Lesson 8-2: Special Right Triangles

AGENDA:

- Check HW - 8.1

- Notes 8. 2 with Applications and Guided Practice

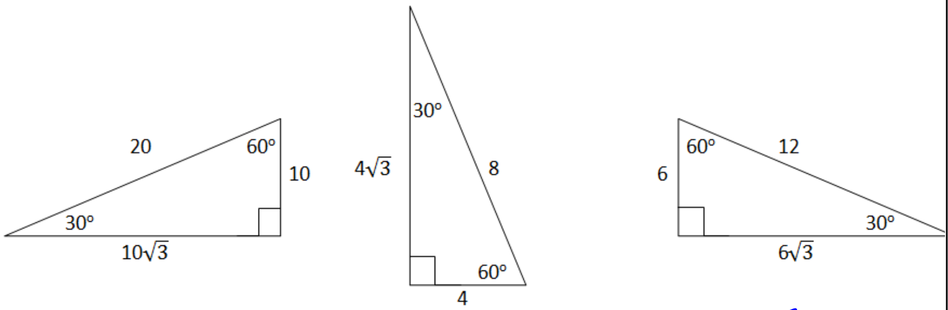
HOMEWORK:

- Text p. 360 #9-10, 14-16, 19-21

- CR#7 is Due Monday 3/20

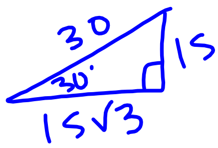
12. In class, we investigated a skateboarding ramp that was built at a 30° angle such that the ramp was 20 feet long for the rider and 10 feet high off the ground. We found that the distance of the base of the ramp along the ground was $10\sqrt{3}$ feet using the Pythagorean Theorem. More likely, all we would have probably known was the measurement of the base of the ramp of $10\sqrt{3}$ feet. Could we have found the length and height of the ramp? How?

Using the right triangles below, see if you can discover a pattern or ratio that exists between the sides and their locations (hint, redraw with the same orientation if you need):



a) My conjecture about the ratio of short leg: long leg: hypotenuse is n : $n\sqrt{3}$: $2n$
for a triangle whose angles are 30° - 60° - 90°

b) Using your conjecture, find the length and height of a ramp that is 30° with a base length of 15√ feet.



13. Construct equilateral triangle ABC with a side length of 2 inches in the space below. Then construct the altitude $\overline{CD}$ (perpendicular from a point off the line) where D is the point of intersection with the base $\overline{AB}$.

Fill in the measures of the following:

i. $m\measuredangle ADC =$ ______ ° and $m\measuredangle BDC =$ ______ °

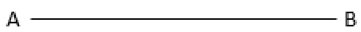
ii. $m\measuredangle A =$ ______ ° and $m\measuredangle B =$ ______ °

iii. $m\measuredangle ACD =$ ______ ° and $m\measuredangle BCD =$ ______ °

iv. AC = ______ inches = BC

v. AD = ______ inches = BD

vi. CD = ______ inches (use the Pythagorean Theorem to answer in simplest radical form)



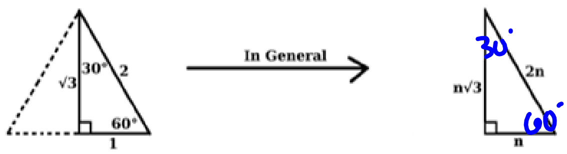
Geometry Name: ________________________ Section: __________ Date: ________

8-2 Notes: Discovering and Applying Special Right Triangles; Rationalizing Denominators

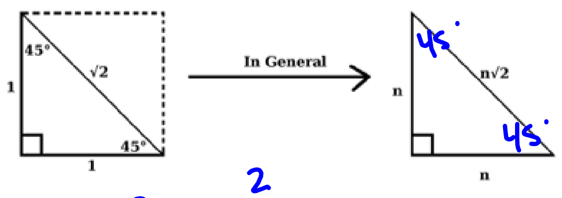
Two types of right triangles by angle measures are common and have easy-to-use ratios among the sides. We call these special right triangles.

30° - 60° - 90°
Constructed by slicing an equilateral triangle down its altitude.

Remember, the short leg is opposite the smallest angle!



45° - 45° - 90°
Constructed by slicing a square down its diagonal.



$1^2+1^2=x^2$

$2=x^2$

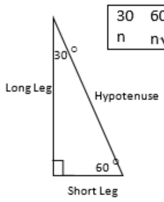
$\pm\sqrt{2}=x$

Special right triangles:

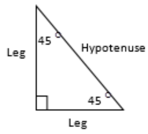
Steps:

1. Write the appropriate box.
2. Look at diagram. Place given information in the appropriate column under the box.
3. Write equation (2nd row of column =3rd row of column)
4. Solve for n

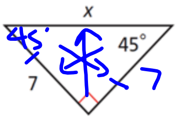
30	60	90
n	$n\sqrt{3}$	2n



45	45	90
n	n	$n\sqrt{2}$



1)



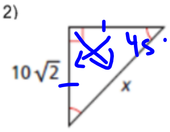
45	45	90
n	n	$n\sqrt{2}$
7	7	x

$n = 7$

$x = n\sqrt{2}$

$\boxed{x = 7\sqrt{2}}$

2)



45	45	90
n	n	$n\sqrt{2}$
$10\sqrt{2}$		x

$n = 10\sqrt{2}$

$x = n\sqrt{2}$

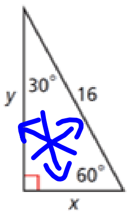
$x = (10\sqrt{2})\sqrt{2}$

$x = 10(\underbrace{\sqrt{2}\cdot\sqrt{2}}_{2})$

$x = 10 \cdot 2$

$\boxed{x = 20}$

1)



30	60	90
n	$n\sqrt{3}$	2n
x	y	16

$2n = 16$

$n = 8$

$x = n$

$x = 8$

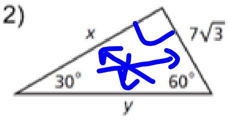
$\boxed{x = 8}$

$y = n\sqrt{3}$

$y = (8)\sqrt{3}$

$\boxed{y = 8\sqrt{3}}$

2)



30	60	90
n	$n\sqrt{3}$	2n
$7\sqrt{3}$	x	y

$n = 7\sqrt{3}$

$x = n\sqrt{3}$

$x = (7\underbrace{\sqrt{3})\sqrt{3}}_{3}$

$x = 7 \cdot 3$

$\boxed{x = 21}$

$y = 2n$

$y = 2(7\sqrt{3})$

$\boxed{y = 14\sqrt{3}}$

$\sqrt{3}\sqrt{3} =$

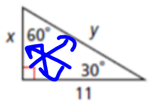
$3 \; \cancel{\sqrt{9}}$

$(\sqrt{3})^2 = 3$

Rationalizing the Denominator

1. Divide both sides by the radical to isolate the variable.
2. Rationalize the side with a radical in the denominator:
   a. Multiply both the numerator and denominator by the radical such as $\frac{\sqrt{3}}{\sqrt{3}}$ or $\frac{\sqrt{2}}{\sqrt{2}}$ the two common cases you will see in our course (essentially, this is multiplying by 1)
   → this will "undo" the radical symbol in the denominator
   b. Simplify by cancelling any like factors

1)



30	60	90
n	$n\sqrt{3}$	2n
x	11	y

$$\frac{11}{\sqrt{3}} = \frac{n\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$$

$$\left(\frac{\sqrt{3}}{\sqrt{3}}\right)\frac{11}{\sqrt{3}} = n$$

$$\frac{11\sqrt{3}}{3} = n$$

$x = n$

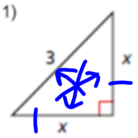
$$\boxed{x = \frac{11\sqrt{3}}{3}}$$

$y = 2n$

$$y = 2\left(\frac{11\sqrt{3}}{3}\right)$$

$$\boxed{y = \frac{22\sqrt{3}}{3}}$$

1)



45	45	90
n	n	$n\sqrt{2}$
x	x	3

$$\frac{3}{\sqrt{2}} = \frac{n\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$$

$$\left(\frac{\sqrt{2}}{\sqrt{2}}\right)\frac{3}{\sqrt{2}} = n$$

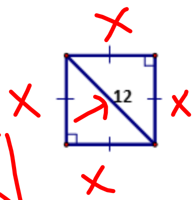
$$\frac{3\sqrt{2}}{2} = n$$

$x = n$

$$\boxed{x = \frac{3\sqrt{2}}{2}}$$

Comparing Pythagorean Theorem Vs. Special Right Triangles

1. Find the area of the square using both methods:



PYTHAG

$a^2 + b^2 = c^2$

$x^2 + x^2 = 12^2$

$2x^2 = 144$

$x^2 = 72$

$x = \pm\sqrt{72}$

$x = \sqrt{36}\sqrt{2}$

$x = 6\sqrt{2}$

45	45	90
n	n	$n\sqrt{2}$
x	x	12

$$\frac{12}{\sqrt{2}} = \frac{n\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$$

$$\left(\frac{\sqrt{2}}{\sqrt{2}}\right)\frac{12}{\sqrt{2}} = n$$

$$\frac{12\sqrt{2}}{2} = n$$

$6\sqrt{2} = n$

$x = n$

$x = 6\sqrt{2}$

$A = bh = x \cdot x$

$= x^2$

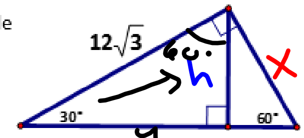
$(\sqrt{72})^2 = 72$

$(6\sqrt{2})^2 = 36 \cdot 2$

$= 72$

Can you use the Pythagorean Theorem with a 30°-60°-90° triangle given only one side length? NO

2. Find the area of the largest triangle



b
24

30°	60°	90°
n	$n\sqrt{3}$	2n
h	y	$12\sqrt{3}$

$2n = 12\sqrt{3}$

$\cancel{2}n = \frac{12\sqrt{3}}{2}$

$n = 6\sqrt{3}$

$h = n \quad h = 6\sqrt{3}$

$A = \frac{bh}{2} = \frac{(24)\,6\sqrt{3}}{2}$

$\boxed{A = 72\sqrt{3}}$

b:

30°	60°	90°
n	$n\sqrt{3}$	2n
x	$12\sqrt{3}$	b

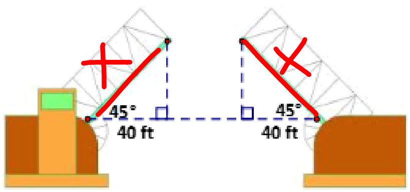
$\frac{12\sqrt{3}}{\sqrt{3}} = \frac{n\sqrt{3}}{\sqrt{3}}$

$12 = n$

$b = 2n = 2(12)$
$= 24$

Mixed Practice:

3. Find the exact length of the drawbridge in feet. $= 2x$



45°	45°	90°
n	n	$n\sqrt{2}$
40		x

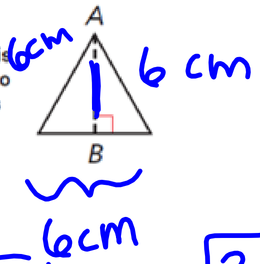
$n = 40$

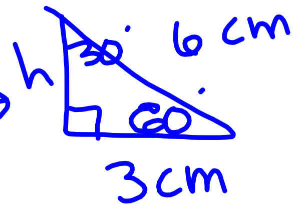
$x = n\sqrt{2}$

$x = 40\sqrt{2}$

DRAWBRIDGE $= 80\sqrt{2}$ FT
2X
drawbridge

4. An ornamental pin is in the shape of an equilateral triangle. The length of each side is 6 centimeters. Josh will attach the fastener to the back along $\overline{AB}$. Will the fastener fit if it is 4 centimeters long?





YES

$$\boxed{\begin{array}{ccc} 30 - & 60 - & 90 \\ n & n\sqrt{3} & 2n \end{array}}$$

$3 \quad h \quad 6$

$n = 3 \qquad h = n\sqrt{3}$

$= 3\sqrt{3}$ cm

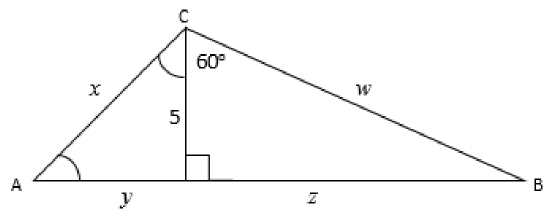
≈ 5.1961 cm

What is the total area of the pin?

$$A = \frac{bh}{2} = \frac{(6\text{ cm})(3\sqrt{3}\text{ cm})}{2}$$

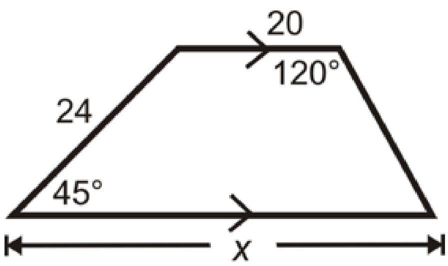
$$A = 9\sqrt{3}\text{ cm}^2$$

EXIT PASS: Determine the perimeter of ΔABC. Show all your work.

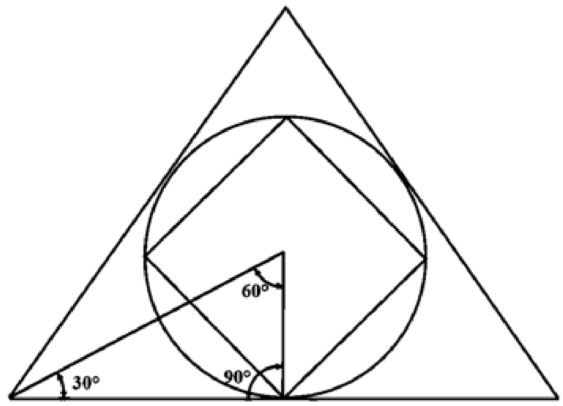


Extra Credit:

A) Determine the value of x and the exact area of the trapezoid.



B) The equilateral triangle has side length 6 cm. If the inscribed circle of the equilateral triangle contains an inscribed square whose diagonals intersect at the vertex of 60° angle of the right triangle, determine the exact length of the diagonal of the square.



Attachments

Bridge to 8.docx