Lesson 7-1: Dilations

Agenda

- Check & Review Bridge to Unit 7
- 7-1 Exploration and Guided Notes
- Need compass and ruler

Homework:

- Problem Set 7.1
- CR#6 due next Th. 2/16

Answers to Bridge to Unit 7

Ex 1) Largest angle = 117°

Ex 2) Shortest side = 24 cm

3) No, since 18=16

4) Yes, since 48=48

5) y=+3 or -3

6) z = -6 or 14

7) x = 9 or -4

8. $15\sqrt{2}$

9. 28√6

10. $\frac{1}{5}$

11. $\frac{3}{2}$

12. $\frac{1}{2}$

13. 3/5

14. 1/3

15. 2/90

- A ratio compares two numbers by division, ex: 1/2, 3:5, 8 to 3
- An extended ratio compares several numbers, ex: 8:3:2
- If you know 2 numbers are in a certain ratio, say 4:2, you can represent the 2 numbers as 2x and 4x

Try These:

EX 1) The ratio of angle measures in a triangle is 1:6:13. What is the measure of the largest angle?

1x + 6x +13x=180

20x =180

x =9

$13(9)$

117^o

EX 2) The ratios of the side lengths of a triangle is 4:7:5, and its perimeter is 96 cm. What is the length of the shortest side?

$$4x+7x+5x=96$$
$$x=6$$
$$4(6)=24cm.$$

Is each of the following a true proportion?

3) $\frac{2}{3}=\frac{6}{8}$

$2(8)=6(3)$

$16\neq 18$

NO

4) $\frac{6}{4}=\frac{12}{8}$

$6(8)=12(4)$

$48=48$

YES

Solve the following Proportions:

5) EX3) $\frac{2y}{9}=\frac{8}{4y}$

$$2y(4y)=9(8)$$
$$8y^2=72$$
$$y^2=9$$
$$y=\pm 3$$

a = 1

b = -8

c = -84

6) EX4) $\frac{z-4}{5}=\frac{20}{z-4}$

$$(z-4)(z-4)=5(20)$$
$$z^2-4z-4z+16=100$$
$$z^2-8z+16-100=0$$
$$z^2-8z-84=0$$
$$(z-14)(z+6)=0$$
$$z=14; z=-6$$

7) EX5) $\frac{x-7}{4}=\frac{11}{2x+4}$

$$(x-7)(2x+4)=4(11)$$
$$2x^2+4x-14x-28=44$$
$$2x^2-10x-28-44=0$$
$$\frac{2x^2-10x-72}{2}=\frac{0}{2}$$
$$2(x^2-5x-36)=0$$
$$2(x-9)(x+4)=0$$
$$x=9; x=-4$$

Express in simplest radical form:

8. $3\sqrt{50}$

$3\sqrt{25}\sqrt{2}$

$3\cdot 5\cdot\sqrt{2}$

$15\sqrt{2}$

9. $7\sqrt{96}$

$7\sqrt{16}\sqrt{6}$

$7\cdot 4\cdot\sqrt{6}$

$28\sqrt{6}$

Simplify the following:

10. $\frac{\cancel{8}\cancel{\sqrt{2}}}{\cancel{40}\cancel{\sqrt{2}}} = \frac{1}{5}$

11. $\frac{\sqrt{144}}{\sqrt{64}} = \frac{12}{8} = \frac{3}{2}$

12. $\sqrt{\frac{20}{80}}$ $\sqrt{\frac{20}{80}} = \sqrt{\frac{2}{8}}$

$= \sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$

13. $\frac{\sqrt{27}}{\sqrt{75}} = \sqrt{\frac{27}{75}} = \sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$

14. $\frac{\sqrt{13}}{\sqrt{117}}$ $\sqrt{\frac{13}{117}} = \sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}}$

$= \frac{1}{3}$

15. $\frac{\sqrt{40}}{30\sqrt{90}} = \frac{1}{30}\sqrt{\frac{40}{90}} = \frac{1}{30}\sqrt{\frac{4}{9}} = \frac{1\sqrt{4}}{30\sqrt{9}} = \frac{1\cdot 2}{30\cdot 3} = \frac{2}{90}$

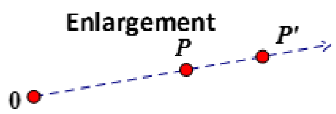
DEFINITION OF DILATION

A dilation with center O and a scale factor of k is a transformation that maps every point P in the plane to point P' so that the following properties are true.

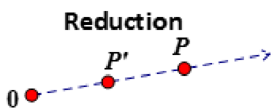
1. **If P is NOT the center O**, then the P' lies on $\overline{OP}$.

The scale factor k is a **positive** number where k = $\frac{OP'}{OP}$ and $k \neq 1$.

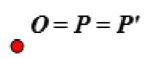
2. **If P is the center point O**, then P = P'. The center of dilation is the only point in the plane that does not move.



$\frac{OP'}{OP} > 1$



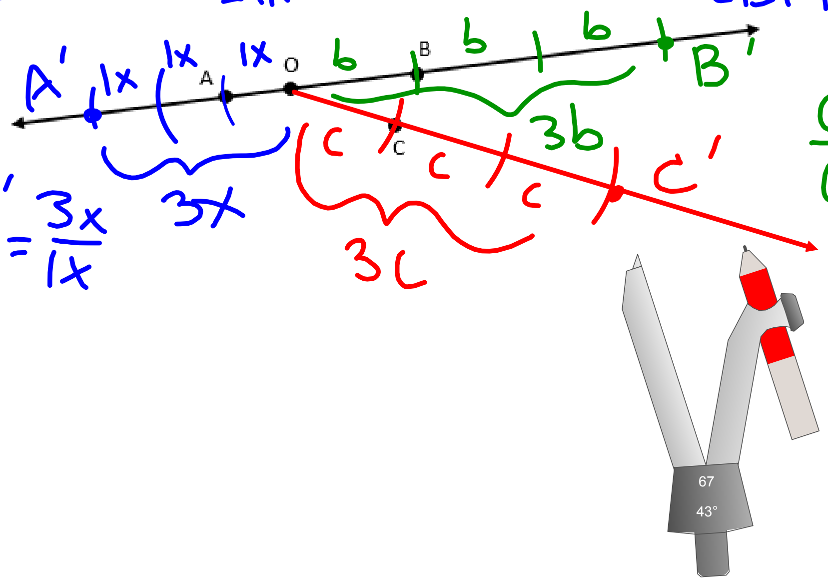
$0 < \frac{OP'}{OP} < 1$



NOTATION $D_{O,k}(x, y)$ **where O is the center of dilation, k is the value of the scale factor**

CONSTRUCTION

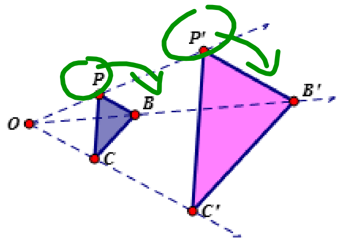
Dilate points A and B by a scale factor of 3 centered at O. What was important? COME FROM CENTER OF DILATION and BE COLLINEAR Now dilate C centered at O with k=3.



$$\frac{OA'}{OA} = \frac{3x}{1x}$$

$$\frac{OB'}{OB} = \frac{3b}{1b}$$

TRANSFORMATION PROPERTIES – The following properties are present in dilation:

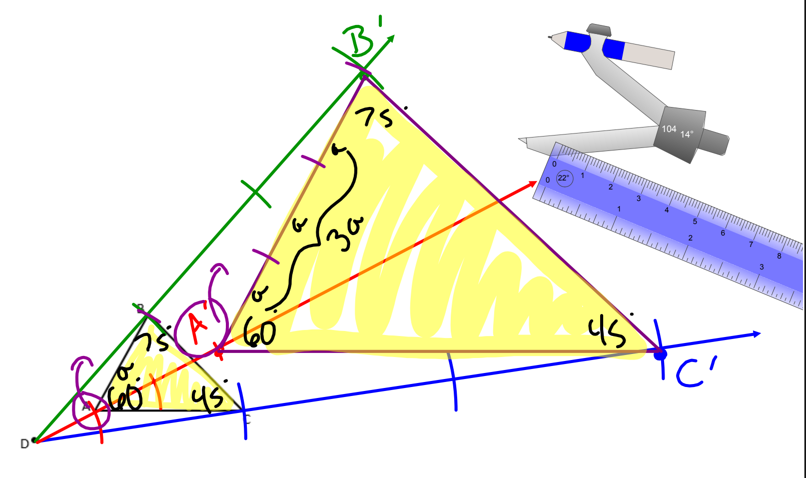


- **DISTANCES ARE DIFFERENT (PROPORTIONAL)** – The distance points move during dilation depend on their distance from the center of dilation - points closer to the center of dilation will move a shorter distance than those farther away. In our example $PP' \neq BB' \neq CC'$ and point B is farther away from the center of dilation O than point P, thus $BB' > PP'$.
- **ORIENTATION IS THE SAME** – The orientation of the shape is maintained.
- **SPECIAL POINTS** – The center of dilation is an invariant point and does not move in a dilation. If the pre-image (P) = image (P') after a dilation then point P was the center of dilation.
- **ANGLE MEASURE? PARALLELISM? PERPENDICULARITY? SLOPE?** Let's find out!

Given the Center of Dilation D, dilate the given triangle by a scale factor of three and label the vertices of the image.

To dilate a polygon with a scale factor $k = 3$:

1. Draw a ray from the center of dilation through a chosen vertex and way beyond the vertex.
2. Put the compass needle on the center of dilation.
3. Measure the distance to the chosen vertex and swipe an arc. Keep this span!
4. Copy the span 3 times (the scale factor) on the ray you drew, starting from the center of dilation and making all the spans adjacent and collinear.
5. Repeat for all vertices of the polygon.



Using your universal angle maker and compass only, investigate the following:

How do the corresponding angles compare between the pre-image and image?

$m\angle A$ 60° $m\angle A'$ 60°
$m\angle B$ 75° $m\angle B'$ 75° Are the pre-image and image angles ≅ ? YES
$m\angle C$ 45° $m\angle C'$ 45° Do dilations preserve angle measure? YES

Do dilations preserve orientation? YES

How do the corresponding side lengths compare between the pre-image and image?

How many AB lengths fit on A'B'? 3 So A'B' = 3 AB. Do dilations preserve distance? NO

Confirm this by comparing the other sides: B'C' = 3 BC; C'A' = 3 CA. Is this an isometric transformation? NO

Dilations produce proportional images.

Fill in the proportions: $\frac{A'B'}{AB} = \left(\frac{\text{Image}}{\text{Pre-image}}\right)$ OR $\frac{AB}{A'B'} = \left(\frac{\text{Pre-image}}{\text{Image}}\right)$

(Scale Factor) **(Similarity Ratio)**

How do the scale factor and similarity ratio compare? $\frac{3}{1}$ $\frac{1}{3}$ RECIPROCALS

PRESERVED PROPERTIES UNDER A DILATION

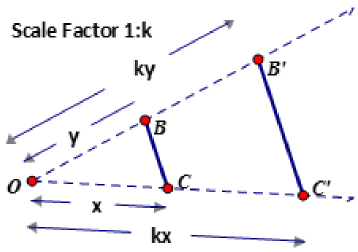
Dilation is NOT an isometric transformation so its properties differ from the ones we saw with reflection, rotation and translation. The following properties are preserved between the pre-image and its image when dilating:

- **Angle measure**
- **Parallelism** (things that were parallel are still parallel)
- **Perpendicularity**
- **Distance IS NOT preserved!!!**
- **Orientation**
- **Slope**
- **Collinearity** (points on a line remain on the line)

COMPARING LENGTHS AFTER A DILATION & DETERMING THE SCALE FACTOR

After a dilation, the pre-image and image have the same shape but not the same size.

The distance from the center of the dilation to each point of the image is equal to the distance from the center of the dilation to each corresponding point of the pre-image figure times the scale factor, OB' = k • OB and OC' = k • OC.



Scale Factor of k:1

OB = y	OB' = ky	ky:y	k:1
OC = x	OC' = kx	kx:x	k:1

The dilation centered at O with a scale factor of $\frac{1}{2}$. OA = 6 cm, OB = 8 cm and AB = 4 cm.

a) A'B' = 2 cm

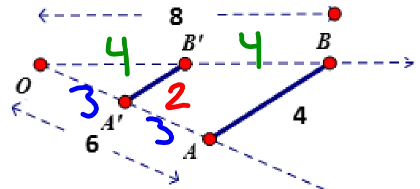
b) OB' = 4 cm

c) OA' = 3 cm

d) AA' = 3 cm

e) What is the ratio of OA':AA'? 3 : 3

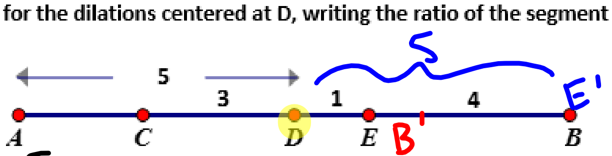
1 : 1



$\frac{1}{2}(AB) = A'B'$

$\frac{1}{2}(4) = A'B'$

Determine the scale factors for the dilations centered at D, writing the ratio of the segment lengths by name:



a) C maps to A $\frac{DA}{DC} = \frac{5}{3}$ $k = \frac{5}{3}$

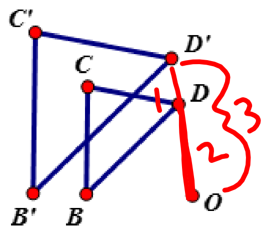
b) E maps to B $\frac{DB}{DE} = \frac{5}{1}$

c) B maps to E $\frac{DE}{DB} = \frac{1}{5}$

Given that triangle BCD was dilated centered at O and DO:D'O is 2:3, what is the scale factor?

Was this a reduction or an enlargement?

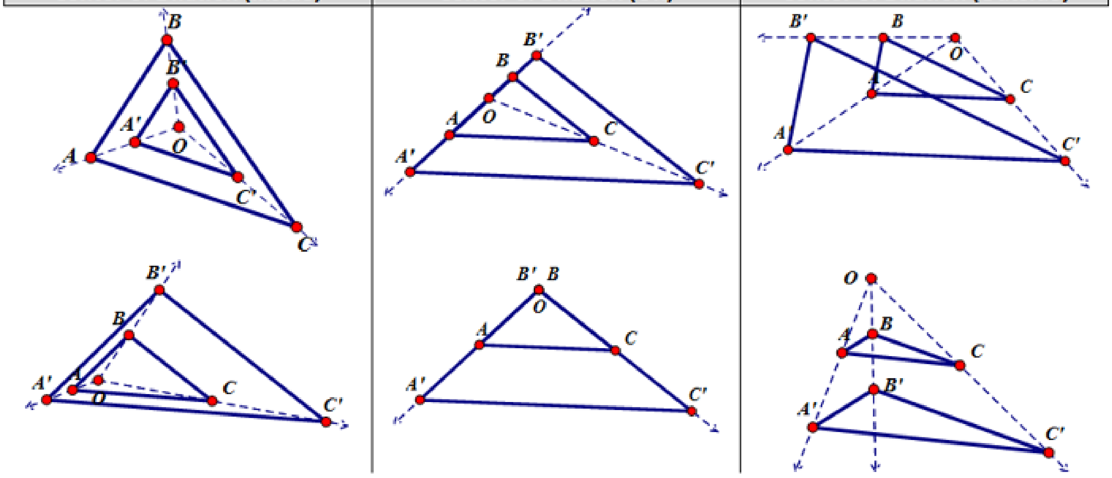
$k = \frac{3}{2} = \frac{D'O}{DO}$



CENTER OF DILATION

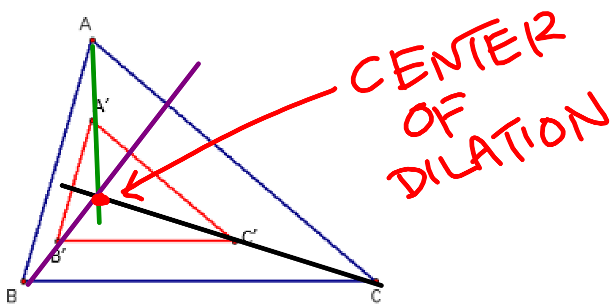
The center of dilation can be located inside, on, or outside of the pre-image.

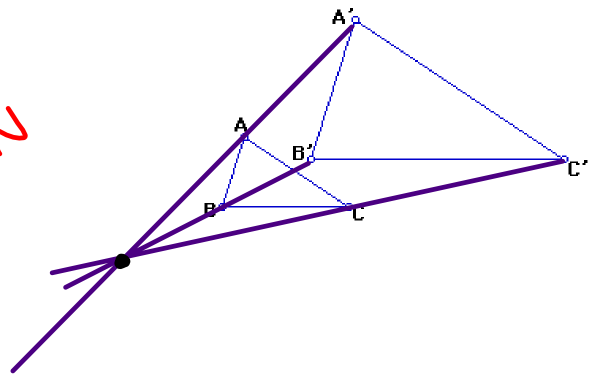
Center of Dilation (Inside)	Center of Dilation (On)	Center of Dilation (Outside)

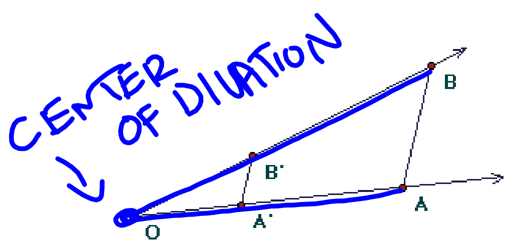


To determine the center of dilation, connect each image to its corresponding pre-image point such that they are collinear. The point of intersection is the center of dilation.

Examples:







PRACTICE

Circle all of the following which are ENLARGEMENTS.

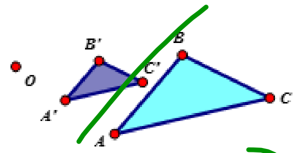
$D_{O,5}(A) = A'$ $K = 5$

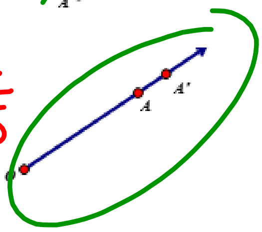
Scale Factor of 2:5
(image : pre-image)

$\frac{\text{IMAGE}}{\text{PRE}} = K = \frac{2}{5}$

$D_{O,0.5}(H) \neq H'$

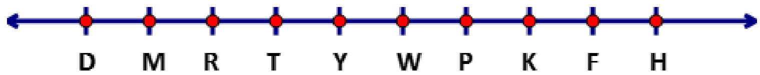
$K = 0.5 < 1$





PROBLEM SET 7-1

1. Determine the missing point.



a) $D_{T,3}(Y) = (\_\_\_\_\_)$ b) $D_{R,\frac{1}{2}}(F) = (\_\_\_\_\_)$ c) $D_{P,4}(\_\_\_\_\_) = (R)$

d) $D_{H,3}(K) = (\_\_\_\_\_)$ e) $^{**}D_{T,-2}(M) = (\_\_\_\_\_)$ f) $D_{F,\frac{1}{8}}(\_\_\_\_\_) = (K)$

**Negative means go the opposite direction from the center of dilation from the center of dilation

2. Determine whether each of the following is a reduction or an enlargement:

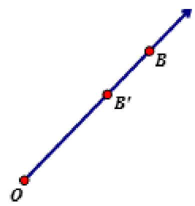
a) Scale Factor of 2:3 (image : pre-image)

Reduction or Enlargement

b) $D_{O,\frac{5}{3}}(G) = G'$

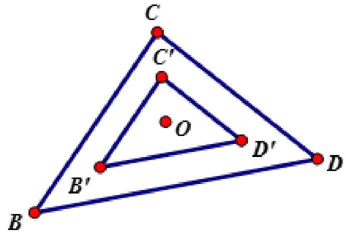
Reduction or Enlargement

c)



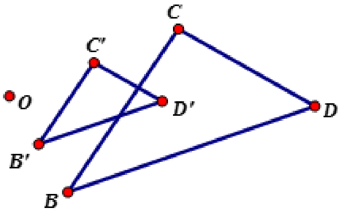
Reduction or Enlargement

d)



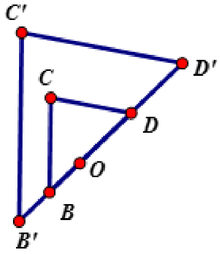
Reduction or Enlargement

e)



Reduction or Enlargement

f)



Reduction or Enlargement

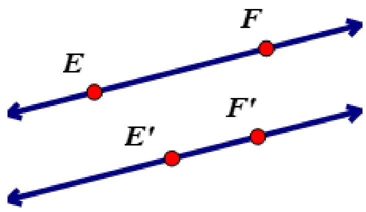
3. Given the dilation of $\overline{EF}$ onto $\overline{E'F'}$,

a. Construct the center of dilation O

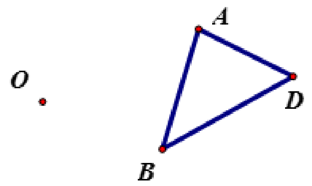
b. Using your compass, determine the scale factor $k = \frac{E'F'}{EF} = \text{——}$

c. Which of the following statements is false?

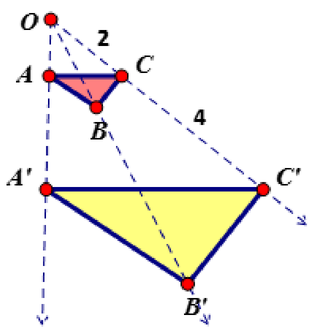
1. $\overline{EF} \parallel \overline{E'F'}$
2. $m\measuredangle EFO = 2(m\measuredangle E'F'O')$
3. $E'F' = \frac{1}{2}(EF)$
4. E and F remain collinear



4. Construct the dilation centered at O of triangle ABD with k=4. (Extra credit: dilate again with k=2.5)



5. Tiffany sees this given dilation and claims that the scale factor is 2 because 4 is twice as big as 2. Is this a scale factor of 2? Explain.



6. Construct $D_{O,\frac{1}{2}}(\Delta ADB)$ (Extra credit: construct again with k=1/4)