Trig. Identities

① $\sec\theta = \frac{1}{\cos\theta}$

② $\csc\theta = \frac{1}{\sin\theta}$

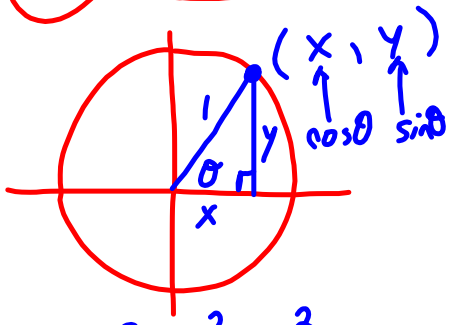
③ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

④ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

OR

$\cot\theta = \frac{1}{\tan\theta}$

⑤ Unit Circle



$x^2 + y^2 = 1^2$

$\cos^2\theta + \sin^2\theta = 1$

$-\cos^2\theta \quad -\cos^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

⑥ $\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1 + \tan^2\theta = \sec^2\theta$

⑦ $\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\cot^2\theta + 1 = \csc^2\theta$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\left.\right) \frac{s}{c} \cdot \frac{s}{c}$

$\sqrt{\tan^2\theta} = \frac{\sqrt{\sin^2\theta}}{\sqrt{\cos^2\theta}}$

$\sqrt{\sin^2\theta} = \sqrt{1-\cos^2\theta}$

$\sqrt{\sin^2\theta} = \sqrt{1+\cos^2\theta}$

$\sin\theta = 1-\cos\theta$

$\sqrt{9+16} = \sqrt{25}$

$\sqrt{9}+\sqrt{16} = \sqrt{25}$

$3+4 = 5$

Identity Rules

① You can only change one side of the equation.

* ② Steps
  - 1st Make some changes
  - 2nd Do some math (Cancel fractions, complex Factor, A/S/M/D Fractions)
  - 3rd Make more changes
  - ⋮
  - etc.

Examples (Verify each identity)

① $\frac{\sin\theta}{\tan\theta} = \cos\theta$

$\frac{\sin\theta \cdot \cos\theta}{\frac{\sin\theta}{\cos\theta}}$

$\sin\theta \cos\theta \div \sin\theta$

$\cos\theta = \cos\theta$ ✓

② $\cos\theta = \frac{\cot\theta}{\csc\theta}$

$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$\cos\theta = \cos\theta$ ✓

③ $\sec^2\theta - \tan\theta\cot\theta = \tan^2\theta$

$1 + \tan^2\theta - \tan\theta\cot\theta$

$1 + \tan^2\theta - \tan\theta \cdot \frac{1}{\tan\theta}$

$1 + \tan^2\theta - 1$

$\tan^2\theta = \tan^2\theta$ ✓

$\frac{1}{\cos^2\theta} - \frac{1(\cos^2\theta)}{1(\cos^2\theta)}$

$\frac{1-\cos^2\theta}{\cos^2\theta}$

$\frac{\sin^2\theta}{\cos^2\theta}$

$\tan^2\theta =$

④ $\sin\theta + \cos\theta \cdot \cot\theta = \csc\theta$

$\sin\theta + \frac{\cos\theta}{1} \cdot \frac{\cos\theta}{\sin\theta}$

4+2·3

$\frac{(\sin\theta)\sin\theta}{(\sin\theta)1} + \frac{\cos^2\theta}{\sin\theta}$

$\frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

$\frac{1}{\sin\theta}$

$\csc\theta = \csc\theta$

(5) $\dfrac{7\sin\theta + 5\cos\theta}{\sin\theta\cos\theta} = 7\sec\theta + 5\csc\theta$

$\dfrac{7}{1}\cdot\dfrac{1}{\cos\theta} + \dfrac{5}{1}\cdot\dfrac{1}{\sin\theta}$

$\dfrac{(\sin\theta)7}{(\sin\theta)\cos\theta} + \dfrac{5(\cos\theta)}{\sin\theta(\cos\theta)}$

$\dfrac{7\sin\theta + 5\cos\theta}{\sin\theta(\cos\theta)}$

(6) $\sin\theta\csc\theta - \sin^2\theta = \cos^2\theta$

$\sin\theta\cdot\dfrac{1}{\sin\theta} - \sin^2\theta$

$1 - \sin^2\theta$

$\cos^2\theta = \cos^2\theta$

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∴

$\sec\theta = \frac{1}{\cos\theta}$ $\sec^2\theta = \frac{1}{\cos^2\theta}$

$\sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta}$

$(1-\sin\theta)(1-\sin\theta)$

$1-2\sin\theta+\sin^2\theta$

$1-\sin\theta \neq \cos\theta$