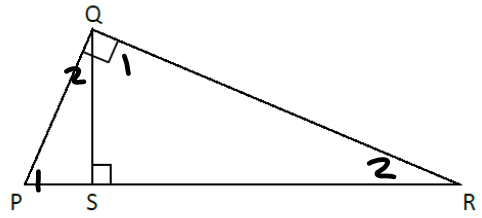
Lesson 8-1: Pythagorean Theorem
Pythagorean Triplets

Algebra Skills:
- ❑ Simplifying Radicals
- ❑ Arithmetic with Radicals
- ❑ FOIL (double distribution)
- ❑ Solving Quadratic Equations
  - ○ by taking square roots
  - ○ by subtraction prop of eq
  - ○ by factoring

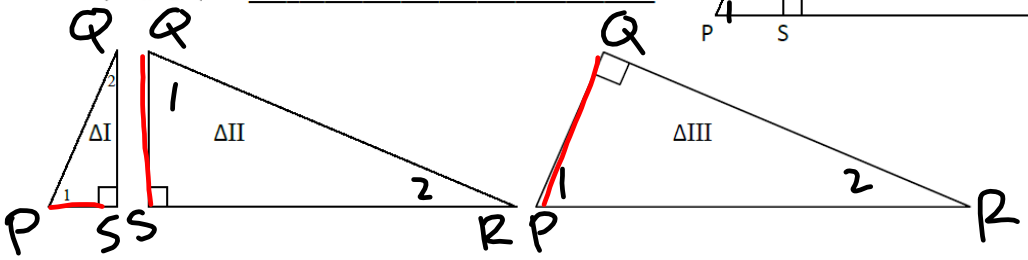
AGENDA:
- Check Bridge to Unit 8 (will review later)
- Notes with Applications and Guided Practice

HOMEWORK:
- Complete Problem Set in Notes
- CR#7 is Due Monday 3/20

Right Triangle Investigation – using similarity and proportions from Unit 7

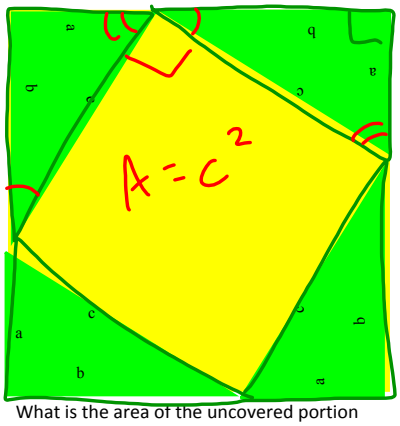


1. Since you know $\measuredangle PQR$ is a right angle, what is the relationship between $\measuredangle P$ and $\measuredangle R$? _____

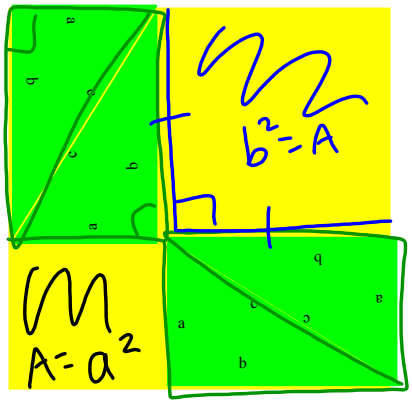


2. Label the vertices of the redrawn triangles ΔPSQ (I), ΔQSR (II), and ΔPQR (III).
3. Label all the acute angles in ΔII and ΔIII that are congruent to $\measuredangle QPS$ as 1's.
4. Label all the acute angles in ΔII and ΔIII that are congruent to $\measuredangle PQS$ as 2's.
5. Color the short sides connecting angles labeled as 1's to right angles RED.

Why does the Pythagorean Theorem work? Prove it!



What is the area of the uncovered portion of the main square?



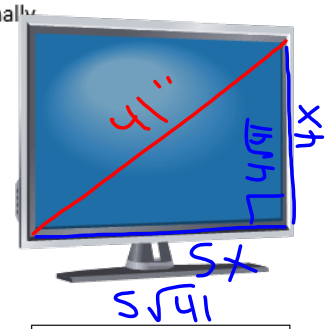
Now what is the area of the uncovered portions of the main square?

Bring it together:
Since you didn't change the area of the four triangles or the overall area of the main square, what does this mean about the uncovered areas you found in part 1 and part 2?

_____.

How does this create an equation that becomes a formula? _____.

TV problem: A customer wants to buy a television set to fit into an entertainment center with an opening 2.5 ft tall and 3 feet wide. The height and width of the television are not printed on the box, but the screen is 41", measured diagonally from edge to edge, and the aspect ratio is 5:4. Will it fit?



$5\sqrt{41}$

$5x : 4x$

$a^2 + b^2 = c^2$

$(4x)^2 + (5x)^2 = 41^2$

$16x^2 + 25x^2 = 41^2$

$\frac{41x^2}{41} = \frac{(41)(41)}{41}$

$x^2 = 41 \rightarrow x = \pm\sqrt{41}$

Algebra Skills:
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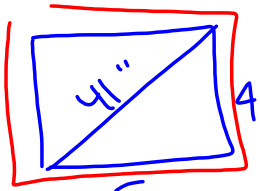


$4\sqrt{41} \approx 25.6124$ IN

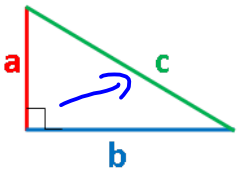
2.5 FT = 30 IN

$5\sqrt{41} \approx 32.0156$ IN

3 FT = 36 IN

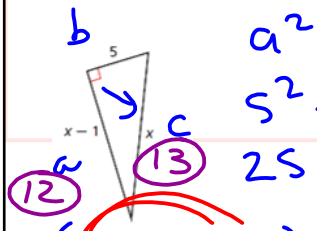
YES

Pythagorean Theorem: In a right triangle, the sum of the squared lengths of the legs is equal to the square of the hypotenuse.



$a^2 + b^2 = c^2$

Example 1: Find the value of the variable and all side the side lengths in each triangle



$a^2+b^2=c^2$

$5^2+(x-1)^2=x^2$

$25+x^2-2x+1=x^2$

$(x-1)(x-1)$

$x^2-1x-1x+1$

x^2-2x+1

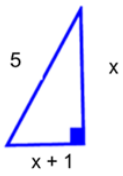
$x^2-2x+26=x^2$

$-x^2 \quad -x^2$

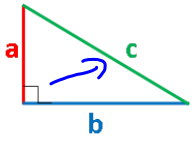
$-2x+26=0$

$26=2x$

$13=x$



Pythagorean Theorem: In a right triangle, the sum of the squared lengths of the legs is equal to the square of the hypotenuse.



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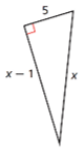
Example 1: Find the value of the variable and all side the side lengths in each triangle



$a^2+b^2=c^2$

$(x+1)^2+x^2=5^2$

$x^2+2x+1+x^2=25$

$2x^2+2x-24=0$

$2(x^2+1x-12)=0$

$\frac{2}{2}$ $\frac{2}{2}$

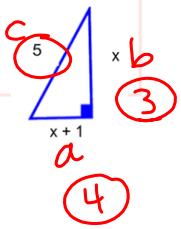
$x^2+1x-12=0$

$(x+4)(x-3)=0$

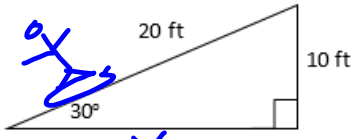
$x+4=0 \quad | \quad x-3=0$

$x=-4 \quad | \quad x=3$

ONE SIDE X



Example 2 (Application): A skateboard ramp is built at a 30° angle such that the ramp is 20 feet long for the rider and has a height of 10 feet. How long, exactly, is the base along the ground?



$a^2 + b^2 = c^2$

$x^2 + 10^2 = 20^2$

$x^2 + 100 = 400$

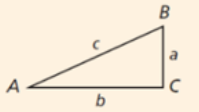
$x^2 = 300$

$x = \pm\sqrt{300}$

$x = \sqrt{100}\sqrt{3}$

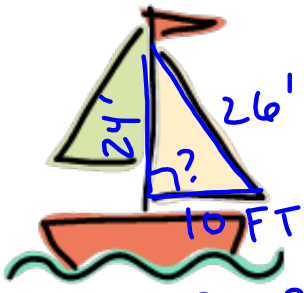
$x = 10\sqrt{3}$ FT

Theorems 5-7-1 Converse of the Pythagorean Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.	 $a^2 + b^2 = c^2$	$\triangle ABC$ is a right triangle.

Example 3: Determining if a triangle is a right triangle

A sail has the dimensions as follows: the base is 10 feet, the height is 24 feet, and the back side (hypotenuse) is 26 feet.



a) Verify that the sail is a right triangle.

$a^2 + b^2 \stackrel{?}{=} c^2$

$10^2 + 24^2 \stackrel{?}{=} 26^2$

$100 + 576 = 676$

$676 = 676$ ✓

SINCE $a^2 + b^2 = c^2$, THEN IT IS A RT $\triangle$.

b) Is this triangle similar to a right triangle with sides 5-12-13? How did you decide?

10-24-26 SIM $\frac{1}{2}$

$k = 2$ DILATION RATIO 2

SSS~

c) What would you guess the hypotenuse is for a right triangle with legs 15 and 36? Why?

5-12-13

15-36-39

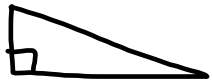
$k = 3$ $k = 3$ $13 \cdot 3 = 39$

PYTHAGOREAN TRIPLES

A set of whole numbers that satisfy the Pythagorean Theorem.

*A Pythagorean triple in which the greatest common factor the numbers share is only 1 is called a primitive triple. Otherwise, look for a scale factor.

{6-8-10}
K=2

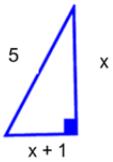


Common Pythagorean Triples
$\{3,4,5\}\{5,12,13\}\{8,15,17\}\{7,24,25\}$

Example 4: Verifying & Identifying Pythagorean Triples

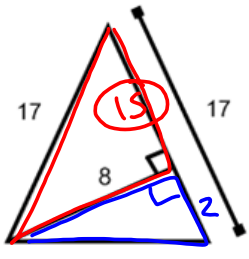
Solve for x and determine the side lengths and perimeter of the largest triangle

a)



{3-4-5}?
x x+1 5
IF X=3, THEN 3+1=4
4=4 ✓

b)



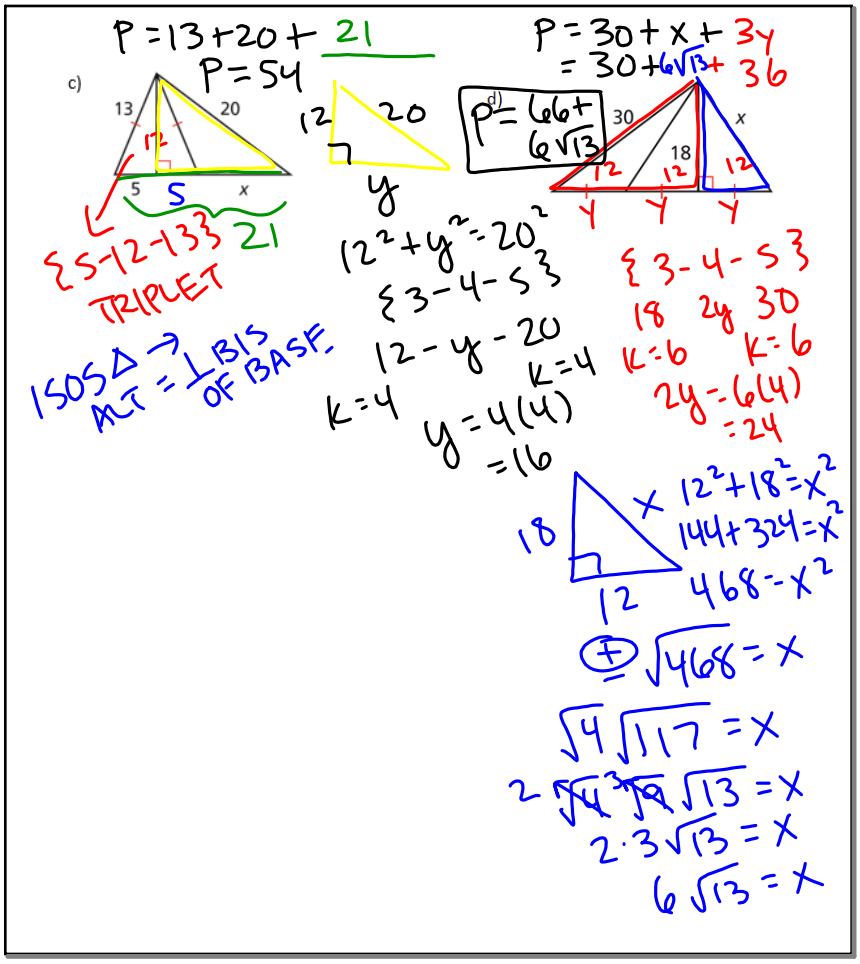
{8-15-17}
TRIPLET

$a^2+b^2=c^2$
$2^2+8^2=x^2$
$68=x^2$ → $x=\pm\sqrt{68}$
$=2\sqrt{17}$

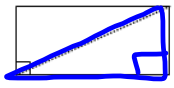
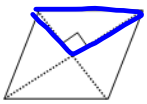
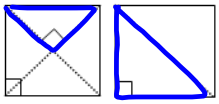
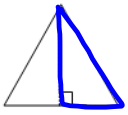
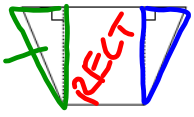
$8^2+b^2=17^2$
$64+b^2=289$
$b^2=225$
$b=15$

$P=17+17+2\sqrt{17}$
$P=34+2\sqrt{17}$

x + x + 2y



APPLYING THE PYTHAGOREAN THEOREM TO DIAGONALS AND ALTITUDES

RECTANGLE	RHOMBUS	SQUARE	TRIANGLE	TRAPEZOID
Rt. △'s are formed by diagonals 	Diagonals are ⊥ forming Rt. △'s 	Rt. △'s are formed by diagonals 	Altitudes are ⊥ to base forming Rt. △'s 	Altitudes are ⊥ to bases forming Rt. △'s 

PRACTICE: Draw a picture to justify your work. Show all work.

 = Keep this problem in mind during the unit...

PROBLEM SET 8-1R: Draw a picture to justify your work. Show all work.

1. Which set of numbers could be the lengths of sides of a right triangle?
   a. 4, 6, $\sqrt{40}$
   b. 2, 6, $\sqrt{40}$
   c. 2, 18, 20
   d. 4, 36, 40

2. The ratio of the lengths of the legs of a right triangle is 5: 12. What is the ratio of the length of the shorter leg to that of the hypotenuse?
   a. 13 : 5 b. $\sqrt{119}$: 5 c. 5 : 13 d. 5 : $\sqrt{119}$

3. If the length of a side of a square ABCD is 5, find the length of AC.

4. A rectangle has a diagonal of length 12 and one side of length 6. What is its perimeter?

5. The length of each side of a rhombus is 13. If the length of the shorter diagonal is 10, find the length of the longer diagonal.

Draw, label, and set up equations for 6-8. You do not need to solve.

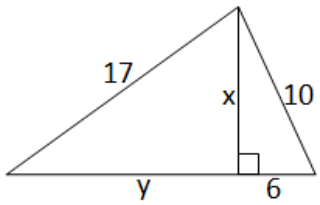
6. In a right triangle ΔABC, $\measuredangle C$ is the right angle. If AC is 1 cm more than BC and AB is 2 cm more BC, find the lengths of the three sides.

7. In rectangle ABCD, AB = x, BC = x + 7, and diagonal BD = x + 8. Find BD.

8. The length of a rectangle is 7 inches more than its width. If the diagonal has a length of 17 inches, find the dimensions of the rectangle.

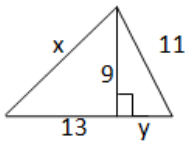
Solve for x , then solve for y .

9.

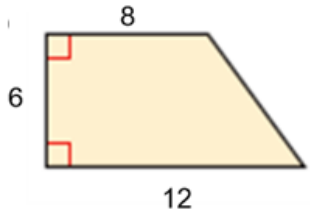


Find the Perimeter and Area of each figure. Give your answers in simplest radical form.

10.

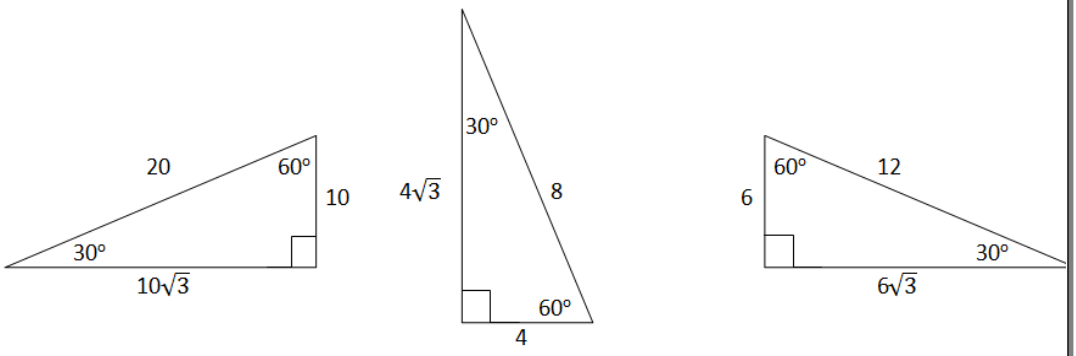


11.



12. In class, we investigated a skateboarding ramp that was built at a 30° angle such that the ramp was 20 feet long for the rider and 10 feet high off the ground. We found that the distance of the base of the ramp along the ground was $10\sqrt{3}$ feet using the Pythagorean Theorem. More likely, all we would have probably known was the measurement of the base of the ramp of $10\sqrt{3}$ feet. Could we have found the length and height of the ramp? How?

Using the right triangles below, see if you can discover a pattern or ratio that exists between the sides and their locations (hint, redraw with the same orientation if you need):



a) My conjecture about the ratio of short leg: long leg: hypotenuse is _____:_____:_____
for a triangle whose angles are 30° - 60° - 90°

b) Using your conjecture, find the length and height of a ramp that is 30° with a base length of $15\sqrt{}$ feet.

13. Construct equilateral triangle ABC with a side length of 2 inches in the space below. Then construct the altitude $\overline{CD}$ (perpendicular from a point off the line) where D is the point of intersection with the base $\overline{AB}$.

Fill in the measures of the following:

i. $m\angle ADC =$ _____ ° and $m\angle BDC =$ _____ °

ii. $m\angle A =$ _____ ° and $m\angle B =$ _____ °

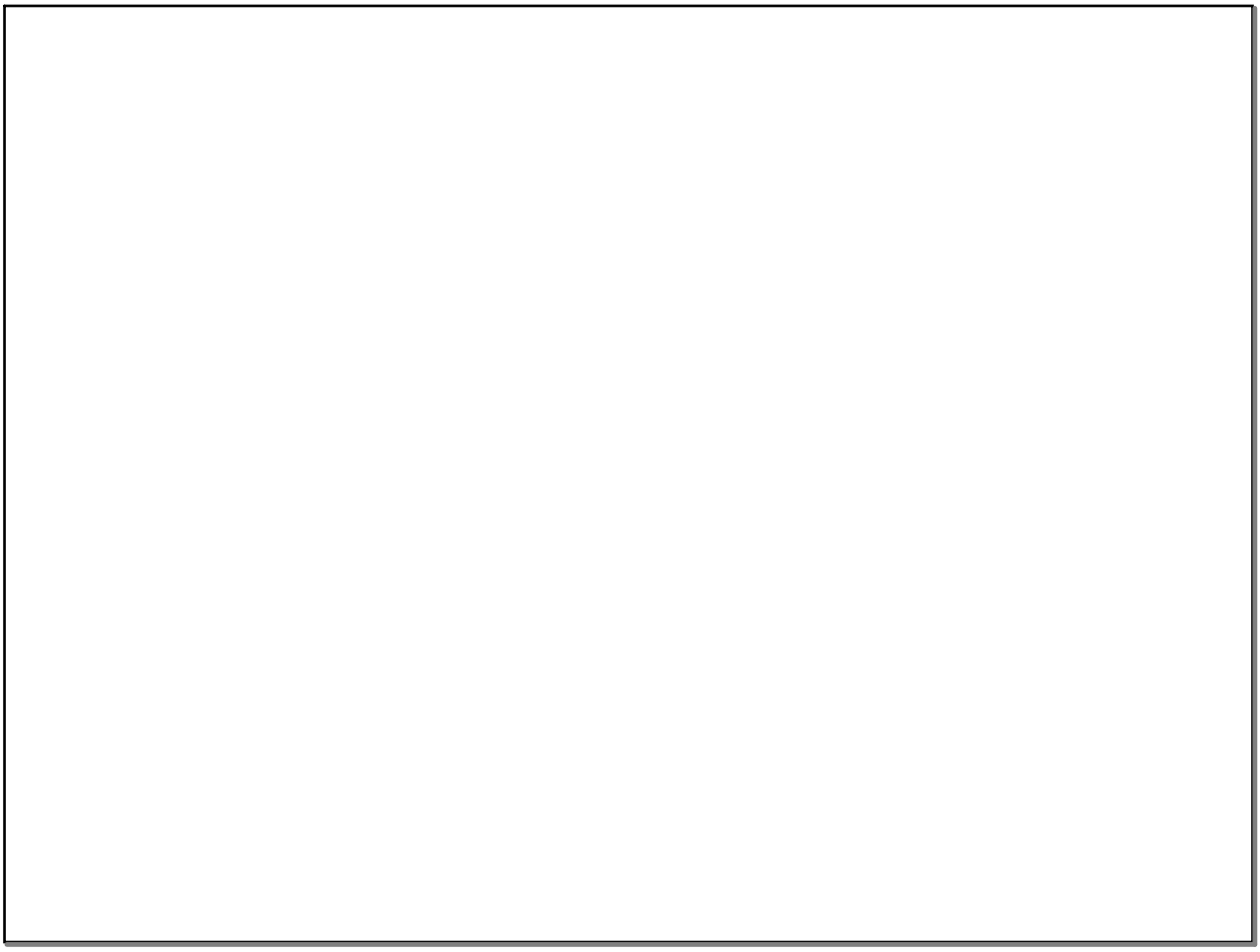
iii. $m\angle ACD =$ _____ ° and $m\angle BCD =$ _____ °

iv. AC = _____ inches = BC

v. AD = _____ inches = BD

vi. CD = _____ inches (use the Pythagorean Theorem to answer in simplest radical form)

A _____ B



Attachments

Bridge to 8.docx