=c^2$

$30^2+10^2=l^2$

$900+100=l^2$

$1000=l^2$

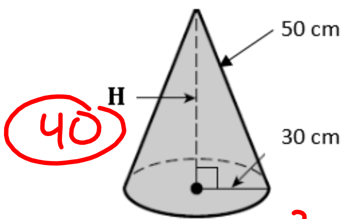
$\pm\sqrt{1000}=l$

$\sqrt{100}\sqrt{10}=l$

$10\sqrt{10}=l$

cm

2. Altitude

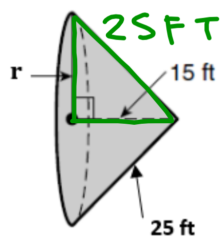


{3-4-5}

K=10

$30^2+H^2=50^2$

3. Diameter



{3-4-5}

{15-r-25}

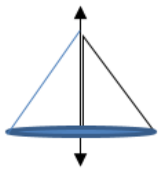
K=5 K=5

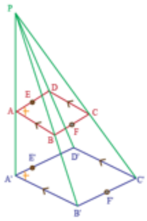
4(5)=r

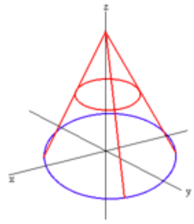
20 FT

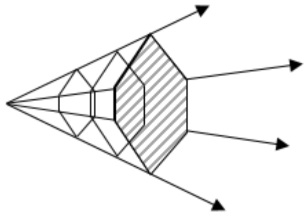
FORMATION:

Cones and pyramids can both be formed by DILATING a circle or a polygon from a point or STACKING similar circles or polygons. Also, a cone may be formed by ROTATING a planar figure around an axis/line.



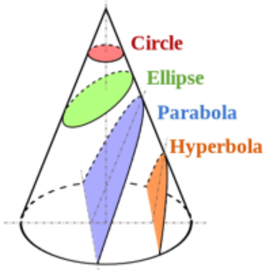






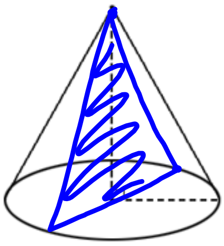
CROSS SECTIONS AND SLICES

Cone:



Note: circle will be similar to the base when the intersecting plane/slice is PARALLEL to the base.

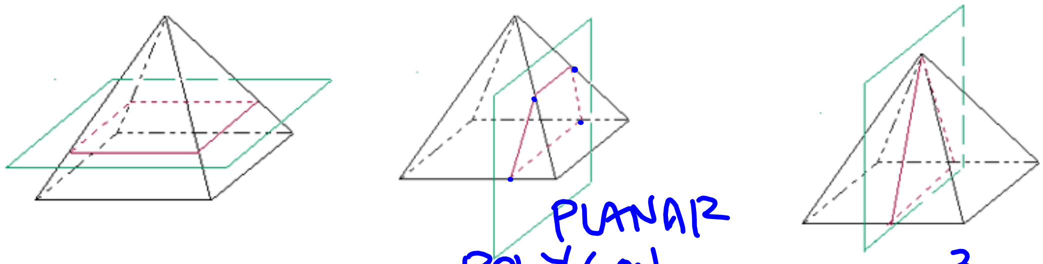
A slice through the vertex and the base will be a △.



http://www.shodor.org/interactivate/activities/CrossSectionFlyer/

Pyramid:

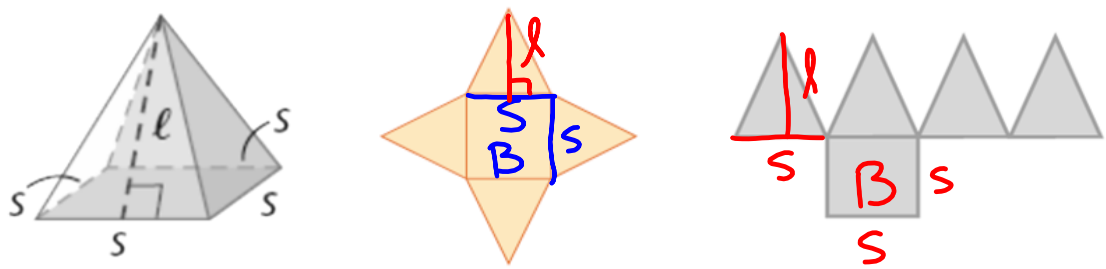
Through Lateral Faces — Through Apex ⟷ Through Lateral Face & Base



Summary: The slice/cross section will be a PLANAR POLYGON with vertices ranging from 3 to the number of sides of the BASE. Slices/cross sections will never be CURVED/CONICAL. If parallel to base, then the slice will be ~ to the base.

http://www.shodor.org/interactivate/activities/CrossSectionFlyer/

USING A NET TO FIND SURFACE AREA OF A PYRAMID

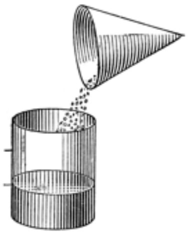


1B + 4 LAT FACE Δ's

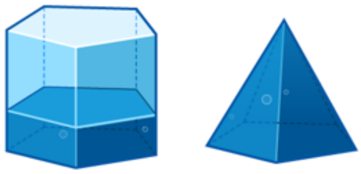
VOLUME OF A CONE OR PYRAMID

How does the volume of a cone relate to the volume of a cylinder or a pyramid relate to a prism?

You can pour the sand or the water from the cone to the cylinder and you will find that the relationship is exactly one-third the volume of the cylinder.



Therefore, the formula for the volume of a cone follows directly from the volume of a cylinder. The cone will always be 1/3 the volume of the cylinder with the same base radius and height. The same is true with a pyramid and prism with the same base polygon.

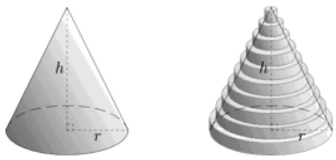


$$\text{VOLUME}_{\text{CONE}} = \frac{1}{3}BH = \frac{1}{3}\pi r^2 H$$

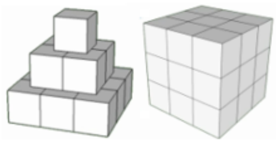
$$\text{VOLUME}_{\text{PYRAMID}} = \frac{1}{3}BH$$

Where B = AREA OF BASE

and H = ALTITUDE OF SOLID

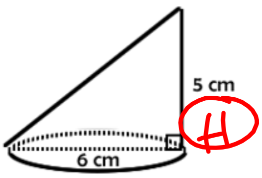


You can also use the same argument about approximating the volumes by stacking cylinders or cubes upon each other can compare those to the volume of the cylinder or prism. This limit argument will again display the relationship when the number of layers becomes very large and the sides become less bumpy. Thus due to **Cavalieri's Principle,** the volume of a cone will be the same for both right and oblique cones.



PRACTICE:

1) Determine the capacity of the oblique cone, in terms of π.



$$V = \frac{BH}{3} = \frac{(9\pi)(5)}{3}\ cm^3$$

$$\boxed{V = 15\pi\ cm^3}$$

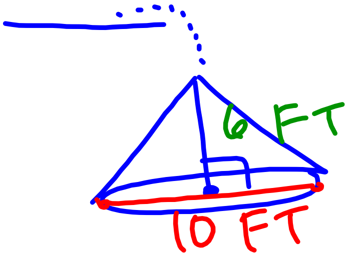
C

$A = \pi r^2 = \pi 3^2 = 9\pi\ cm^2$

$r = 3\ cm$ (B)

$\boxed{D = 6\ cm}$

2) Sand falls from a conveyor belt and forms a pile on a flat surface. The diameter of the pile is approximately 10 feet, and the height is approximately 6 feet. Estimate the volume of the pile of sand. State your assumptions used in the modeling. → RT CONE



$V = \frac{BH}{3} = \frac{(25\pi)(6)}{3}\ FT^3$

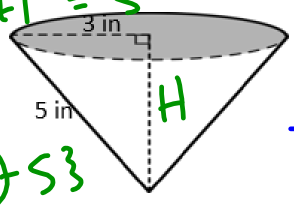
$V = 50\pi\ FT^3$

C
$A = \pi r^2 = \pi 5^2 = 25\pi\ FT^2$ (B)
$R = 5\ FT$
$D = 10\ FT$

3) How could this be harder? Solving for a missing dimension is a little more complicated, which usually involves a right triangle. Find the volume of the cone to the nearest tenth of a cubic centimeter.

a. Given the slant height

$3^2 + H^2 = 5^2$



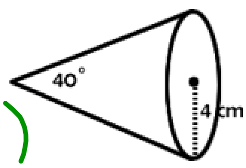
{3 (4) 5}

$V = \frac{BH}{3} = \frac{(9\pi)(4)}{3} = 12\pi\ IN^3 = 37.6991\ IN^3$

$V = 37.7\ IN^3$

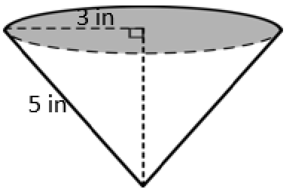
C
$A = \pi r^2 = \pi 3^2 = 9\pi\ IN^2$ (B)
$R = 3\ IN$
D

b. Given the vertex angle

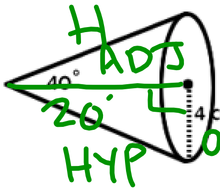


3) How could this be harder? Solving for a missing dimension is a little more complicated, which usually involves a right triangle. Find the volume of the cone to the nearest tenth of a cubic centimeter.

a. Given the slant height



b. Given the vertex angle



C

$A = \pi r^2 = \pi 4^2 = 16\pi$ cm^2

R = 4 cm

D

(B)

$\tan 20° = \frac{4}{H}$

$H = \frac{4}{\tan 20°}$

$= 10.9899$ cm

$V = \frac{BH}{3}$

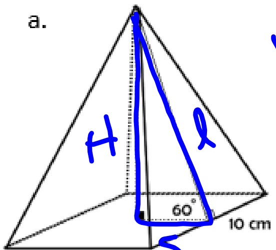
$= \frac{(16\pi)(10.9899)}{3}$

$= 184.1375$

$= 184.1$ cm^3

4) Find the volume of the right square pyramids to the nearest tenth of a cubic centimeter.

a.



30° 60° 90°

n n√3 2n

5 H

$H = 5\sqrt{3}$

B 10

10

$B = 10 \cdot 10 = 100$

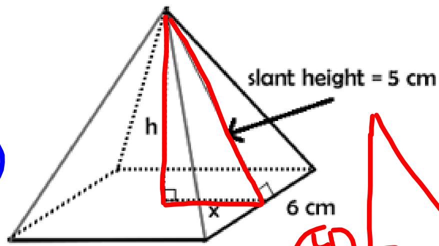
$V = \frac{BH}{3}$ cm^3

$= \frac{(100)(5\sqrt{3})}{3}$

$= 288.6751$

$= 288.7$ cm^3

b.



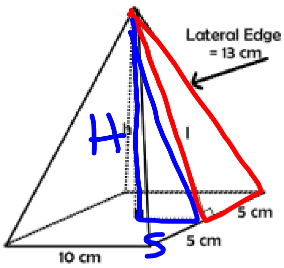
{3-4-5} 3

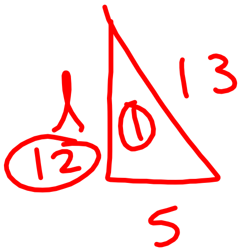
$V = \frac{BH}{3}$ cm^3

$= \frac{(36)(4)}{3}$

$= 48.0$ cm^3

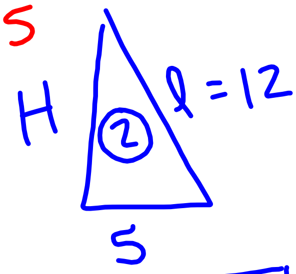
5) Find the altitude of the right square pyramid to the nearest tenth.





{5-12-13}

$5^2 + l^2 = 13^2$



$5^2 + H^2 = 12^2$

$25 + H^2 = 144$

$H^2 = 119$

$H = \sqrt{119}$

10.9087

$H \approx 10.9$ cm