

Angular and Linear Velocity

The field of biomechanics involves measuring the angular and linear velocities of an athlete's head to determine if the athlete has sustained a concussion.

$2\pi \text{ radians} = 360^\circ$

(1) Angular Displacement ( $\theta$ ): Amount of angle that object "cuts out" of a circle in radians.  
 (multiply revolutions by  $2\pi$  radians)

Ex:  $\pi \text{ rad.}, 7 \text{ rad.}, (2 \text{ rev.}) \cdot (2\pi) = 4\pi \text{ radians}$

(2) Angular Velocity ( $\omega$ ): Amount of angle cut out of a circle over a period of time.  

$$\omega = \frac{\theta \text{ (Ang. displ.)}}{t \text{ (time)}}$$

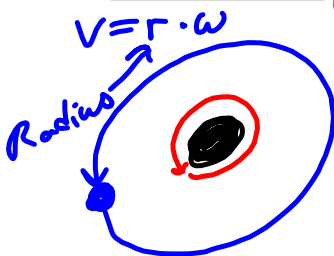
Ex:  $\frac{2\pi \text{ radians}}{1 \text{ sec.}}$

$\frac{\pi \text{ rad/hr.}}{6}$

3 rev. in 2 sec. =  $\frac{3 \cdot 2\pi \text{ radians}}{2 \text{ sec.}}$   
 =  $3\pi \text{ rad/sec.}$

(3) Linear Velocity ( $v$ ): Amount of distance travelled in a circular path over a period of time.  

$$v = r \cdot \omega$$



Ex:  $50 \text{ mi/hr.}$   
 $10 \text{ ft/sec.}$

For each of the following situations, find the object's angular velocity and linear velocity to the nearest hundredth: mph

- (1) Mr. Smith moves a smartboard marker in a circular path with a radius of 26.5 in. and completes  $4\frac{1}{8}$  revolutions in 5 seconds.

Ang. Vel.

$$\omega = \frac{\theta}{t} = \frac{(4\frac{1}{8}) \cdot 2\pi \text{ rad}}{5 \text{ sec.}} = 5.18 \text{ rad/sec.}$$

Lin. Vel.

$$v = r \cdot \omega$$

$$v = (26.5 \text{ in.}) (5.18 \text{ rad/sec.})$$

$$v = \frac{137.27 \text{ in.} \cdot 60 \text{ sec.} \cdot 60 \text{ min.} \cdot 1 \text{ ft} \cdot 1 \text{ mi}}{1 \text{ sec.} \cdot 1 \text{ min.} \cdot 1 \text{ hr} \cdot (12 \text{ in.} \cdot 5280 \text{ ft})} \approx 137.27 \text{ in/sec.}$$

- (2) A 20 inch tire rotates at 4 rps.

$$\omega = \frac{\theta}{t} = \frac{(4 \text{ rev}) (2\pi \text{ rad/rev})}{1 \text{ sec.}} = 8\pi \text{ rad/sec.}$$

$$\approx 7.8 \text{ mph}$$

$$v = r \cdot \omega = (10 \text{ in.}) (8\pi \text{ rad/sec.}) = 80\pi \text{ in/sec.}$$

- (3) A 40 inch diameter clothes dryer is rotating at 300 rpm.

$$\omega = \frac{\theta}{t} = \frac{(300) (2\pi) \text{ rad.}}{1 \text{ min.}} = 600\pi \text{ rad/min.}$$

$$v = r \cdot \omega = (20) (600\pi) = 12000\pi \text{ in/min.}$$

(4) Duct tape rotation competition:

Mr. Smith

vs.

Haley

Radius 18 in.

Radius 18 in.

50 revolutions

43 revolutions

15 seconds

15 seconds

$$\omega = \frac{\theta}{t} = \frac{50(2\pi)}{15 \text{ sec.}} = 20.94 \dots \text{ rad/sec.}$$

$$v = r \cdot \omega = (18 \text{ in.})(20.94 \dots)$$

$$v = \frac{376.9911184 \text{ in.}}{1 \text{ sec.}} \cdot \frac{1 \text{ ft.}}{12 \text{ in.}} \cdot \frac{3600 \text{ sec.}}{1 \text{ hr.}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft.}}$$

$$v \approx 21.42 \text{ mph}$$

HW

$$\omega = \frac{43(2\pi)}{15 \text{ sec.}}$$

$$\omega = 18.01 \dots \text{ rad/sec.}$$

$$v = r \cdot \omega$$

$$v = 18(18.01 \text{ rad/sec})$$

$$v = \frac{324.21 \text{ in.}}{1 \text{ sec.}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft.}} \cdot \frac{3600 \text{ sec.}}{1 \text{ hr.}}$$

$$v \approx 18.42 \text{ mph}$$

(5) A 30 inch tire rotates at 6 rps

$$35.70 \text{ mph}$$

Pg. 355-356  
# 13-34  
Every third prob.