Verify the identities:

① $(\sec\theta - \tan\theta)(1+\sin\theta) \stackrel{?}{=} \cos\theta$

$\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)(1+\sin\theta)$

$\left(\frac{1-\sin\theta}{\cos\theta}\right)\left(\frac{1+\sin\theta}{1}\right)$

$\frac{1-\sin^2\theta}{\cos\theta}$

$\frac{\cos^2\theta}{\cos\theta}$

$\cos\theta = \cos\theta$ ✓

② $2-\cos^2\theta = 1+\sin^2\theta$

$2-(1-\sin^2\theta) \quad 1+1-\cos^2\theta$

$2-(1-\sin^2\theta)$

$2-1+\sin^2\theta$

$1+\sin^2\theta = 1+\sin^2\theta$ ✓

③ $\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta} = \csc^2\theta - \cot^2\theta$

$\frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$

$\frac{1-\cos^2\theta}{\sin^2\theta}$

$\frac{\sin^2\theta}{\sin^2\theta}$

1

$\sin^2\theta + \cos^2\theta$

$\frac{\sin\theta}{1/\sin\theta} + \frac{\cos\theta}{1/\cos\theta}$

$\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta}$ ✓

④ $\csc\theta - \cot\theta = \frac{1}{(\csc\theta + \cot\theta)} \cdot \frac{(\csc\theta - \cot\theta)}{(\csc\theta - \cot\theta)}$

$$\frac{\csc\theta - \cot\theta}{\csc^2\theta - \csc\theta\cot\theta + \csc\theta\cot\theta - \cot^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\frac{\csc\theta - \cot\theta}{1}$$

$$\boxed{\csc^2\theta = 1 + \cot^2\theta}$$

⑤ $\frac{\sin^2\theta}{1-\cos\theta} = 1+\cos\theta$

FACTOR

$\frac{(1-\cos^2\theta)}{(1-\cos\theta)}$

$\frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$

$1+\cos\theta = 1+\cos\theta$

⑥ $\frac{\csc^2\theta - \cot^2\theta}{1-\sin^2\theta} = \sec^2\theta$

$\frac{\frac{1}{1}\cdot\frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}\cdot\frac{\sin^2\theta}{1}}{1-\sin^2\theta}$

$\frac{\frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{1}}{1-\sin^2\theta}\cdot\frac{\sin^2\theta}{1}$

$\frac{1-\cos^2\theta}{(1-\sin^2\theta)(\sin^2\theta)}$

$\frac{\sin^2\theta}{\cos^2\theta(\sin^2\theta)}$

$\frac{1}{\cos^2\theta}$

$\sec^2\theta = \sec^2\theta$ ✓

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⑦ $\sin\theta+\cos\theta = \frac{2\sin^2\theta-1}{\sin\theta-\cos\theta}$

$\frac{2\sin^2\theta-(\sin^2\theta+\cos^2\theta)}{\sin\theta-\cos\theta}$

$\frac{2\sin^2\theta-\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$

$\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$

$\frac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta}$

$\sin\theta+\cos\theta = \sin\theta+\cos\theta$ ✓