Angular and Linear Velocity

The field of biomechanics involves measuring the **angular and linear velocities** of an athlete's head to determine if the athlete has sustained a concussion.

2π radians $= 360°$

(1) **Angular Displacement** (θ): Amount of angle that object "cuts out" of a circle in radians.

(Multiply revolutions by 2π radians)

Ex: π rad., 7 rad., $(2 \text{ rev.}) \cdot (2\pi) = 4\pi$ radians

(2) **Angular Velocity** (ω): Amount of angle cut out of a circle over a period of time.

$\omega = \frac{\theta \text{ (Ang. displ.)}}{t \text{ (time)}}$

Ex: $\frac{2\pi \text{ radians}}{1 \text{ sec.}}$

$\frac{\pi}{6}$ rad/hr.

3 rev. in 2 sec. $= \frac{3 \cdot 2\pi \text{ radians}}{2 \text{ sec.}} = 3\pi$ rad/sec.

(3) **Linear Velocity** (v): Amount of distance travelled in a circular path over a period of time.

$v = r \cdot \omega$

Ex: 50 mi/hr.

10 ft/sec.

For each of the following situations, find the object's angular velocity and linear velocity to the nearest hundredth: mph

(1) Mr. Smith moves a smartboard marker in a circular path with a radius of 26.5 in. and completes $4\frac{1}{8}$ revolutions in 5 seconds.

Ang. Vel.

$\omega = \frac{\theta}{t} = \frac{(4\frac{1}{8})\cdot 2\pi \text{ rad}}{5 \text{ sec.}} \approx 5.18 \text{ rad/sec.}$

Lin. Vel.

$v = r \cdot \omega$

$v = (26.5 \text{ in})(5.18 \text{ rad/sec})$

$v \approx 137.27 \text{ in/sec.}$

$v = \frac{137.27 \text{ in}}{1 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{(12 \text{ in} \cdot 5280 \text{ ft})} \approx 7.8 \text{ mph}$

(2) A 20 inch tire rotates at 4 rps.

$\omega = \frac{\theta}{t} = \frac{(4 \text{ rev})(2\pi \text{ rad/rev})}{1 \text{ sec}} = 8\pi \text{ rad/sec.}$

$v = r \cdot \omega = (10 \text{ in})(8\pi \text{ rad/sec}) = 80\pi \text{ in/sec.}$

(3) A 40 inch diameter clothes dryer is rotating at 300 rpm.

$\omega = \frac{\theta}{t} = \frac{(300)(2\pi) \text{ rad.}}{1 \text{ min.}} = 600\pi \text{ rad/min.}$

$v = r \cdot \omega = (20)(600\pi) = 12000\pi \text{ in/min.}$

(4) Duct tape rotation competition:

Mr. Smith	vs.	Haley
Radius 18 in.		Radius 18 in.
50 revolutions		43 revolutions
15 seconds		15 seconds

$\omega = \frac{\theta}{t} = \frac{50(2\pi)}{15 \text{ sec.}} = 20.94\ldots$ rad/sec.

$V = r \cdot \omega = (18 \text{ in})(20.94\ldots)$

$V = \frac{376.9911184 \text{ in}}{1 \text{ sec.}} \cdot \frac{1 \text{ ft.}}{12 \text{ in}} \cdot \frac{3600 \text{ sec.}}{1 \text{ hr.}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}}$

$V \approx 21.42$ mph

$\omega = \frac{43(2\pi) \text{ rad}}{15 \text{ sec.}}$

$\omega = 18.01\ldots$ rad/sec.

$V = r \cdot \omega$

$V = 18(18.01 \text{ rad/sec})$

$V = \frac{324.21 \text{ in}}{1 \text{ sec.}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{3600 \text{ sec.}}{1 \text{ hr.}} \cdot \frac{1 \text{ ft}}{12 \text{ in}}$

$V \approx 18.42$ mph

HW

Pg. 355-356

13-34

Every third prob.

(5) A 30 inch tire rotates at 6 rps

35.70 mph