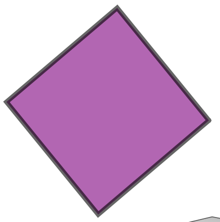
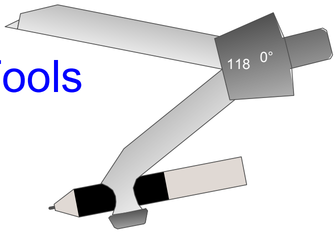
AGENDA - Unit 9 -5

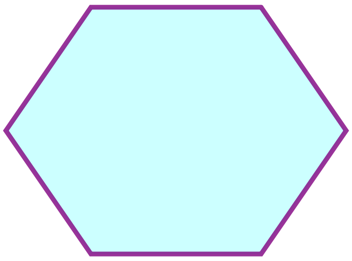
Constructions and Area of Regular Polygons

Go over HW 9.4
- Notes 9.5
- **Need Construction Tools**





HW - 9.5
- Worksheet 9.5- Need Construction Tools
- CR #8 - Due 4/7



GEOMETRY + LAB **Name:** ______________________________ **Section:** __________ **Date:** _______

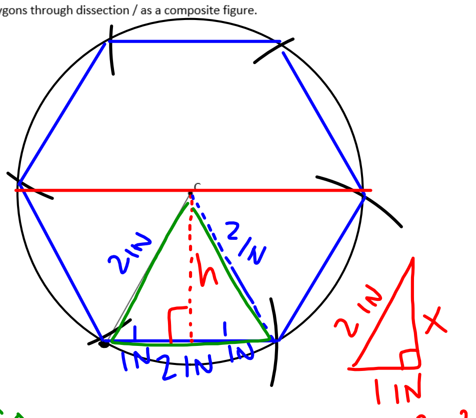
9-4R & 9-5L Notes: Regular Polygon as a Composite Figure; Construction of Inscribed Regular Polygons

In previous lessons, we found the area of non-rectangular figures and composite figures by using dissection and simple area formulas. Let's look at regular polygons through dissection / as a composite figure.

Discovery: Area of a Regular Polygon

A. **Construct a regular hexagon inscribed in a circle centered at A with the given radius.** Place the sharp point at C and measure the span of the radius. Draw a point on the circle. From that point, swipe an arc the span of the radius to locate the next point on the circle. Repeat until you have marked 6 points. Connect these points as the vertices of the hexagon.

B. **Find the area of the regular hexagon (use in^2) – either exact or nearest tenth.** Consider what you know about regular polygons, central angles and composite areas as discovered in the bridge, right triangle side ratios from Unit 8, and now areas of planar figures and composite figures.
   - How did you compute it?



$A_{HEX} = 6A_\Delta$

$A_\Delta = \frac{bh}{2} = \frac{(2)(\sqrt{3})}{2}$

$1^2 + x^2 = 2^2$

$x^2 = 3$

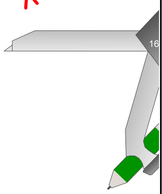
$x = \pm\sqrt{3}$

$A_\Delta = 1.7321 \text{ IN}^2$

$A_{HEX} = 6A_\Delta = 6(1.7321)$

$= 10.3923\ldots$

$\boxed{A_{HEX} = 10.4 \text{ IN}^2}$

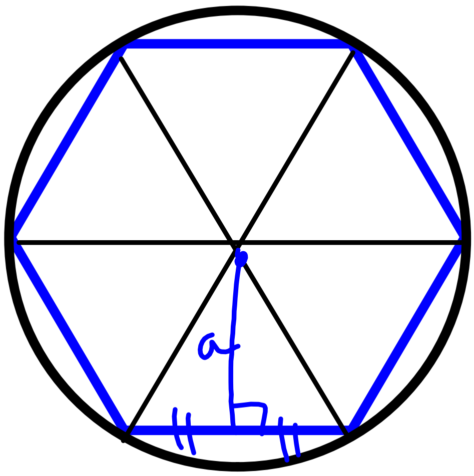


B. Find the area of the regular hexagon (use in²) – either exact or nearest tenth.
Consider what you know about regular polygons, central angles and composite areas as discovered in the bridge, right triangle side ratios from Unit 8, and now areas of planar figures and composite figures.

- How did you compute it?

- What significant segment did you use to find the area? HEIGHT OF ISOS △

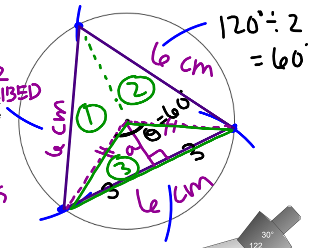
Definition of an apothem: the segment from the CENTER of a regular polygon that is the ⊥ BISECTOR of a side of the polygon.

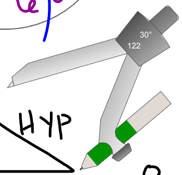


$CA = \frac{360^\circ}{n} = \frac{360^\circ}{3} = 120^\circ$

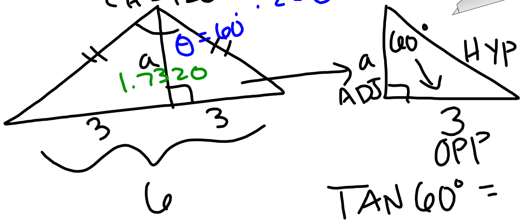
Equilateral Triangles

- **Construct a regular triangle inscribed in the circle.**
- What kind of triangle is it? EQUILATERAL
- How did you modify the inscribed hexagon construction? CONNECTED EVERY OTHER
- What is the center of this circle called? CIRCUMSCRIBED because it is equidistant to the VERTICES, and the circle is CIRCUMSCRIBED around the triangle.
- **Construct the apothem.**
  Is the altitude of the equilateral triangle the same as the apothem? NO Why not? COME FROM DIFFERENT POINTS
  Is it double the length of the apothem? NO
- **Compute the length in simplest radical form of the apothem if the side length of the triangle is 6 cm.**





$CA = 120^\circ \div 2 = \theta$



T $\frac{O}{A}$

$\tan 60^\circ = \frac{3}{a}$

$\frac{a(\cancel{\tan 60^\circ})}{\cancel{\tan 60^\circ}} = \frac{3}{\tan 60^\circ}$

$a = \frac{3}{\tan 60^\circ}$

$a = 1.7320$

$\text{AREA}_{EQ\Delta} = 3A_\Delta$

$A_\Delta = \frac{bh}{2}$

$= \frac{(6)(1.7320)}{2}$

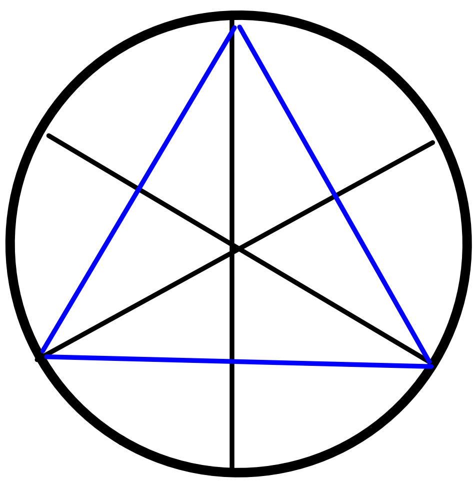
$= 5.1960$

$A_{EQ\Delta} = 3(5.1960)$

$= 15.5880$

$\boxed{\approx 15.6 \text{ cm}^2}$

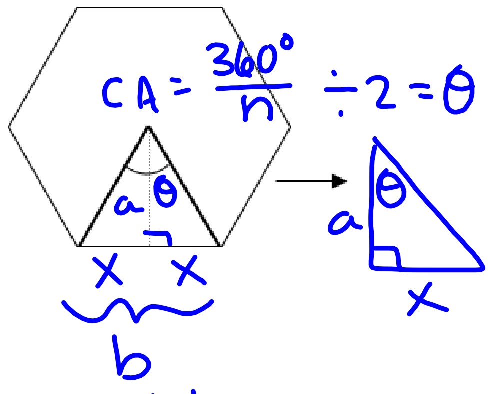
- **Compute the area of the triangle using a composite/dissection approach. Leave your answer in simplest radical form or round to the nearest tenth.**



To find the area of a regular polygon:

1. Draw in the isosceles triangle.
2. Find the central angle and divide by 2.
3. Pull out, draw, and label the right triangle:
   -Angle measure, apothem a, base x.
4. Find missing value: apothem a and/or base piece x.
5. Compute the area of the either the right triangle or the isosceles triangle.
6. Multiply by the number of triangles needed.
7. Round at the end or simplify any radicals.

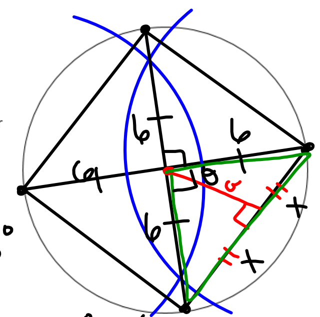
$CA = \frac{360^\circ}{n} \div 2 = \theta$



$A = \frac{bh}{2}$ ISOS Δ

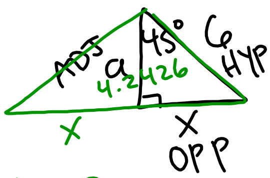
SQUARES

- **Construct a square inscribed in the circle.** ≅ ⊥ DIAG BIS EACH OTHER
- Draw in the apothem. Classify the triangles created:
  ISOS RT
- Compute the length of the apothem if the length of the diameter is 12 in using trigonometry. Leave in terms of a trig ratio.



$CA = \frac{360^\circ}{4} = 90^\circ$

$\theta = 90^\circ \div 2 = 45^\circ$



$a: \cos 45^\circ = \frac{x}{6}$

$6(\cos 45^\circ) = x$

$4.2426 = x$

$b = 2(4.2426)$
$= 8.4852$

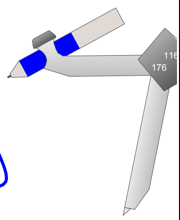
$A_{ISOS\,\Delta} = \frac{bh}{2} = \frac{(8.4852)(4.2426)}{2}$

$= \frac{35.9993}{2} = 17.9996$

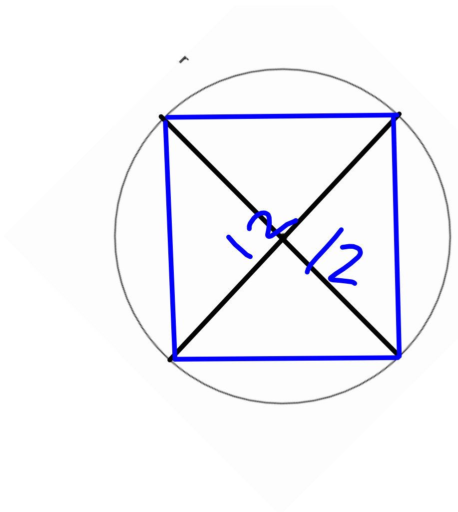
$A_{SQ} = 4A_{\Delta} = 4(17.9996)$

$= 71.9984$

$\boxed{72.0\ \text{IN}^2}$



- Compute the area of the square, rounding to the nearest hundredth.



$\frac{12 \cdot 12}{2} = 72$

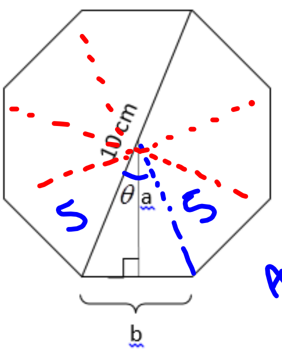
$\frac{d_1 d_2}{2} = A$

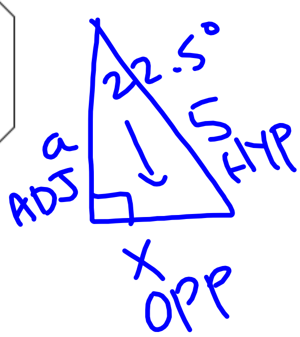
Practice

Find the missing dimension(s) in each of the regular polygons (this sets you up to find the area later). Remember that you need the Central Angle, θ, apothem a, base piece x, base of the isosceles triangle b = side length of polygon.

Octagon (given diagonal)

$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

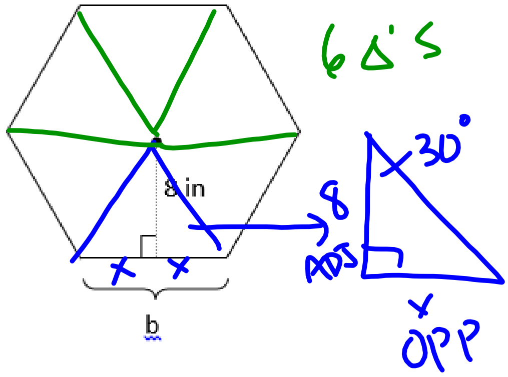




ISOS Δ'S = 8

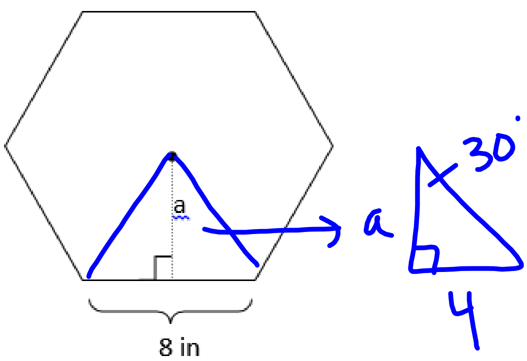
Central Angle	$\frac{360^\circ}{8} = 45^\circ$
θ	$\frac{CA}{2} = 45^\circ \div 2 = 22.5^\circ$
a	$\cos 22.5^\circ = \frac{a}{5}$ $5(\cos 22.5^\circ) = a$ 4.6194
x	$\sin 22.5^\circ = \frac{x}{5}$ $5(\sin 22.5^\circ) = x$ 1.9134
b = side length	$2x = 2(1.9134)$ $= 3.8268$

Hexagon (given apothem)



Central Angle	$\frac{360^\circ}{6} = 60^\circ$
θ	$60^\circ \div 2 = 30^\circ$
a	8 IN
x	$\tan 30^\circ = \frac{x}{8}$ $8(\tan 30^\circ) = x$
b = side length	$2(\quad) = b$

Hexagon (given side length)



Central Angle	
θ	
a	$\tan 30^\circ = \frac{4}{a}$ $a = \frac{4}{\tan 30^\circ}$
x	4 IN
b = side length	8 IN