Agenda- 6.8

More Quadrilateral Proofs in the Coordinate Plane

- Check HW 6.7
- Guided Notes 6.8

HW - Worksheet 6.8

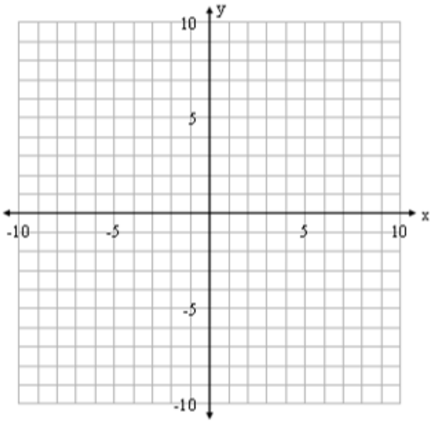
8) Given T(-4,5) and R(8,-1) are two vertices of trapezoid TRAP, which of the following equations could contain the base $\overline{AP}$? Explain why the others cannot.

a. $y = -\frac{1}{2}x + 3$

b. $y - 1 = -\frac{1}{2}(x - 4)$

c. $y + 1 = \frac{2}{1}(x - 8)$

d. $y = -\frac{1}{2}x + 8$



9) Given R(-2,1), locate the vertex O of rhombus SOCR if the line containing diagonal $\overline{SC}$ has the equation:

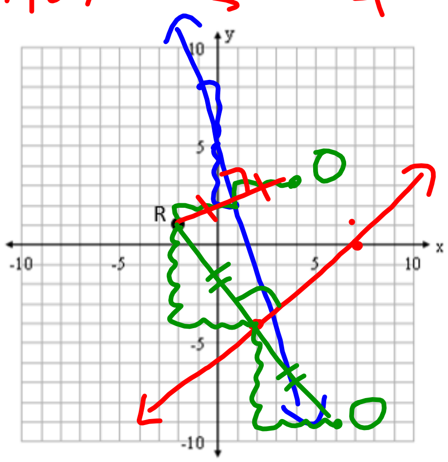
⊥ BIS ↗ LINE OF REFL $\overline{\text{OR}}$ DIAG.

A) $y = -\frac{3}{1}x + 5$

$-\frac{3}{1} \perp +\frac{1}{3}$

B) $y - 0 = \frac{4}{5}(x - 7)$

(7,0) $\frac{4}{5} \perp -\frac{5}{4}$



Lesson 6-8: Quadrilateral Proofs in the Coordinate Plane

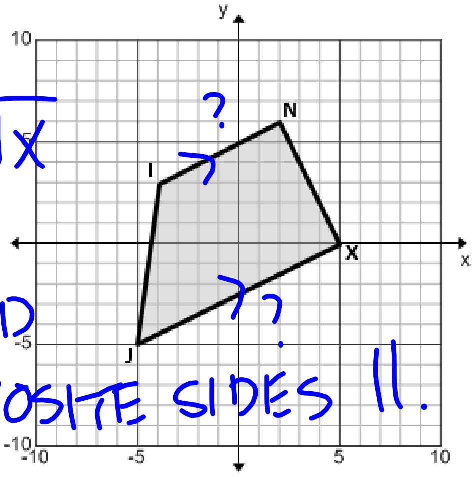
COMPLETE ON LESSON SUMMARY

Use the coordinate plane tools of slope, distance, and midpoint in order to prove relationships among sides and angles that satisfy conditions for quadrilaterals (fill in your Lesson Summaries).

- Slope: prove two sides/segments are parallel or perpendicular. Remember, you need to connect perpendicular to a RIGHT ∡ for a rectangle or right triangle.
- Distance: prove two sides/segments are congruent.
- Midpoint: locate or prove a point is the midpoint of a segment (→ segment bisector).

Ways to prove a quadrilateral is a parallelogram:		
Theorem/Condition	Diagram	Formula
Show 2 pairs of opposite sides **Parallel**	→ Parallelogram	4 SLOPES
Show 2 pairs of opposite sides **Congruent**	→ Parallelogram	4 DISTANCES
Show 1 pair of opposite sides **Both Parallel and Congruent**	→ Parallelogram	2 SLOPES 2 DISTANCES } SAME PAIR
Show 2 pairs of opposite angles **Congruent**	→ Parallelogram	n/a
Show diagonals **Bisect Each Other**	→ Parallelogram	2 MIDPOINTS
Show an angle is supplementary to **Both Its Consecutive Angles**	3 SUPP → Parallelogram	n/a

1. The vertices of quadrilateral JINX are J(-5,-5), I(-4,3), N(2,6), X(5,0). Prove by means of coordinate geometry that the JINX is a trapezoid.



$m_{\overline{IN}} = \frac{3}{6} = \frac{1}{2}$

$m_{\overline{JX}} = \frac{5}{10} = \frac{1}{2}$ } $\overline{IN} \parallel \overline{JX}$

JINX IS A TRAPEZOID SINCE 1 PAIR OPPOSITE SIDES ∥.

2. Prove using coordinate geometry that the quadrilateral PQRS with vertices P(-8,-6), Q(-5,-1), R(1,-5), S(-2,-10) is a parallelogram by each of the following methods:

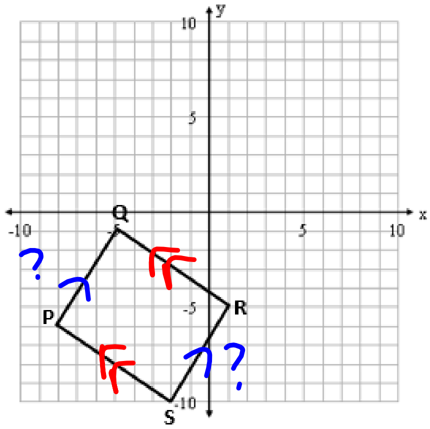
A. Definition – BOTH sets of opposite sides are parallel

$m\overline{PQ} = \frac{5}{3}$

$m\overline{RS} = \frac{5}{3}$

$m\overline{PS} = -\frac{2}{3}$

$m\overline{QR} = -\frac{2}{3}$



Since $m\overline{PQ} = \frac{5}{3} = m\overline{SR}$, then $\overline{PQ} \parallel \overline{SR}$.

Since $m\overline{QR} = -\frac{2}{3} = m\overline{PS}$, then $\overline{QR} \parallel \overline{PS}$.

Since both sets of opposite sides are parallel, then quadrilateral PQRS is a PARALLELOGRAM

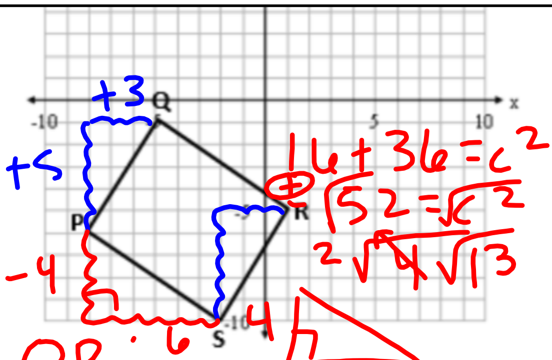
B. BOTH sets of opposite sides are congruent

PQ: RS: 3, 5

$3^2 + 5^2 = c^2$

$9 + 25 = c^2$

$\sqrt{34} = \sqrt{c^2}$



QR: PS: 6, 4

$4^2 + 6^2 = c^2$

$16 + 36 = c^2$

$\sqrt{52} = \sqrt{c^2}$

$2\sqrt{4}\sqrt{13}$

Since $PQ = \sqrt{34} = SR$, then $\overline{PQ} \cong \overline{SR}$.

Since $QR = 2\sqrt{13} = PS$, then $\overline{QR} \cong \overline{PS}$.

Since both sets of opposite sides are congruent, then quadrilateral PQRS is a PARALLELOGRAM

C. One set of opposite sides is BOTH parallel and congruent

Since $m\overline{PQ} = \frac{5}{3} = m\overline{SR}$, then $\overline{PQ} \parallel \overline{SR}$.

Since $PQ = \sqrt{34} = SR$, then $\overline{PQ} \cong \overline{SR}$

Since one set of opposite sides is both parallel and congruent, then quadrilateral PQRS is a parallelogram.

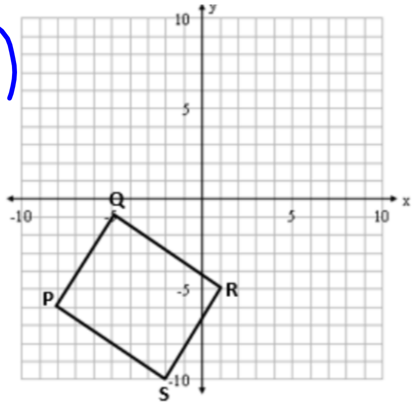
D. Diagonals bisect each other

MDPT $\overline{QS}$: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(-\frac{7}{2}, -\frac{11}{2}\right)$

Since the midpoint of diagonal $\overline{QS}$ is $\left(-\frac{7}{2}, -\frac{11}{2}\right)$, and the midpoint of diagonal $\overline{PR}$ is $\left(-\frac{7}{2}, -\frac{11}{2}\right)$, then the diagonals have a common midpoint and thus BISECT EACH OTHER.
Therefore, quadrilateral PQRS is a parallelogram.

MDPT $\overline{PR}$: $\left(\frac{-8+1}{2}, \frac{-6+-5}{2}\right)$

$\left(-\frac{7}{2}, -\frac{11}{2}\right)$



COMPLETE ON LESSON SUMMARY

APPLYING SPECIAL CONDITIONS

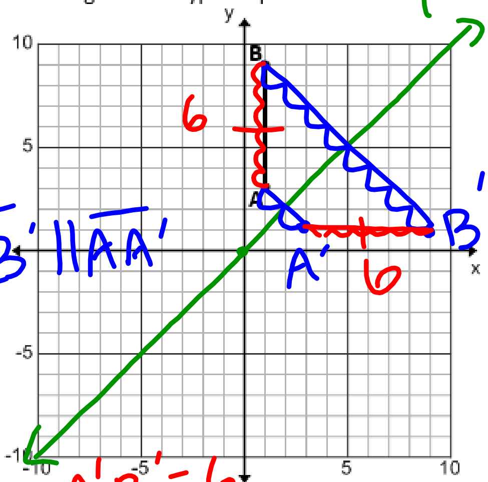
Ways to prove a quadrilateral is a rectangle:		$\boxed{P}$ +
Show it's a parallelogram w/ **One Right Angle**	**P** + → **Rectangle**	2 SLOPES
Show it's a parallelogram w/ **Congruent Diagonals**	**P** + → **Rectangle**	$\boxed{P}$ + 2 DISTANCES
Ways to prove a quadrilateral is a rhombus:		
Show it has 4 **Congruent Sides** ONE TIME!	**P** + → **Rhombus**	4 DISTANCES
Show it's a parallelogram w/ **1 Pair of Consecutive Congruent Sides**	**P** + → **Rhombus**	$\boxed{P}$ + 2 DISTANCES
Show it's a parallelogram w/ **Perpendicular** diagonals	**P** + → **Rhombus**	$\boxed{P}$ + 2 SLOPES
Ways to prove a quadrilateral is a square:		
Show it is a parallelogram that is both **A Rectangle + A Rhombus**	**P** **+ Rect + Rhom → Square**	$\boxed{P}$ CHOICE RECT CHOICE RHOM CHOICE

Isosceles trapezoid: prove a trapezoid + congruent diagonals, congruent legs, or 1 pair of congruent base angles.

2 SLOPES 2 DISTANCES

1) Reflect $\overline{AB}$ into the line y=x where A(1,3) and B(1,9). Graph and label the image. What type of quadrilateral is formed by ABB'A'? ______________________. Prove it using coordinate geometry:

y=x



$y = x \quad m = \frac{1}{1} \quad \perp \; -\frac{1}{1}$

$m\overline{BB'} = -\frac{1}{1}$
$m\overline{AA'} = -\frac{1}{1}$ } $\overline{BB'} \parallel \overline{AA'}$

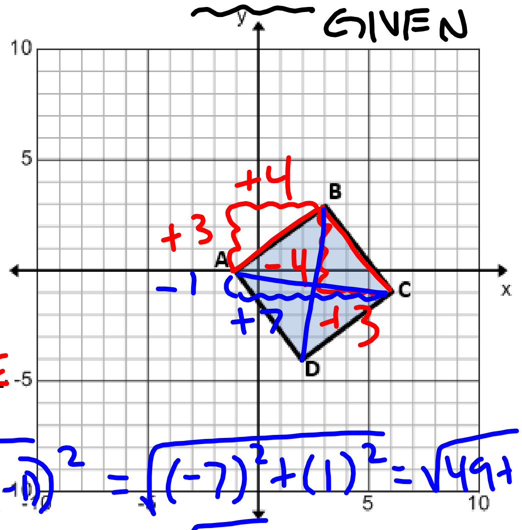
TRAPEZOID

ISOS TRAPEZOID: AB = A'B' = 6

≅ LEGS $\overline{AB} \cong \overline{A'B'}$

2) Quadrilateral ABCD has vertices A(-1, 0), B(3, 3), C(6, -1), and D(2, -4). Prove that parallelogram ABCD is a square using coordinate geometry.

GIVEN



✓ Rectangle – Sides

$m\overline{AB} = \frac{3}{4}$

$m\overline{BC} = -\frac{4}{3}$

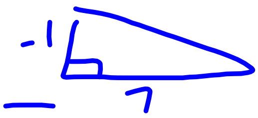
Since $m\overline{AB}$= $\frac{3}{4}$ and $m\overline{BC}$= $-\frac{4}{3}$ are OPP RECIPROCALS, then $\overline{AB} \perp \overline{BC}$. Therefore ∡B is a RIGHT ∡. Since parallelogram ABCD has a right angle, then ABCD is a RECTANGLE

Rectangle – Diagonals

$AC = \sqrt{(-1-6)^2 + (0-(-1))^2} = \sqrt{(-7)^2 + (1)^2} = \sqrt{49+1}$

$= \sqrt{50}$

$BD = \sqrt{50}$



$(-1)^2 + (7)^2 = c^2$

$\sqrt{50} = \sqrt{c^2}$

$5 \sqrt{25}\sqrt{2} = c$

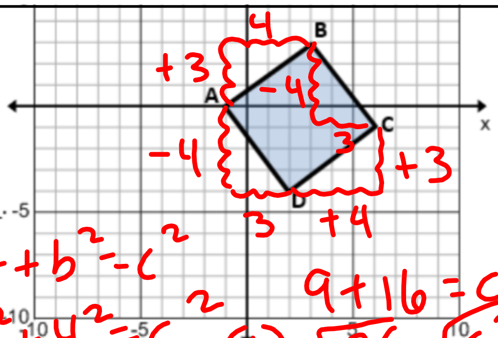
$5\sqrt{2} = c$

Since AC= $5\sqrt{2}$ =BD, then $\overline{AC} \cong \overline{BD}$.

Since parallelogram ABCD has congruent diagonals, then ABCD is a RECTANGLE

"WRECKED ANGLE"

✓ Rhombus - Sides



AB, BC, CD, DA

3 4

4 3

$a^2+b^2=c^2$

$3^2+4^2=c^2$

$9+16=c^2$

$\sqrt{25}=\sqrt{c^2}$

$5=c$

Since AB=BC=CD=DA= 5, then $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$.

Since parallelogram ABCD has 4 congruent sides, then ABCD is a RHOMBUS.

Rhombus - Diagonals $m\overline{AC} = -\frac{1}{7}$ $m\overline{BD} = \frac{7}{1}$

Since $m\overline{AC}$= $-\frac{1}{7}$ and $m\overline{BD}$= $\frac{7}{1}$ are OPPOSITE RECIPROCALS, then $\overline{AC} \perp \overline{BD}$

Since parallelogram ABCD has perpendicular diagonals, then ABCD is a RHOMBUS.

✓ Square: Since parallelogram ABCD is both a RECTANGLE and a RHOMBUS then ABCD is a SQUARE.