Midterm Review – Unit 4 (Proofs)

Commonly Used Reasons in Proofs

Right ∡s

- ⊥ lines → right ∡s
- All right ∡s are ≅
- DEFN: Right angle measures 90°
- Right ∡s → Right Δ's
- Defn: A triangle with 1 right ∡ is a right Δ

Complementary/ Supplementary

- DEFN: 2 ∡s whose measures add to 90 ↔ complementary ∡s
- DEFN: 2 ∡s whose measures add to 180 ↔ supplementary ∡s
- DEFN: 2 Adjacent ∡s whose noncommon sides form opposite rays ↔ Linear pair
- Linear pair → supplementary angles
- ≅ Linear pair → right angles
- ≅ Linear pair → ⊥ lines
- Congruent Complements Theorem OR Complements of the same angle (or ≅ ∡s) are ≅
- Congruent Supplements Theorem OR Supplements of the same angle (or ≅ ∡s) are ≅

Bisectors

- Midpoint↔ 2 ≅ collinear segments
- Segment bisector ↔ 2 ≅ collinear segments
- Angle bisector↔ 2 ≅ adjacent angles
- ⊥ bisector→right angles AND ⊥ bisector → midpoint
- DEFN: A median is a segment from a vertex to the midpoint of the opposite side
- DEFN: A midsegment is a segment whose endpoints are the midpoints of two sides in a Δ

≅ ↔ =

- ≅ segments ↔ segments with = measure
- ≅ angles ↔ angles with = measure

Vertical ∡s

- DEFN: Non-adjacent ∡s formed by intersecting lines are vertical ∡s
- Vertical ∡s are ≅ (theorem)

Properties

- Reflexive property of equality/congruence
- Symmetric property of equality/congruence
- Transitive property of equality/congruence

Note: Algebraic Proofs may also contain:

- *Addition or Subtraction*
- *Division or Multiplication*
- *Simplification*

Sum of parts

- Segment addition postulate
- ∡ addition postulate
- Common Segment Theorem *(remember to state reflexive piece)*
- Common Angle Theorem *(remember to state reflexive piece)*
- Halves of congruent angles are congruent
- Halves of congruent segments are congruent
- Consecutive adjacent angles on a line sum to 180°
- Angles at a point sum to 360°
- ∥ lines ↔ ≅ corresponding ∡s

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∥ lines

- ∥ lines ↔ ≅ alternate interior ∡s
- ∥ lines ↔ ≅ alternate exterior ∡s
- ∥ lines ↔ supplementary same side interior ∡s
- Midsegment of a Δ → ½ the length of the side it is parallel to

⊥ lines

- A line ⊥ to 1 of 2 ∥ lines → ⊥ to the other
- 2 lines ⊥ to the same line → ∥ lines

  } *Parallel lines are perpendicular to the same line*
- DEFN: An altitude is ⊥ from a vertex to the opposite side

Δ congruence

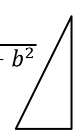
- SSS ≅ SSS → ≅ Δ*s*
- SAS ≅ SAS → ≅ Δ*s*
- ASA ≅ ASA → ≅ Δ*s*
- AAS ≅ AAS → ≅ Δ*s*
- RtΔ HL ≅ RtΔ HL → ≅ Δ*s*

After proving ≅ Δ*s*,
- CPCTC
- ≅ Δ*s* ⟶ corresponding angles ≅
- ≅ Δ*s* ⟶ corresponding sides ≅

Isosceles Triangles

- Isosceles triangle → 2 ≅ base angles (Isosceles triangle theorem)
- DEFN of an isosceles triangle ⟷ 2 ≅ sides in a triangle
- Converse of the isosceles triangle thm: if 2 ∡'s in a Δ are ≅ → their opposite sides are ≅

Algebraic tools used in Coordinate Proofs:

	Purpose & Implication(s)	Concept	Formula
Slope Formula	• Prove lines/segments are parallel: $m_1 = m_2$ →establish corr/alt int/ alt ext angles are congruent • Prove lines/segments are perpendicular: $m_1 \bullet m_2 = -1$ → get right angles/altitudes	$\frac{Rise}{Run}$	$m = \frac{\triangle y}{\triangle x} = \frac{y_2 - y_1}{x_2 - x_1}$
Distance Formula	Find the lengths of segments → conclude congruent segments or sides	Pythagorean Theorem hypotenuse length = distance  c = $\sqrt{a^2 + b^2}$ $b = y_2 - y_1$ $a = x_2 - x_1$	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint Formula	Find the coordinates of the midpoint of a segment →Use midpoint OR →Establish a midsegment	(Avg *x* values, Avg *y* values)	$mdpt = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Point-Slope Equation of a line: $y - y_1 = m(x - x_1)$ where (x_1, y_1) is a point on the line

Slope-Intercept Equation of a line: $y = mx + b$ where (0, b) is the y-intercept

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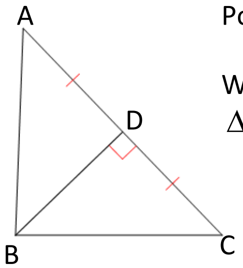
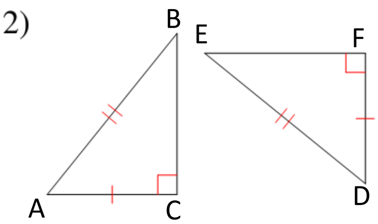
Exercises 1-8: State the postulate or theorem you would use to prove the triangles below congruent. If not possible, state ***not possible***. Then identify the rigid motion that would map one triangle onto the other.

1)


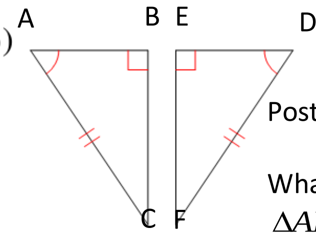
Postulate:

What rigid motion maps ΔABD to ΔCBD?

2)


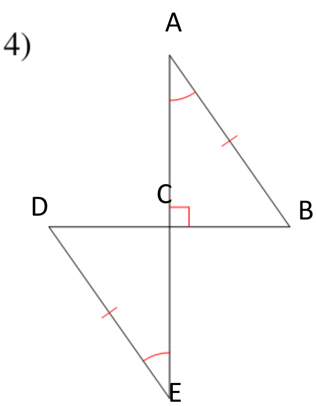
Postulate:

What rigid motion maps ΔABC to ΔDEF?

3)


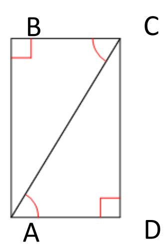
Postulate:

What rigid motion maps ΔABC to ΔDEF?

4)


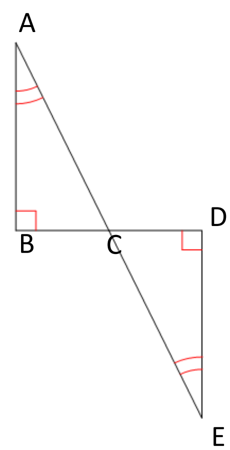
Postulate:

What rigid motion maps ΔABC to ΔEDC?

5)


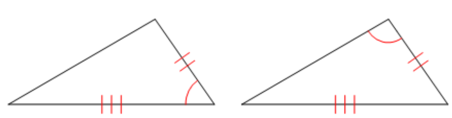
Postulate:

What rigid motion maps ΔABC to ΔCDA?

6)


Postulate:

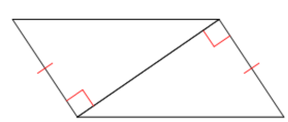
What rigid motion maps ΔABC to ΔEDC?

7)


C

Postulate:

What rigid motion maps ΔABC to ΔDEF?

8)


Postulate:

What rigid motion maps ΔABD to ΔCDB?

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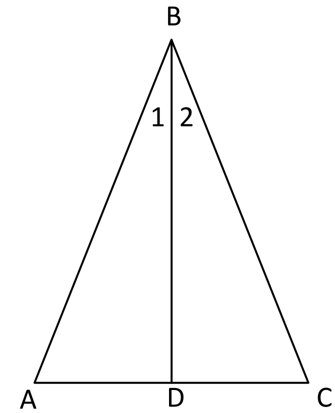
Complete the following Proofs.

1. Given: $\triangle$ ABC is isosceles w/ vertex B
   $\overline{BD}$ is an altitude of $\triangle$ABC

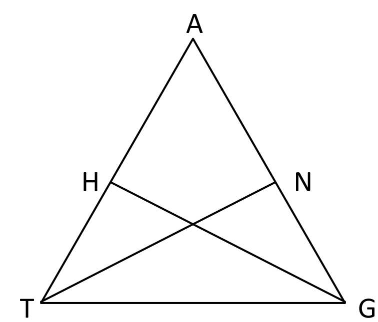
   Prove: $\angle 1 \cong \angle 2$



STATEMENTS	REASONS

2. Given: $\overline{GH} \perp \overline{TA}, \overline{TN} \perp \overline{GA}, \Delta TAG \text{ is isosceles } w/\text{base } \overline{TG}$

   Prove: $\overline{HG} \cong \overline{NT}$



STATEMENTS	REASONS

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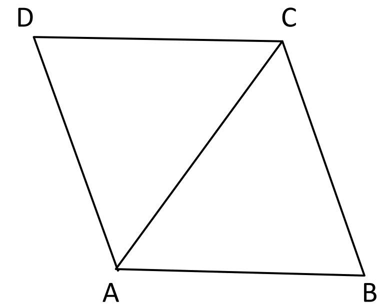
3. Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AC}$ bisects $\angle DAB$

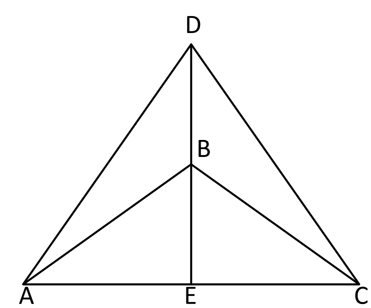
   Prove: $\overline{AD} \cong \overline{CD}$



STATEMENTS	REASONS

4. Given: E is midpoint of $\overline{AC}$, $\overline{AB} \cong \overline{CB}$

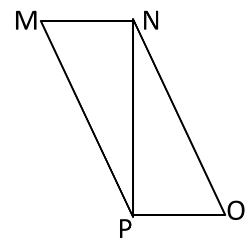
   Prove: $\Delta AED \cong \Delta CED$



STATEMENTS	REASONS

5. Given: $\overline{MP} \cong \overline{ON}$, $\overline{MN} \perp \overline{NP}$, $\overline{OP} \perp \overline{NP}$

Prove: $\overline{MN} \cong \overline{OP}$



STATEMENTS	REASONS

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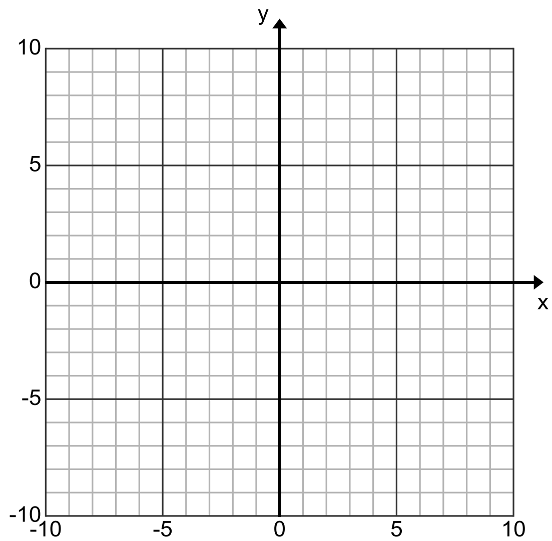
Tips for Coordinate Proofs:

1. Sketch in the coordinate plane.
2. Determine which tool to use:
   - **Slope** – prove line relationships:
     - Equal → || lines
     - Product = -1 → ⊥ lines
     - Opposite reciprocals → ⊥ lines
   - **Distance** – prove lengths are equal and therefore segments are ≅
   - **Midpoint** – show segment is bisected or segments bisect each other
3. Write concluding statements.

6. Given: A(-5,6), B(3,6), and C(-1,0) are vertices of ΔABC.
   E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$, and G is the midpoint of $\overline{CA}$.

   Prove: The midsegment ΔEFG is an isosceles triangle.



7. **Given**: ΔCOR with $\overline{OE}$ drawn.

Prove: $\overline{OE}$ is the median of ΔCOR.