Lesson 7-2: Definition of Similar Polygons;

Triangle Similarity Criteria

Agenda

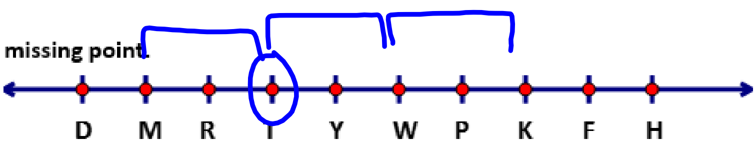
- Check & Review HW 7.1
- Turn in midterm remediation packets
- 7-2 Exploration and Guided Notes - Need compass and ruler

Homework:

- Text: . 465 # 9,10, 19, 20, 23, 24
- p. 475 #11, 12, 22, 23, 24, 31, 32, 36
- CR#6 due next Th. 2/16

PROBLEM SET 7-1

1. Determine the missing point.



a) $D_{T,3}(Y)=(\_\_\_\_\_\_)$ b) $D_{R,\frac{1}{2}}(F)=(\_\_\_\_\_\_)$ c) $D_{P,4}(\_\_\_\_\_\_)=(R)$

d) $D_{H,3}(K)=(\_\_\_\_\_\_)$ e) $^{**}D_{T,-2}(M)=(K)$ f) $D_{F,\frac{1}{8}}(\_\_\_\_\_\_)=(K)$

**Negative means go the opposite direction from the center of dilation from the center of dilation

2. Determine whether each of the following is a reduction or an enlargement:

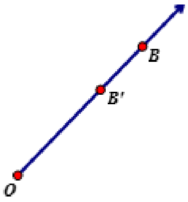
a) Scale Factor of 2:3 (image : pre-image)

Reduction or Enlargement

b) $D_{O,\frac{5}{3}}(G) = G'$

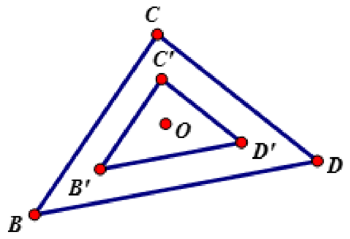
Reduction or Enlargement

c)



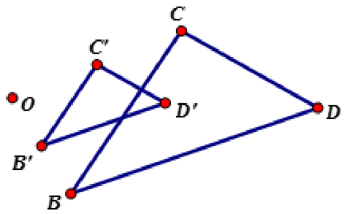
Reduction or Enlargement

d)



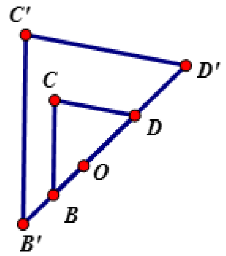
Reduction or Enlargement

e)



Reduction or Enlargement

f)



Reduction or Enlargement

3. Given the dilation of $\overline{EF}$ onto $\overline{E'F'}$,

a. Construct the center of dilation O

b. Using your compass, determine the scale factor k = $\frac{E'F'}{EF} = \_\_\_\_$

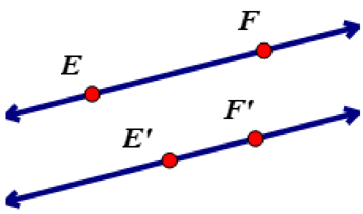
c. Which of the following statements is false?

1. $\overline{EF} \parallel \overline{E'F'}$

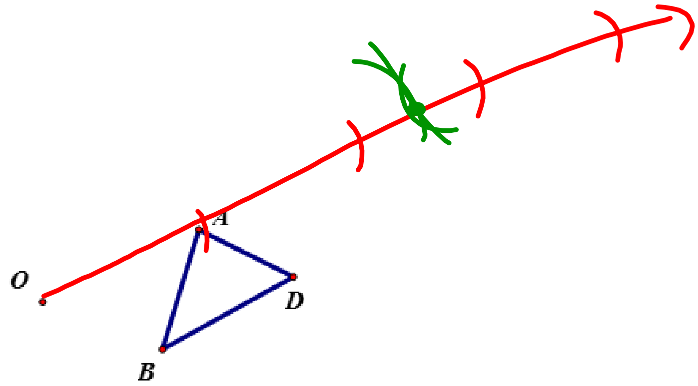
2. $m\measuredangle EFO = 2(m\measuredangle E'F'O')$

3. $E'F' = \frac{1}{2}(EF)$

4. E and F remain collinear

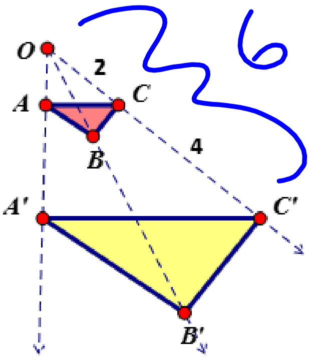


4. Construct the dilation centered at O of triangle ABD with k=4. (Extra credit: dilate again with k=2.5)

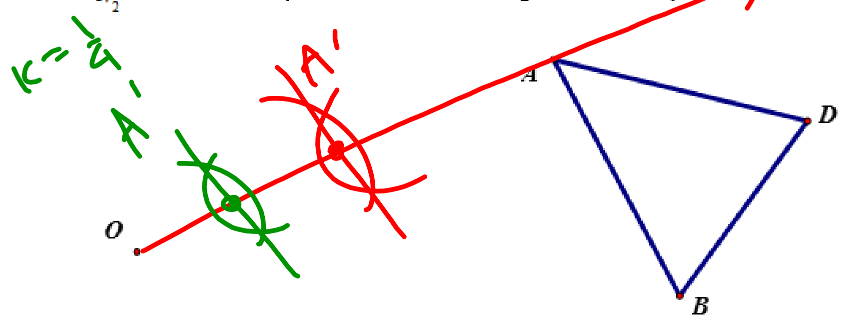


5. Tiffany sees this given dilation and claims that the scale factor is 2 because 4 is twice as big as 2. Is this a scale factor of 2? Explain.

$$\frac{OC'}{OC} = \frac{6}{2}$$



6.   Construct $D_{O,\frac{1}{2}}(\Delta ADB)$ (Extra credit: construct again with k=1/4)

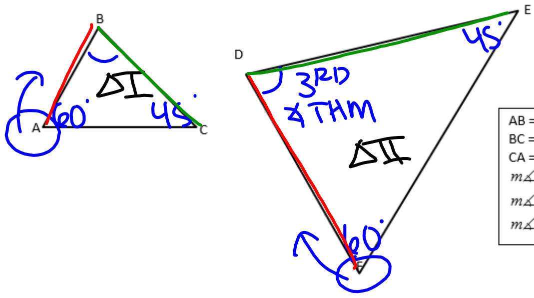


Lesson 7-2R: Similar Polygons & Triangle Similarity Criteria

Definition: Two polygons are similar if and only if their corresponding angles are congruent and their corresponding sides are proportional in the same similarity ratio.

Recall *Scale Factor*: $\frac{\text{Image}}{\text{Pre-image}}$ vs *Similarity Ratio*: $\frac{\text{Pre-image}}{\text{Image}}$

Given the following triangles, use your universal angle maker to record the angle measures. Then determine if the triangles are similar by definition. If so, write the similarity statement and identify the similarity ratio.



AB = 3 cm	DE = 7.4 cm
BC = 3.7 cm	EF = 8 cm
CA = 4 cm	FD = 6 cm
$m\angle A$ = ___	$m\angle D$ = ___
$m\angle B$ = ___	$m\angle E$ = ___
$m\angle C$ = ___	$m\angle F$ = ___

Compare the angles. Are they congruent? How does this set up your correspondence?

YES ΔABC CORRESPONDS TO ΔFDE

Compare the sides. Are they proportional?

$\frac{\Delta I}{\Delta II}: \frac{AB}{FD} = \frac{3}{6} = \frac{1}{2}$

$\frac{\Delta I}{\Delta II}: \frac{BC}{DE} = \frac{3.7}{7.4} = \frac{1}{2}$

$\frac{\Delta I}{\Delta II}: \frac{CA}{EF} = \frac{4}{8} = \frac{1}{2}$

YES ALL = SAME SIM RATIO

Write the similarity statement for the triangles: ΔABC ~ ΔFDE

ΔI ~ ΔII

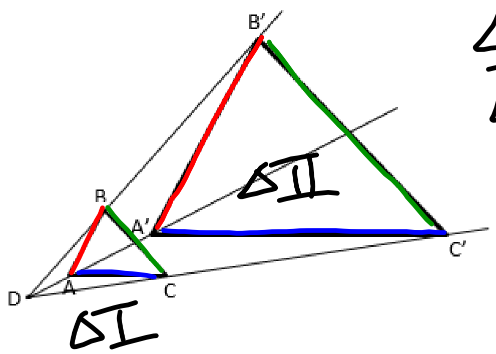
State the similarity ratio which corresponds with your similarity statement: $\frac{\Delta I}{\Delta II}: \frac{1}{2}$

What measurements did you take when determining if the previous triangles were similar? Are their certain criteria (like triangle congruency criteria) that we can use to avoid having to compare all the corresponding sides and angles?

~

Triangle Similarity Criteria: SSS~

Can just proportional sides guarantee similar triangles? Consider that a dilation will preserve angle measure, satisfying half of the definition of similar polygons... If we could get a dilation, we would be sure to have similar polygons....

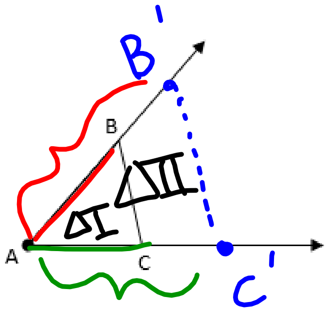


$\frac{\Delta I}{\Delta II}: \boxed{\frac{AB}{A'B'}} = \boxed{\frac{BC}{B'C'}} = \boxed{\frac{AC}{A'C'}}$

→ SIM RATIO →
SCALE FACTOR →
DILATION →
PRESERVED ∡ MEASURE

Triangle Similarity Criteria: SAS~

Consider a triangle that we want to map onto another using the common vertex as the center of dilation (note that we could have used rigid motions to map onto the common vertex first). Explain what would be needed in order to map B to B′ and C to C′ with the center of dilation at A such that $\Delta ABC \sim \Delta A'B'C'$?



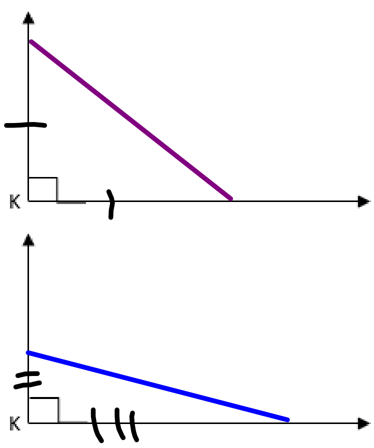
$\frac{\Delta I}{\Delta II}: \quad \frac{AB}{AB'} = \frac{AC}{AC'}$

WITH INCLUDED ∡ $\cong$

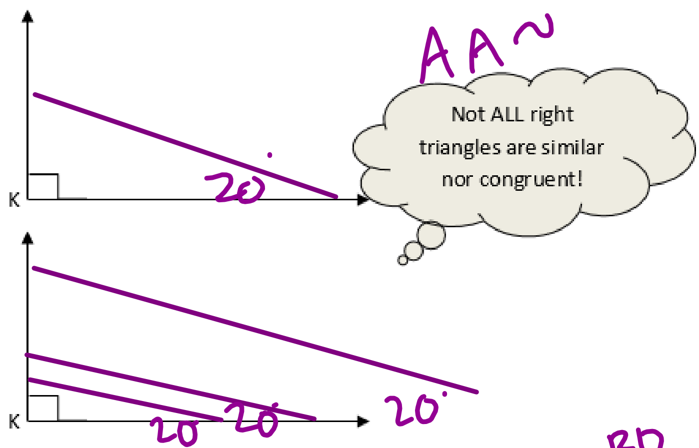
Triangle Similarity Criteria: AA~ Having one angle measure alone was not enough, as we saw in SAS~.

Given that angle K is a right angle, can you draw two triangles such that they are…

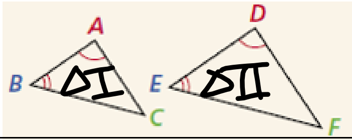
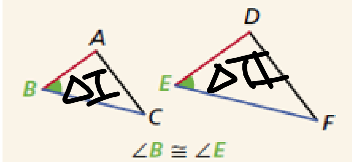
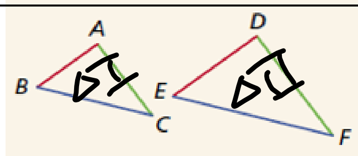
Neither similar nor congruent?



Similar but not congruent? What did you need?



Why does AA~ criteria work? Remember that side-angle relationships in triangles! Why don't we need AAA~? 3RD ∡ THM

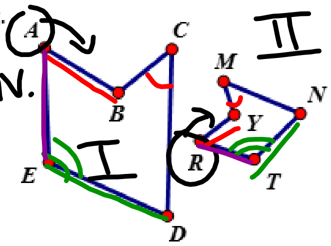
Triangle Similarity Criteria	Example	Similarity Statement	Similarity Ratio
AA~AA		△ABC ~ △DEF ΔI ~ ΔII	$\frac{\Delta I}{\Delta II}$ $\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$
SAS~SAS		△ABC ~ △DEF	
SSS~SSS		△ABC ~ △DEF	

$\frac{1}{3}=\frac{3}{9}$ $\frac{9}{3}=\frac{3}{1}$ $\cancel{\frac{3}{3}=\frac{1}{9}}$

EQUIV. I ~ II

WRITING PROPORTIONS WITH SIMILAR POLYGONS

1) Pentagon ABCDE is similar to Pentagon RYMNT. Complete the following.



∠C ≅ ∠ M $\frac{AB}{RY}=\frac{ED}{TN}$. I SIM / II RATIO $\frac{MN}{RT}=\frac{CD}{AE}$ EQUIV. $\frac{II}{I}=\frac{II}{I}$

∠T ≅ ∠ E $\frac{NT}{DE}=\frac{RT}{AE}$ $\frac{II}{I}$ $\frac{AB}{BC}=\frac{RY}{\ }$

SCALE FACTOR

2) ΔABC is similar to another triangle. Provided is some information about the two triangles, $\frac{BC}{DR}=\frac{AB}{TD}$.

From this information determine the triangle similarity statement.

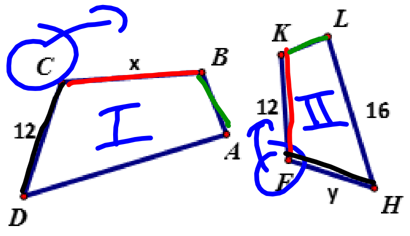
ΔABC ~ Δ TDR

A → T
B → D
C → R

DETERMINING MISSING SIDE LENGTHS

3) Given that the polygons are similar with the given similarity ratio, write two proportions to solve for x & y:

CBAD : FKLH is 3:2 $\quad I \sim II$



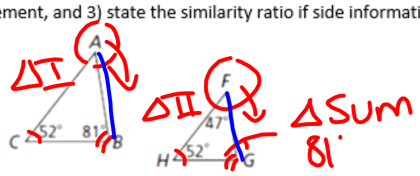
$\frac{I}{II} : \frac{3}{2} = \frac{CB}{FK} \qquad \frac{3}{2} = \frac{CD}{HF}$

$\frac{3}{2} = \frac{x}{12} \qquad \frac{3}{2} = \frac{12}{y}$

DETERMINING AND APPLYING TRIANGLE SIMILARITY CRITERIA

Identify if the sets of triangles meet triangle similarity criteria. If so, 1) state which criteria is met and 2) write a similarity statement, and 3) state the similarity ratio if side information is provided.

1.

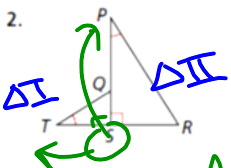


Criteria: AA ~

Statement: $\Delta ABC \sim \Delta FGH$

Similarity Ratio: $\frac{\Delta I}{\Delta II} = \frac{AB}{FG}$

2.

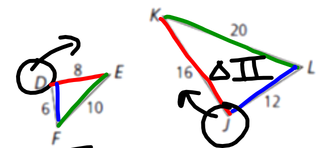


Criteria: AA ~

Statement: $\Delta STQ \sim \Delta SPR$

Similarity Ratio: $\frac{\Delta I}{\Delta II} : \frac{ST}{SP}$

3. $\triangle DEF$ and $\triangle JKL$



ΔI

$\frac{\Delta I}{\Delta II} : \boxed{\frac{8}{16}} = \boxed{\frac{10}{20}} = \boxed{\frac{6}{12}}$

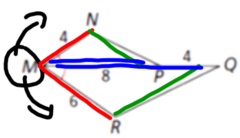
$\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

Criteria: SSS ~

Statement: $\Delta DEF \sim \Delta JKL$

Similarity Ratio: $\frac{1}{2}$

4. $\triangle MNP$ and $\triangle MRQ$



MQ = 12
(8+4)

$\frac{\Delta I}{\Delta II} : \boxed{\frac{4}{6}} = \boxed{\cancel{\frac{NP}{RQ}}} = \boxed{\frac{8}{12}}$

$\frac{2}{3} \quad -- \quad \frac{2}{3}$ ✓

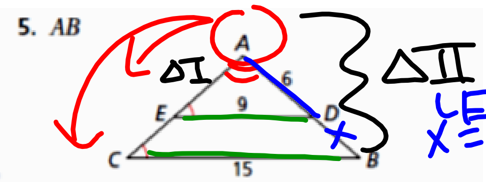
Criteria: SAS ~

Statement: $\Delta MNP \sim \Delta MRQ$

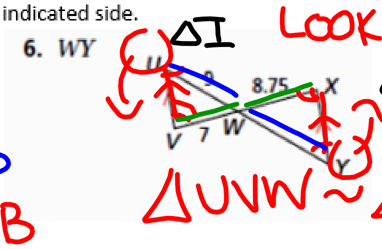
Similarity Ratio: $\frac{2}{3}$

Identify if the sets of triangles meet triangle similarity criteria. If so, 1) state which criteria is met and 2) write a similarity statement, 3) state the similarity ratio, and 4) solve for the indicated side.

5. *AB*



6. *WY*



LOOK FOR ||

AA ~ $\triangle AED \sim \triangle ACB$

$\triangle UVW \sim \triangle YXW$ BY AA ~

$\frac{\Delta I}{\Delta II}$: $\frac{AE}{AC} = \frac{ED}{CB} = \frac{DA}{BA}$

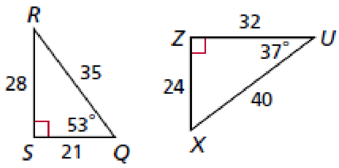
$\frac{\Delta I}{\Delta II}$: $\frac{7}{8.75} = \frac{9}{WY}$

$\frac{9}{15} = \frac{6}{6+x}$

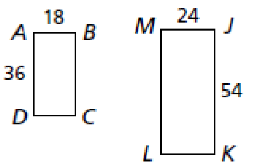
SIM RATIO $\frac{3}{5}$

Multi-Step Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.

9. $\triangle RSQ$ and $\triangle UXZ$

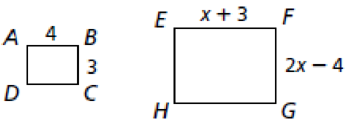


10. rectangles *ABCD* and *JKLM*

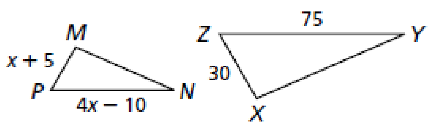


Find the value of *x*.

19. $ABCD \sim EFGH$



20. $\triangle MNP \sim \triangle XYZ$

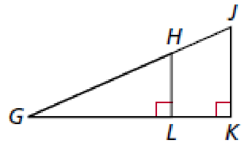


23. $\square JKLM \sim \square NOPQ$. If $m\angle K = 75°$, name two 75° angles in $\square NOPQ$.

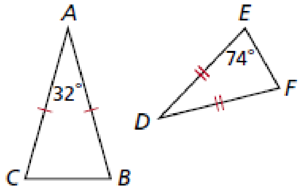
24. A dining room is 18 ft long and 14 ft wide. On a blueprint for the house, the dining room is 3.5 in. long. To the nearest tenth of an inch, what is the width of the dining room on the blueprint?

Explain why the triangles are similar and write a similarity statement.

11.



12.

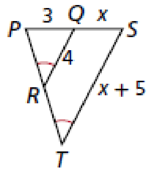


Draw $\triangle JKL$ and $\triangle MNP$. Determine if you can conclude that $\triangle JKL \sim \triangle MNP$ based on the given information. If so, which postulate or theorem justifies your response?

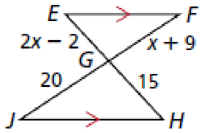
22. $\angle J \cong \angle M, \dfrac{JL}{MP} = \dfrac{KL}{NP}$

Find the value of x.

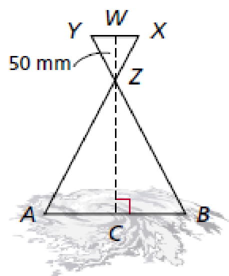
23.



24.



31. Meteorology Satellite photography makes it possible to measure the diameter of a hurricane. The figure shows that a camera's aperture YX is 35 mm and its focal length WZ is 50 mm. The satellite W holding the camera is 150 mi above the hurricane, centered at C.

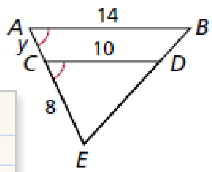


a. Why is $\triangle XYZ \sim \triangle ABZ$? What assumption must you make about the position of the camera in order to make this conclusion?

b. What other triangles in the figure must be similar? Why?

c. Find the diameter AB of the hurricane.

32. **ERROR ANALYSIS** Which solution for the value of y is incorrect? Explain the error.



A

$\triangle ABE \sim \triangle CDE$ by AA $\sim$,
so $\frac{14}{8+y} = \frac{10}{8}$. Then
$10(8 + y) = 8(14)$, or
$80 + 10y = 112$. So
$10y = 32$ and $y = 3.2$.

B

$\triangle ABE \sim \triangle CDE$ by AA $\sim$,
so $\frac{8}{10} = \frac{y}{14}$. Therefore
$8(14) = 10y$, which
means $10y = 112$ and
$y = 11.2$.

36. $\square ABCD \sim \square EFGH$. Which similarity postulate or theorem lets you conclude that $\triangle BCD \sim \triangle FGH$?

Ⓐ AA
Ⓑ SSS
Ⓒ SAS
Ⓓ None of these