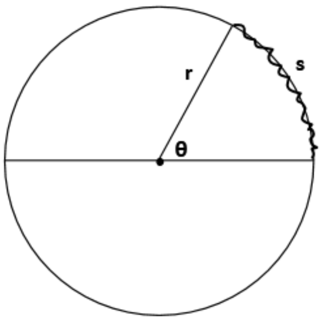
Radian Measure

We are accustomed to measuring angles in degrees. There is another scale by which angles can be measured, **radians**. An angle that measures one radian is an angle whose intercepted arc is exactly as long as the circle's radius.



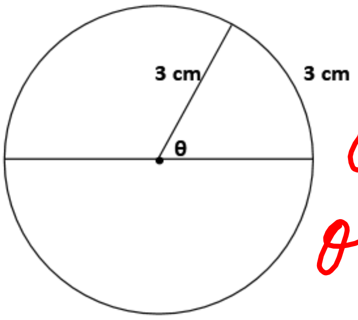
s = length of arc

r = length of radius

θ = measure of central angle in radians

$\theta = \frac{s}{r}$ (Rad.)

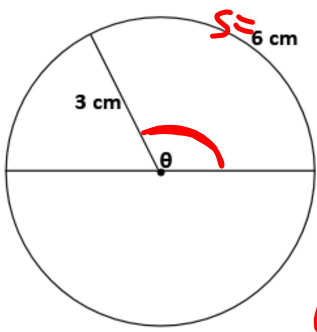
Find the measure of **θ in radians:**



$\theta = \frac{s}{r}$

$\theta = \frac{3cm}{3cm}$

$\theta = 1$ radian



$\theta = \frac{6cm}{3cm}$

$\theta = 2$ radians

How does radian measure relate to degree measure?

The following exploration will help us discover the relationship between radians and degrees.

~~7 radians~~

2 radians

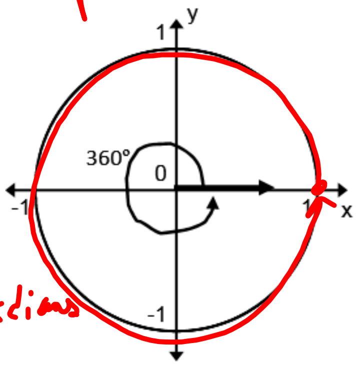
$\sim 120°$

$2 \cdot \pi$ radians

$$\underbrace{2\pi \text{ radians} = 360°}_{\sim 6 \text{ rad.}}$$

Suppose we have a 360° angle in standard position on the **unit circle**. In order to find the measure of that 360° angle in radians, we need to know the length of the arc it intercepts and the length of the radius. The arc length of a 360° angle would also be the circumference of the circle.

$\theta = \frac{s}{r}$ $r = 1$ unit



1. Find the arc length (s) of the whole circle:

$s = C = \pi d$

$C = \pi \cdot 2$

$C = 2\pi$

2. Use $\theta = \frac{s}{r}$ to find the measure of a 360° angle in radians.

$\theta = \frac{s}{r} = \frac{2\pi}{1} = 2\pi$ radians

Conclusion:

Angle in degrees	**Angle in radians**
360	2π
180	π
90	$\pi/2$
30	$\pi/6$
45	$\pi/4$
60	$\pi/3$

÷6 ÷6

1 radian = 60° MC
62 CS
65 CR
61 DS
66 HP
63 NM

$60° = 1.047.....$

$5 \cdot \pi \approx \frac{15}{6}$

Convert the following angles (Radians ↔ Degrees)

(1) 55° Deg → Rad

$\cancel{55}^{11} \cdot \frac{\pi}{\cancel{180}_{36}} = \frac{11\pi}{36}$ radians

(2) $\frac{5\pi}{6}$ Rad → Deg

$\frac{5\cancel{\pi}}{\cancel{6}_1} \cdot \frac{\cancel{180}^{30}}{\cancel{\pi}} = 150°$

(3) 7 radians

$7 \cdot \frac{180}{\pi} = \frac{1260°}{\pi}$

400 MP
415 CS

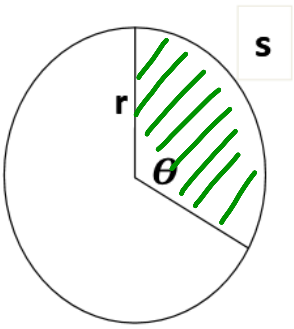
$\frac{\pi \text{ rad}}{180°} = \frac{7 \text{ rad}}{x \text{ deg}}$

$\frac{180(7)}{\pi} = \frac{\pi x}{\pi}$

Area of a Sector

$\theta = \frac{s}{r}$

Radians (radians)



Area of a Sector $= \frac{1}{2}\theta r^2$

1 rad ≈ 57°

Examples:

(1) Find the area of a sector that has a central angle of 120° and a radius of 9 inches.

$A = \frac{1}{2}\theta^{*} r^2$

$^{*}\ \theta = \frac{\cancel{120}^{\,2}° \cdot \pi}{\cancel{180}_{\,3}}$

$\theta = \frac{2\pi}{3}$ rad.

$A = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\pi}{3} \cdot 9^2$

$A = \frac{81\pi}{3} = 27\pi \text{ in}^2 \approx 84.82 \text{ in}^2$

(2) Find the area of a sector that has a central angle of 5 radians and an arc length of 26 inches.

$A = \frac{1}{2}\theta r^{*2}$

$A = \frac{1}{2}(5)\cdot(5.2)^2$

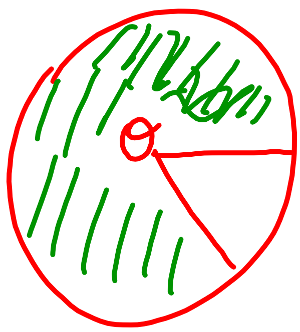
$\boxed{A = 67.6 \text{ in}^2}$

$^{*}\ \theta = \frac{s}{r}$

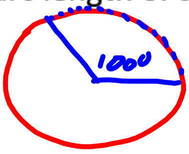
$\frac{5}{1} = \frac{26}{r}$

$\frac{5r}{5} = \frac{26}{5}$

$r = 5.2$ in.



(3) Find the arc length of a sector that has an area of 400 in^2 and a central angle of 100°.



$A = \frac{1}{2}\theta r^2$

$\theta = \frac{s}{r}$

$400 = \frac{1}{2}\left(\frac{100\pi}{180}\right) \cdot r^2$

$\frac{400}{\left(\frac{5\pi}{18}\right)} = \frac{\frac{5\pi}{18} \cdot r^2}{\left(\frac{5\pi}{18}\right)}$

$\sqrt{r^2} = \frac{\cancel{400}^{80}}{\cancel{5}\pi} \cdot 18 = \sqrt{\frac{1440}{\pi}}$

$r = 21.40948939...$

$\theta = \frac{s}{r}$

$\frac{5\pi}{9} = \frac{s}{21.40.....}$

$\boxed{s \approx 37.37 \text{ in}}$

(4) If a sector has an area of 550 m^2 and a radius of 10 m, what is the sector's arc length?

(5) A sprinkler sweeps out an angle of 140°. Compute the area covered by the sprinkler if the radius is 15 feet. Also, find the sector's arc length.

4 $s = 110\ m$

5 a $A = 87.5\pi\ ft^2$

b $s \approx 36.65\ ft.$

HW

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