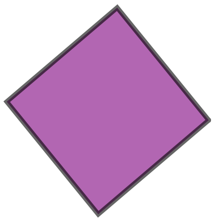
AGENDA - Unit 9 -5

Constructions and Area of Regular Polygons

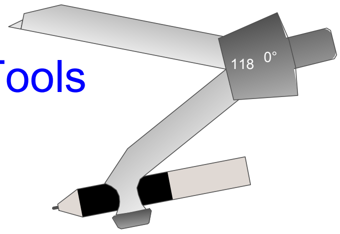
Go over HW 9.4

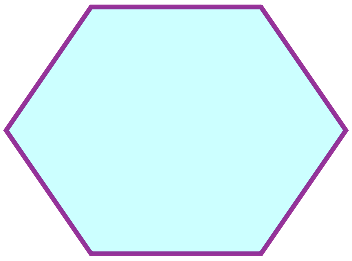
- Notes 9.5
- **Need Construction Tools**



HW - 9.5

- Worksheet 9.5- Need Construction Tools
- CR #8 - Due 4/12





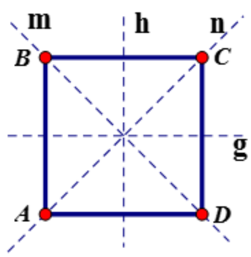
Worksheet 9-3R & 9-4L

Assume all rotations are centered at the center of the polygon.

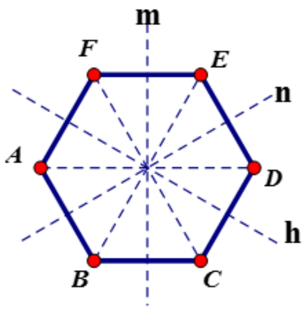
1. What is the minimum number of degrees to map a regular 16-gon onto itself?

2. Name the regular polygon that will map onto itself under a minimum rotation of 90°.

3. Given square ABCD, under a rotation of how many degrees will point C map to point D?



4. Given regular hexagon ABCDEF with lines m, n, and h through midpoints of the sides,
   a. Determine the number of degrees to rotate the regular hexagon such that E maps to A.

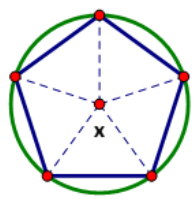


   b. Identify three separate precise transformations that map F to C under a
      i. Point Reflection: ______________________________
      ii. Rotation: ______________________________
      line reflection iii. Rotation: ______________________________

   c. Identify two separate precise transformations that map F to B under a
      i. Line Reflection: ______________________________
      ii. Rotation: ______________________________

5. A rotation of 180° will carry all the following regular polygons onto themselves except:
   a. Square
   b. Octagon
   c. Nonagon
   d. Decagon

6. Which of the following angle measures would rotate a regular pentagon onto itself?
   a. 36°
   b. 90°
   c. 144°
   d. 180°



GEOMETRY + LAB **Name:** ____________________ **Section:** ________ **Date:** ______

9-4R & 9-5L Notes: Regular Polygon as a Composite Figure; Construction of Inscribed Regular Polygons

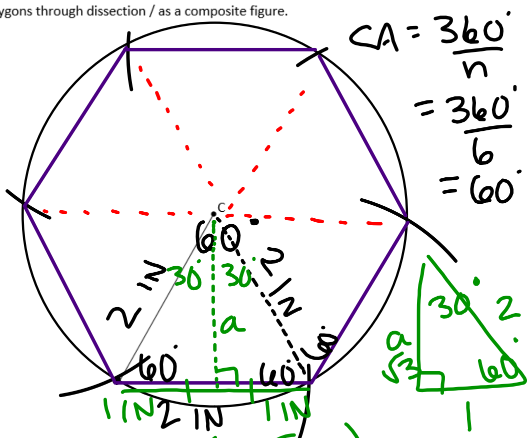
In previous lessons, we found the area of non-rectangular figures and composite figures by using dissection and simple area formulas. Let's look at regular polygons through dissection / as a composite figure.

Discovery: Area of a Regular Polygon

A. **Construct a regular hexagon inscribed in a circle centered at A with the given radius.** Place the sharp point at C and measure the span of the radius. Draw a point on the circle. From that point, swipe an arc the span of the radius to locat[illegible] point on the circle. Repeat [illegible] have m[illegible]ts as the vertices of the hexagon.

B. **Find the area of the regul[illegible] hexago[illegible] (use in²) – either exact or nea[illegible]ten[illegible]** Consider what you know [illegible]ular polygons, central angles [illegible]mposite areas as discovered in t[illegible]e, right triangle side ratios fro[illegible], and now areas of planar figures and composite figures.
- How did you compute it?

$CA = \frac{360°}{n} = \frac{360°}{6} = 60°$



$A_{\Delta} = \frac{bh}{2} = \frac{(2\ IN)(\sqrt{3}\ IN)}{2}$

$= \sqrt{3}\ IN^2$

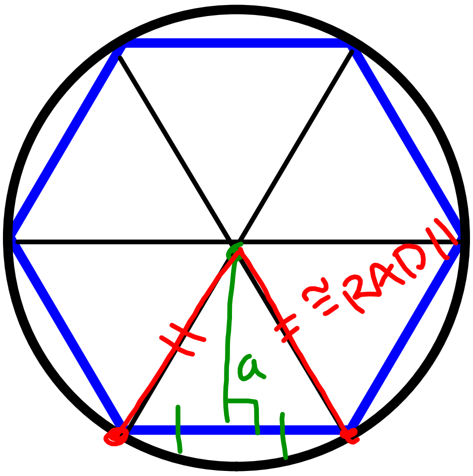
$A_{HEXAGON} = 6(A_{\Delta})$

$\boxed{A_{HEX} = 6\sqrt{3}\ IN^2}$

B. **Find the area of the regular hexagon (use in²) – either exact or nearest tenth.**
Consider what you know about regular polygons, central angles and composite areas as discovered in the bridge, right triangle side ratios from Unit 8, and now areas of planar figures and composite figures.
- How did you compute it?

- What significant segment did you use to find the area?

Definition of an apothem: the segment from the CENTER of a regular polygon that is the ⊥ BISECTOR of a side of the polygon.

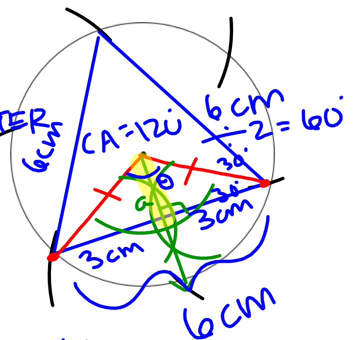


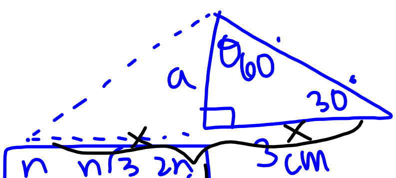
__Equilateral Triangles__

- **Construct a regular triangle inscribed in the circle.**
- What kind of triangle is it? EQUILATERAL
- How did you modify the inscribed hexagon construction? CONNECTED EVERY OTHER
- What is the center of this circle called? CIRCUMCENTER because it is equidistant to the VERTICES, and the circle is CIRUMSCRIBED around the triangle.
- **Construct the apothem.**

  Is the altitude of the equilateral triangle the same as the apothem? NO Why not? COME FROM DIFFERENT POINTS

  Is it double the length of the apothem? NO
- **Compute the length in simplest radical form of the apothem if the side length of the triangle is 6 cm.**





n	$n\sqrt{3}$	2n
30°	60°	90°
a	3	6

$a = n$

$a = \sqrt{3}$

$n\sqrt{3} = 3$

$n = \frac{3}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$= \frac{3\sqrt{3}}{3} = \sqrt{3}$

$n = \sqrt{3}$

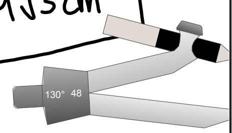
$A_{\Delta\ ISOS} = \frac{bh}{2} = \frac{(6)(\sqrt{3})}{2}$

$A_{ISOS\ \Delta} = 3\sqrt{3}\ cm^2$

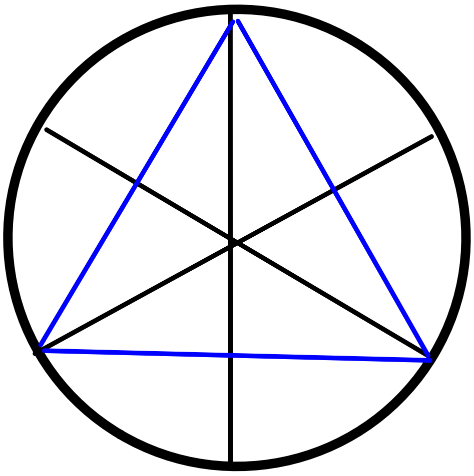
$A_{EQUIL\ \Delta} = 3(A_{ISOS\ \Delta})$

$= 3(3\sqrt{3})$

$\boxed{A_{EQUIL\ \Delta} = 9\sqrt{3}\ cm^2}$



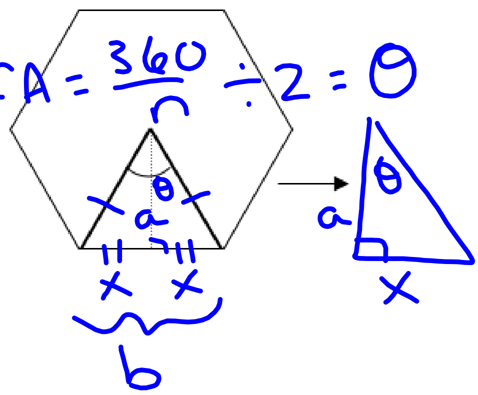
- **Compute the area of the triangle using a composite/dissection approach. Leave your answer in simplest radical form or round to the nearest tenth.**



To find the area of a regular polygon:

1. Draw in the isosceles triangle.
2. Find the central angle and divide by 2.
3. Pull out, draw, and label the right triangle:
   -Angle measure, apothem a, base x.
4. Find missing value: apothem a and/or base piece x.
5. Compute the area of the either the right triangle or the isosceles triangle.
6. Multiply by the number of triangles needed.
7. Round at the end or simplify any radicals.

$CA = \frac{360}{n} \div 2 = \theta$

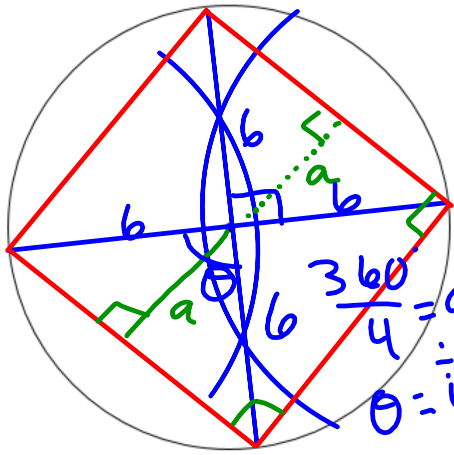


SQUARES

- **Construct a square inscribed in the circle.**
- Draw in the apothem. Classify the triangles created:

ISOS RT △

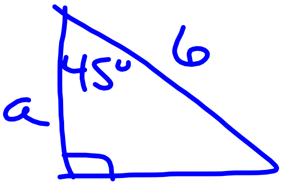
- Compute the length of the apothem if the length of the diameter is 12 in using trigonometry. Leave in terms of a trig ratio.



$\frac{360}{4} = 90°$

$\div 2$

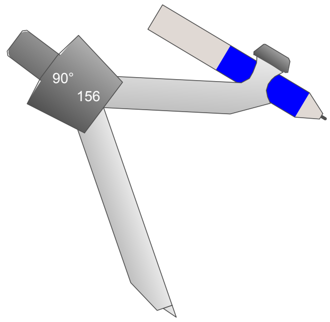
$\theta = 45$



$C\frac{A}{H}$

$\cos 45° = \frac{a}{6}$

$6(\cos 45°) = a$



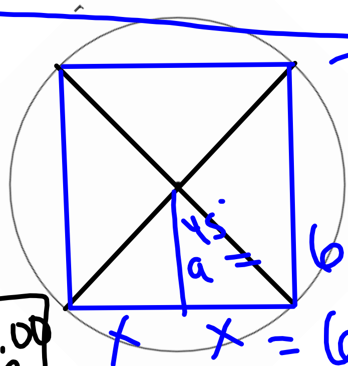
- Compute the area of the square, rounding to the nearest hundredth.

$$A_{\underset{\triangle}{Isos}} = \frac{bh}{2} = \frac{[12(\sin 45°)][6(\cos 45°)]}{2}$$

$$A_{\underset{\triangle}{Isos}} = 18$$

$$A_{SQ} = 4A_{\underset{\triangle}{Isos}}$$

$$= 4((18)) = \boxed{72.00\ IN^2}$$



$6(\cos 45°) = 4.2426$

$x \quad x = 6(\sin 45°)$

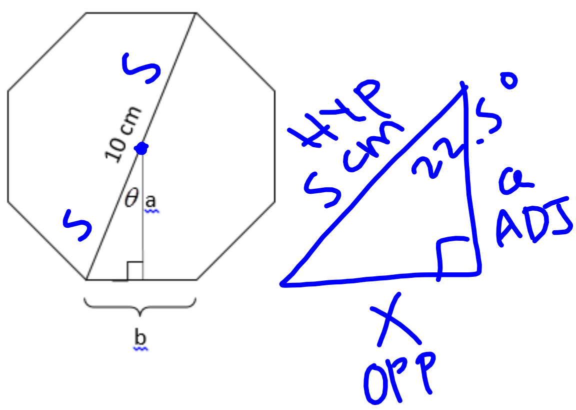
$b = 12\sin 45°$

8.4853

Practice

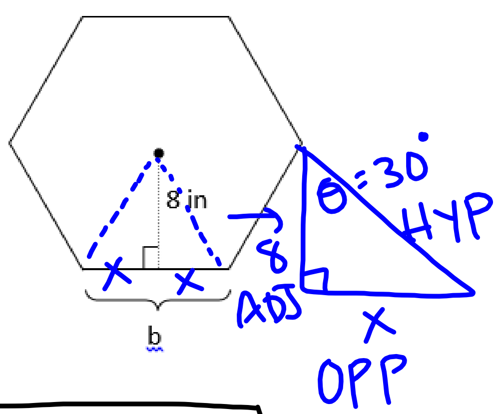
Find the missing dimension(s) in each of the regular polygons (this sets you up to find the area later). Remember that you need the Central Angle, θ, apothem a, base piece x, base of the isosceles triangle b = side length of polygon.

Octagon (given diagonal)



Central Angle	$\frac{360°}{8} = 45°$
θ	$45° \div 2 = 22.5°$
a	$\cos 22.5° = \frac{a}{5}$ $5(\cos 22.5°) = a$
x	$\sin 22.5° = \frac{x}{5}$ $5(\sin 22.5°) = x$
b = side length	$2x = 10(\sin 22.5°)$

Hexagon (given apothem)



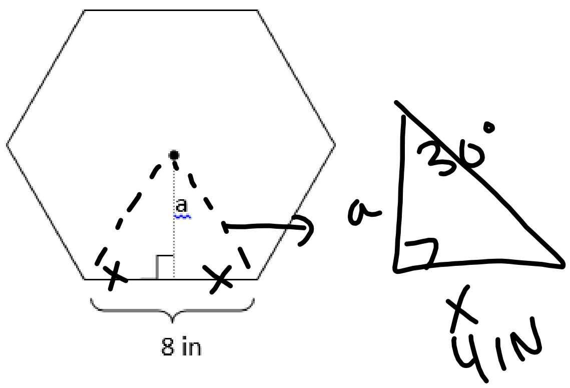
Central Angle	$\frac{360°}{6} = 60°$
θ	$60° \div 2 = 30°$
a	8 IN
x	$\tan 30° = \frac{x}{8}$ $8(\tan 30°) = x$
b = side length	$2x = 16(\tan 30°)$

$$\boxed{\begin{array}{ccc} 30° & 60° & 90° \\ n & n\sqrt{3} & 2n \end{array}}$$

x 8

$n = \frac{8\sqrt{3}}{3} = x$

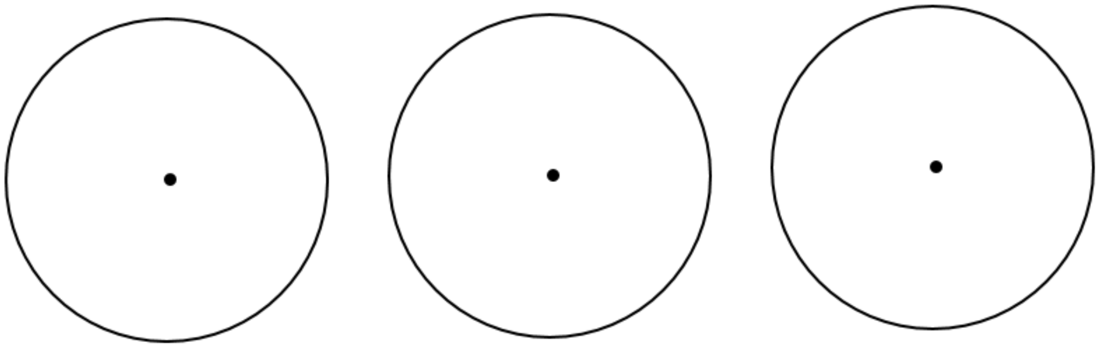
Hexagon (given side length)



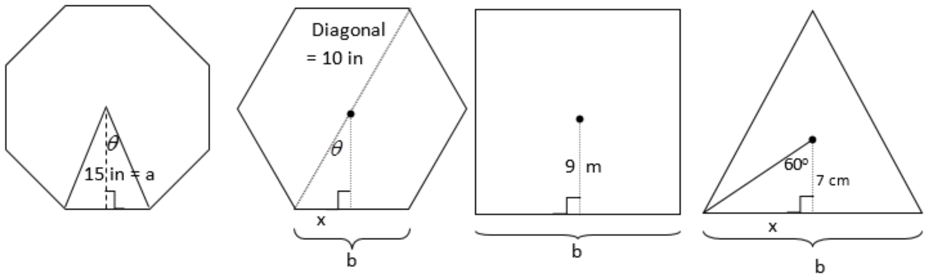
Central Angle	$60°$
θ	$30°$
a	$\tan 30° = \frac{4}{a}$ $a = \frac{4}{\tan 30°}$
x	4 IN
b = side length	8 IN

WORKSHEET 9-4R & 9-5L

1. Construct the following:
   Inscribed Regular Hexagon   Inscribed Square   Inscribed Equilateral Triangle



2. Fill in the chart with all the dimensions for each of the regular polygons using the given information:

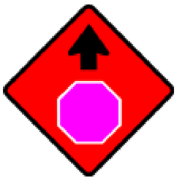


Central Angle				
θ				
a				
x				
b = side length				

3. Look back at both the notes and the worksheet polygons – when was the height of the polygon equal to double the apothem? ____________________

4. A stop sign is a regular octagon which comes in one of two sizes.
   a. Compute the area, to the nearest tenth, of the sign that is 30 inches high (not the diagonal; the height in regular polygon with an even number of sides is equal to twice the apothem length, which is what you hopefully found in #3). USE THE INFORMATION YOU ALREADY PUT IN THE CHART IN #2.

   b. If the entire sign is to be painted with a base coat of red paint, how many gallons of paint will need to be purchased for 50 signs? One gallon of this paint covers 400 square feet.



Challenge: which sign requires less metal to produce – the square sign alerting a "stop ahead" with an apothem of 11 inches or the square bike & pedestrian crossing sign with a diagonal of 30 inches? Express your answer as a percent difference from the sign with a smaller area.