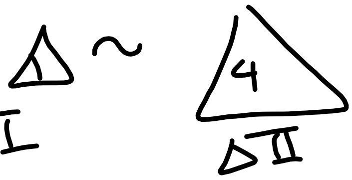
Lesson 7-3R :Triangle Similarity Criteria in Proofs and Constructions



$\frac{\Delta I}{\Delta II}$ = SIM RATIO

Agenda

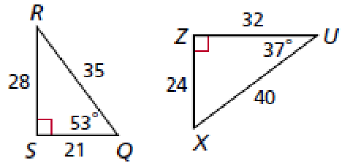
- Check & Review HW 7.2
- 7-3 Exploration and Guided Notes
- Need compass and ruler

Homework:

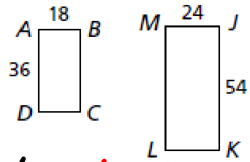
- Proble Set 7.3
- CR#6 due next Th. 2/16

Multi-Step Determine whether the polygons are similar. If so, write the similarity ratio and a similarity statement.

9. $\triangle RSQ$ and $\triangle UXZ$



10. rectangles $ABCD$ and $JKLM$

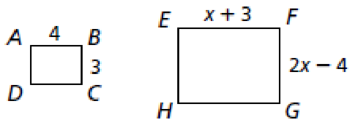


9. yes; $\frac{7}{8}$; $\triangle RSQ \sim \triangle UZX$

10) no

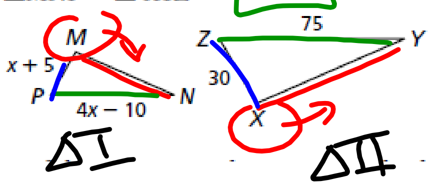
Find the value of x.

19. $ABCD \sim EFGH$ 19) 5



$\frac{\Delta I}{\Delta II}$ $\boxed{\frac{MN}{XY}} = \boxed{\frac{PN}{YZ}} = \boxed{\frac{PM}{XZ}}$ 20) 15

20. $\triangle MNP \sim \triangle XYZ$



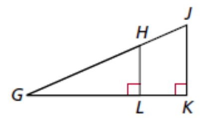
$\frac{4x-10}{75} = \frac{x+5}{30}$

23. $\square JKLM \sim \square NOPQ$. If $m\angle K = 75°$, name two 75° angles in $\square NOPQ$. $\angle O$; $\angle Q$

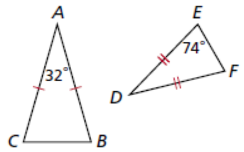
24. A dining room is 18 ft long and 14 ft wide. On a blueprint for the house, the dining room is 3.5 in. long. To the nearest tenth of an inch, what is the width of the dining room on the blueprint? 24) 7.2 in

Explain why the triangles are similar and write a similarity statement.

11.



12.



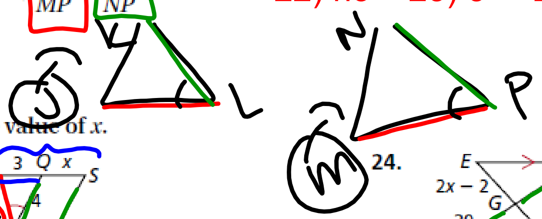
11. It is given that $\angle GLH \cong \angle K$. $\angle G \cong \angle G$ by the Reflex. Prop. of $\cong$. Therefore $\triangle HLG \sim \triangle JKG$ by AA $\sim$.

12. By the Isosc. $\triangle$ Thm., $\angle C \cong \angle B$. By the $\triangle$ Sum Thm. $m\angle C = m\angle B = 74°$. In the same way, $m\angle F = 74°$. So by the def. of $\cong$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$. Therefore $\triangle ABC \sim \triangle DEF$ by AA $\sim$

Draw $\triangle JKL$ and $\triangle MNP$. Determine if you can conclude that $\triangle JKL \sim \triangle MNP$ based on the given information. If so, which postulate or theorem justifies your response?

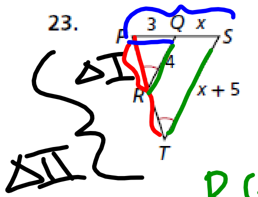
22. $\cancel{\angle J \cong \angle M}$, $\boxed{\frac{JL}{MP}} = \boxed{\frac{KL}{NP}}$

22) no 23) 3 24)7

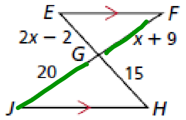


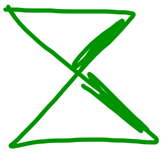
Find the value of x.

23.



24.





$\frac{\triangle I}{\triangle II}$: $\frac{RQ}{TS} = \frac{PQ}{PS}$

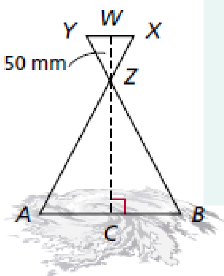
$\frac{4}{x+5} = \frac{3}{x+3}$

31. Meteorology Satellite photography makes it possible to measure the diameter of a hurricane. The figure shows that a camera's aperture YX is 35 mm and its focal length WZ is 50 mm. The satellite W holding the camera is 150 mi above the hurricane, centered at C.

a. Why is $\triangle XYZ \sim \triangle ABZ$? What assumption must you make about the position of the camera in order to make this conclusion?

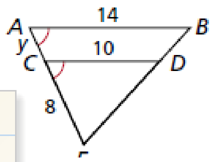
b. What other triangles in the figure must be similar? Why?

c. Find the diameter AB of the hurricane.



31a. The $\triangle$s are $\sim$ by AA assume that the ca the hurricane (that
b. $\triangle YWZ \sim \triangle BCZ$, ar $\triangle ACZ$, also by AA
c. 105 mi

32. **ERROR ANALYSIS** Which solution for the value of y is incorrect? Explain the error.



A

$\triangle ABE \sim \triangle CDE$ by AA $\sim$,
so $\frac{14}{8+y} = \frac{10}{8}$. Then
$10(8 + y) = 8(14)$, or
$80 + 10y = 112$. So
$10y = 32$ and $y = 3.2$.

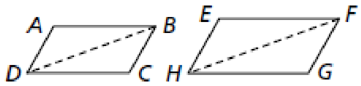
B

$\triangle ABE \sim \triangle CDE$ by AA $\sim$,
so $\frac{8}{10} = \frac{y}{14}$. Therefore
$8(14) = 10y$, which
means $10y = 112$ and
$y = 11.2$.

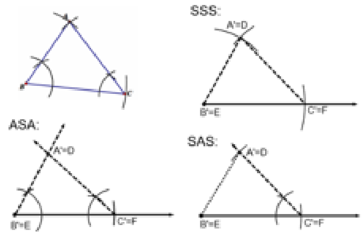
32. Solution B is incorrect. The proportion should be $\frac{8}{10} = \frac{8+y}{14}$.

36. $\square ABCD \sim \square EFGH$. Which similarity postulate or theorem lets you conclude that $\triangle BCD \sim \triangle FGH$?

Ⓐ AA Ⓒ SAS
Ⓑ SSS Ⓓ None of these



CONSTRUCTING SIMILAR TRIANGLES

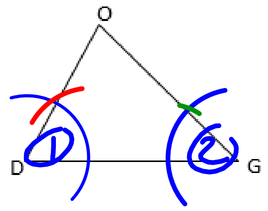


Remember that when we constructed congruent triangles, we used a congruency criteria such as SSS≅, ASA≅, and SAS≅.

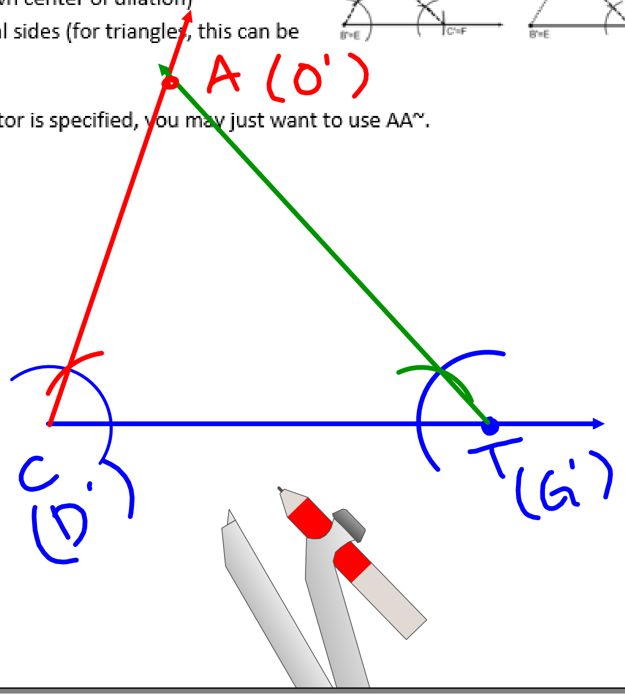
To construct a similar polygon, you can either

- Dilate the given polygon (select your own center of dilation)
- Copy congruent angles and proportional sides (for triangles, this can be a similarity criteria)

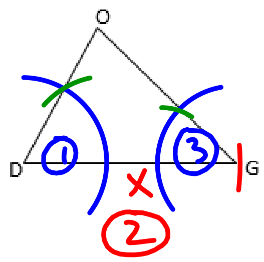
Construct $\Delta CAT \sim \Delta DOG$. Since no scale factor is specified, you may just want to use AA~.



1) COPY ∡D
2) COPY ∡G



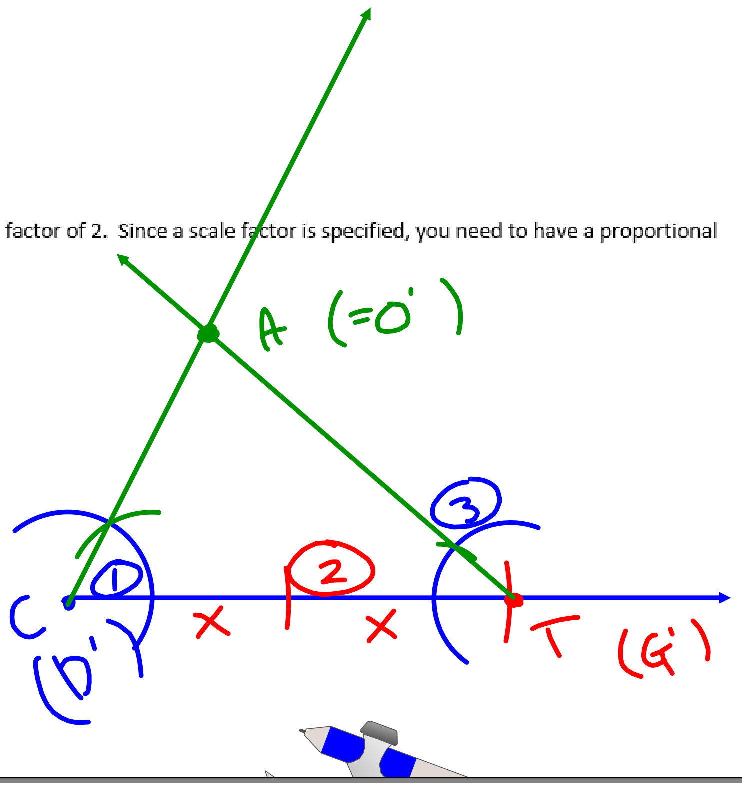
Construct $\Delta CAT \sim \Delta DOG$. Since no scale factor is specified, you may just want to use AA~.

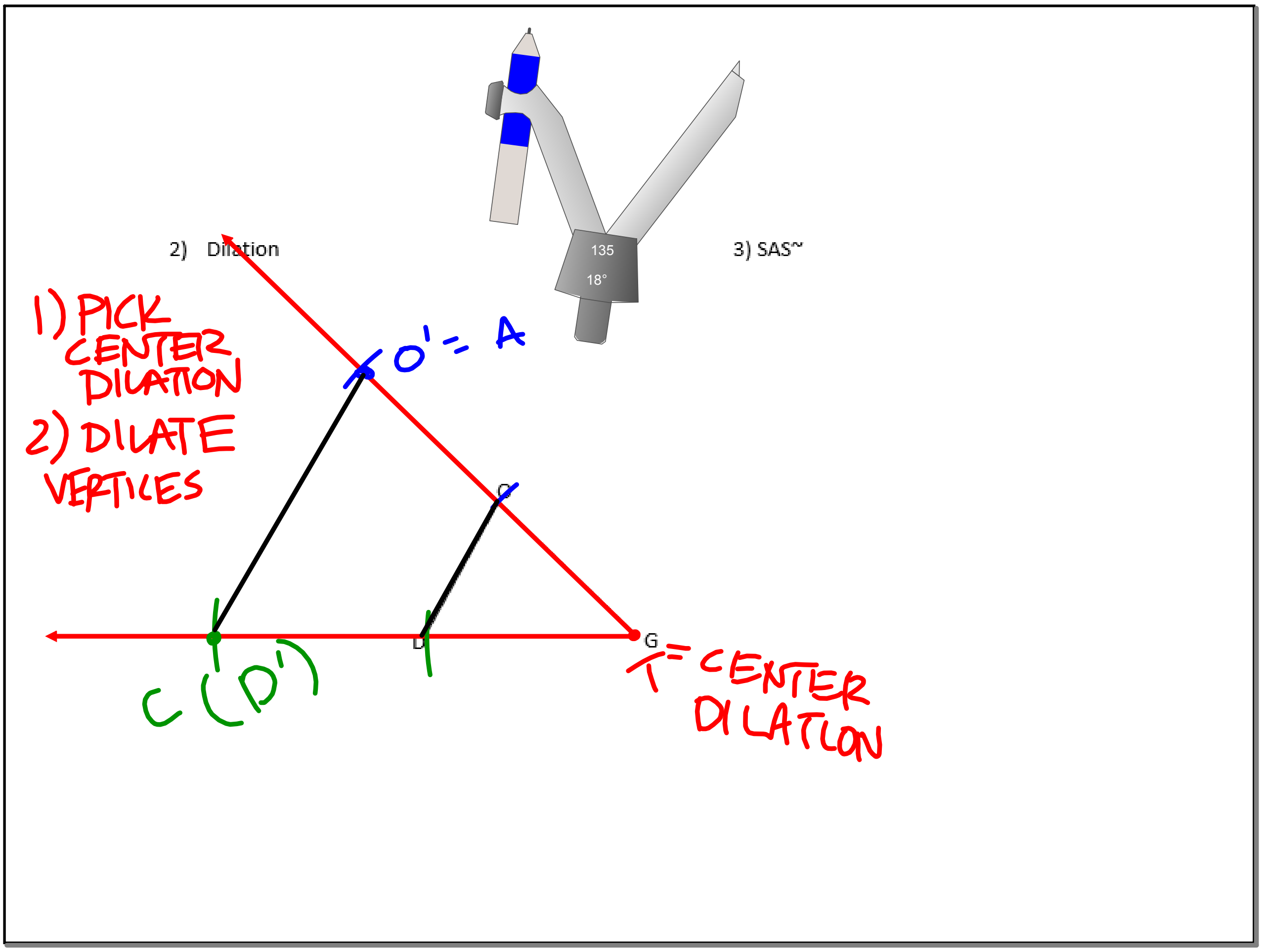


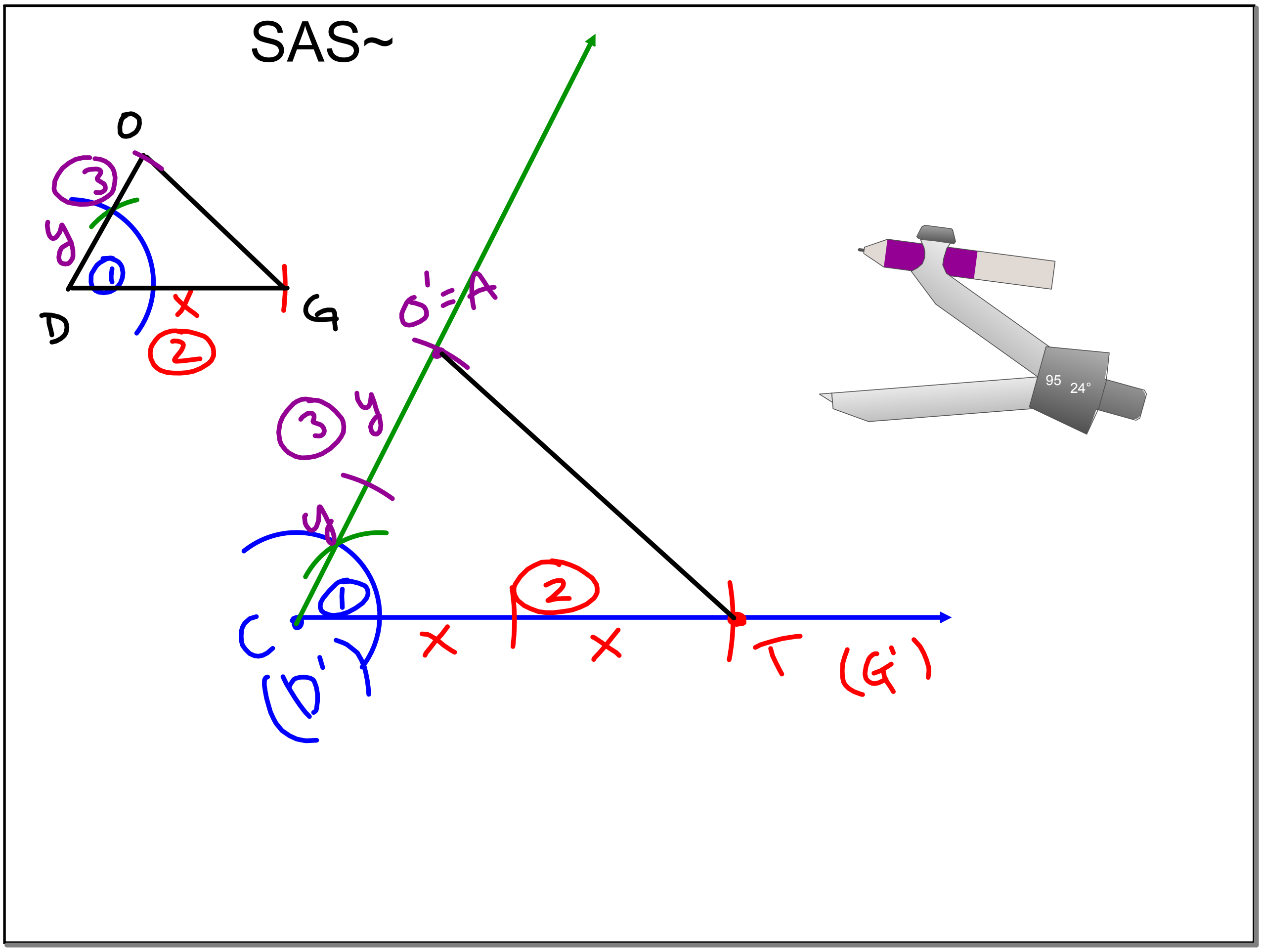
Construct $\Delta CAT \sim \Delta DOG$ with a scale factor of 2. Since a scale factor is specified, you need to have a proportional side length. Options:

1) AA~ with an included side length

1) COPY ∡D
2) DILATE $\overline{DG}$
3) COPY ∡G



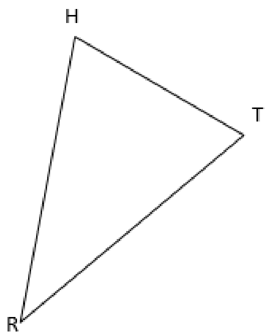

2) Dilation
3) SAS~
135
18°
1) PICK CENTER DILATION
2) DILATE VERTICES
O' = A
O
D
G
C (D')
T = CENTER DILATION


SAS~
O
3
y
1
D
x
2
G
O' = A
3
y
y
95
24°
C
(D')
1
x
2
x
T (G')

Finish for HW

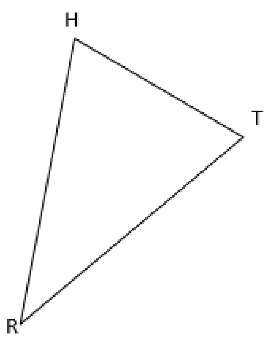
PRACTICE:

1) Construct $\Delta LUV \sim \Delta HRT$ a similar triangle with a scale factor of ½ using your choice of method.



Finish for HW:

2) Construct $\Delta LUV \sim \Delta HRT$ a similar triangle with a scale factor of $\frac{3}{2}$ using a different method from #1.

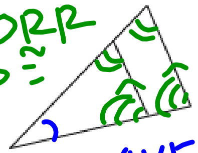


Geometry + LAB Name ______________________________ Section __________ Date _________

7-3R / 7-4L Notes: Proving Triangles Similar with AA ~ , SAS ~ , SSS ~

HINTS FOR PROVING TRIANGLES SIMILAR:

- Since AA ~ is the easiest, try that first. Don't forget to look for:
  - a reflexive angle
  - vertical angle pairs
  - congruent alternate interior or corresponding angles formed by parallel lines cut by a transversal.



Note that these triangles do not have parallel lines!

- If you are given that one side is a factor of another (ie: $AB = 2AD,\ AC = 2AE$), think SAS~ or SSS~ :
  - Isolate the number using the division property of equality $\left(\frac{AB}{AD} = 2,\ \frac{AC}{AE} = 2\right)$

    SIM RATIO

  - Then use substitution property to set the two/three ratios equal to each other $\left(\frac{AB}{AD} = \frac{AC}{AE}\right)$
  - Remember to include the congruent angle pair if you are using SAS~.

GOAL: $\frac{\Delta I}{\Delta II} = \frac{\Delta I}{\Delta II}$

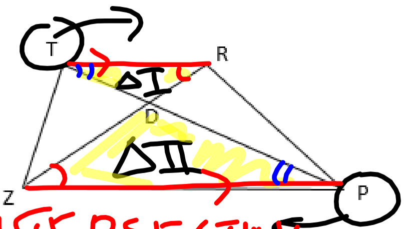
HINTS FOR PROVING PARTS OF SIMILAR TRIANGLES ARE CONGRUENT OR PROPORTIONAL:

- If you are asked to prove that the product of 2 sides is equal to the product of 2 sides $(DC \cdot AC = BC \cdot EC)$
  - Before you start your proof, think backwards using the cross products property to write a proportion with $\frac{\Delta 1}{\Delta 2} = \frac{\Delta 1}{\Delta 2}$ such as $\left(\frac{BC}{DC} = \frac{AC}{EC}\right)$. Now you know which pairs of sides you need to prove proportional.
  - Prove the triangles containing these sides are similar using AA~ (cannot use SAS~ or SSS~ if proving sides)
  - Then use $\sim\Delta s \rightarrow$ proportional sides to state that $\frac{BC}{DC} = \frac{AC}{EC}$
  - Finally, use the cross products property to say that $DC \cdot AC = BC \cdot EC$

- If you are asked to prove that 2 angles are congruent,
  - Prove the triangles are similar by a similarity theorem SSS~ or SAS~
  - Then take out the corresponding angles by $\sim\Delta s \rightarrow\ \cong \measuredangle s$

Congruent Triangles VS **Similar Triangles**

Statement	Reason	Statement	Reason
ΔI≅ΔII	SSS≅, SAS≅, AAS≅, ASA≅, RHL≅	ΔI~ΔII	SSS~, SAS~, AA~
Part≅Part	CPCTC	∡≅∡ or $\frac{Side1}{Side2} = \frac{Side1}{Side2}$	$\sim\Delta s \rightarrow$ corresponding $\measuredangle s \cong$ OR $\sim\Delta s \rightarrow$ corresponding sides proportional
		side1•side2=side2•side1	Cross Products Property



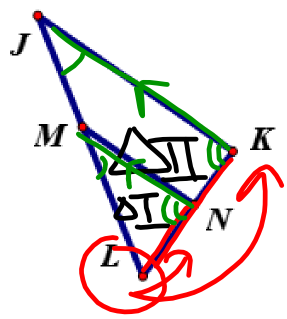
1. Given: Trapezoid TRPZ with diagonals intersecting at D
   Prove: $\Delta TRD \sim \Delta PZD$

SINCE IT'S GIVEN TRPZ IS TRAP W/DIAG INTERSECTING AT D, THEN $\overline{TR} \parallel \overline{PZ}$ BY DEFN OF TRAPEZOID.

SINCE $\parallel \rightarrow$ ALT INT ∡'S ≅, THEN ∡TRD ≅ ∡PZD & ∡RTD ≅ ∡ZPD.

A A

∴ $\Delta TRD \sim \Delta PZD$ BY AA~.



2. Given: $\overline{JK} \parallel \overline{MN}, \overline{JML}, \overline{JNK}$ (could also be given a midsegment)

   Prove: $\Delta LNM \sim \Delta LKJ$

A A ~

$\boxed{JK \parallel MN, JML, JNK}$ GIVEN

↓ ↓

$\boxed{∡LNM ≅ ∡K; ∡LMN ≅ ∡J}$ $\parallel \rightarrow$ CORRESP. ∡'S ≅

$\Delta LNM \sim \Delta LKJ$ BY AA~.

① $\Delta I \sim \Delta II$ BY AA~

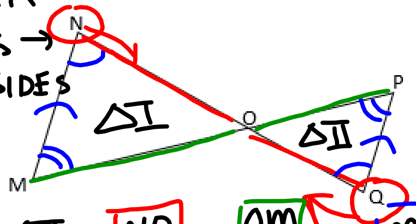
3. Given: $\overline{MN} \parallel \overline{PQ}$

② $\frac{—}{—} = \frac{—}{—}$ ~Δ's → PROP SIDES

Prove: $QO \cdot OM = NO \cdot OP$

How does this help set up your correspondence?

③ CROSS PRODUCTS



$\frac{\Delta I}{\Delta II} : \boxed{\frac{NO}{QO}} = \boxed{\frac{OM}{OP}} = \boxed{\frac{MN}{PQ}}$

1) $\overline{MN} \parallel \overline{PQ}$ 1) GIVEN

2) $\angle N \cong \angle Q$ (A) 2) ∥ → ALT INT ∡'S ≅
$\angle M \cong \angle P$ (A)

3) $\Delta NOM \sim \Delta QOP$ 3) AA~ (STEPS 2, 2)

4) $\frac{NO}{QO} = \frac{OM}{OP}$ 4) SIMILAR TRIANGLES HAVE CORRESP PROPORTIONAL SIDES

5) $QO \cdot OM = NO \cdot OP$ 5) CROSS PRODUCTS PROPERTY

~~AA~~~ ~~SSS~~~ SAS~

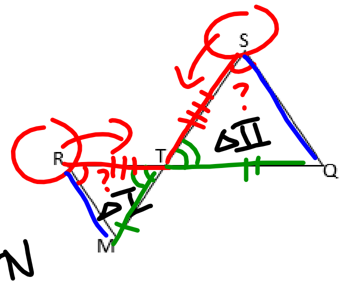
4. Given: $RT \cdot TQ = TM \cdot ST$

$\frac{\Delta I}{\Delta II} : \frac{RT}{ST} = \frac{TM}{TQ}$

Prove: 1) $\Delta RTM \sim \Delta STQ$
2) $\angle R \cong \angle S$

SAS~

~Δ's → ≅ ∡'S



1) $RT \cdot TQ = TM \cdot ST$ 1) GIVEN

2) $\frac{RT}{ST} = \frac{TM}{TQ}$ (S) (S) 2) DIVISION PROP OF EQ.

3) $\angle RTM \cong \angle STQ$ (A) 3) VERTICAL ∡'S ARE ≅

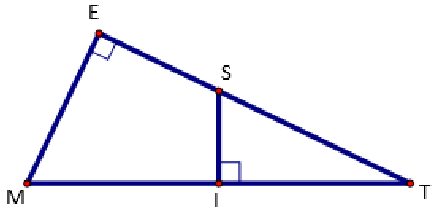
4) $\Delta RTM \sim \Delta STQ$ 4) SAS~ (2, 3, 2)

5) $\angle R \cong \angle S$ 5) SIMILAR Δ'S HAVE CORRESP ∡'S ≅

Problem Set 7-3R / 7-4L

1. Given: $\overline{TE} \perp \overline{EM}, \overline{SI} \perp \overline{IT}$

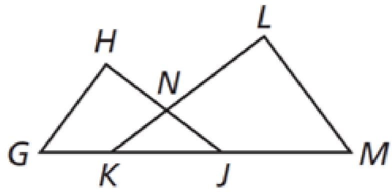
   Prove: $\Delta TIS \sim \Delta TEM$



2. **Given:** $\triangle KNJ$ is isosceles with $\angle N$ as the vertex angle.
   $\angle H \cong \angle L$

   Prove: 1) $\Delta GHJ \sim \Delta MLK$

   2) $\dfrac{GH}{ML} = \dfrac{HJ}{LK}$



3. **Given:** $\frac{PR}{MR} = \frac{QR}{NR}$

Prove: 1) $\Delta PRQ \sim \Delta MRN$
2) $\overline{MN} \parallel \overline{PQ}$