Lesson 5.2 - Angle Bisectors & Incenter

AGENDA
- Homework Check & Review
- Exploration - Get a compass, ruler, and Universal Angle Maker
- Notes 5.2

HOMEWORK:
- p. 311-312: #9,10,11,~~2~~0,28,29,31
- Proof A
- Construction Project – Incenter
- Keep up with lesson summaries - Do Incenter tonight
- Finish CR#4 due 12/14/16

Note: End of Interim is coming soon so back/missing work is due asap!

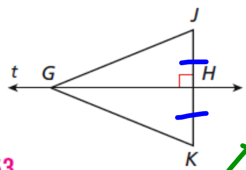
5.1 HW Answers

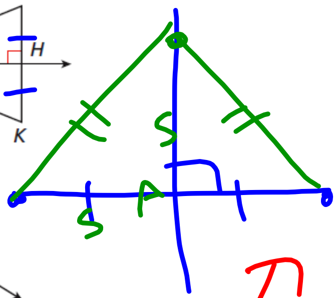
Use the diagram for Exercises 12–14.

12. Given that line t is the perpendicular bisector of $\overline{JK}$ and $GK = 8.25$, find GJ. 8.25

13. Given that line t is the perpendicular bisector of $\overline{JK}$, $JG = x + 12$, and $KG = 3x - 17$, find KG. 26.5

14. Given that $GJ = 70.2$, $JH = 26.5$, and $GK = 70.2$, find JK. 53



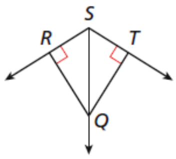


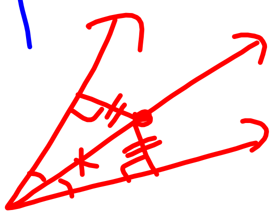
Use the diagram for Exercises 15–17.

15. Given that $m\angle RSQ = m\angle TSQ$ and $TQ = 1.3$, find RQ. 1.3

16. Given that $m\angle RSQ = 58°$, $RQ = 49$, and $TQ = 49$, find $m\angle RST$. 116°

17. Given that $RQ = TQ$, $m\angle QSR = (9a + 48)°$, and $m\angle QST = (6a + 50)°$, find $m\angle QST$. 54°

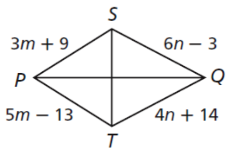




Write an equation in point-slope form for the perpendicular bisector of the segment with the given endpoints.

19. $E(-4, -7)$, $F(0, 1)$ **20.** $X(-7, 5)$, $Y(-1, -1)$ **21.** $M(-3, -1)$, $N(7, -5)$

22. $\overline{PQ}$ is the perpendicular bisector of $\overline{ST}$. Find the values of m and n.
$m = 11$; $n = 8.5$



19. $y + 3 = -\frac{1}{2}(x + 2)$
20. $y - 2 = x + 4$
21. $y + 3 = \frac{5}{2}(x - 2)$

HW 5.1 Answers

12. $GJ = GK$
$GJ = 8.25$

13. $JG = KG$
$x + 12 = 3x - 17$
$12 = 2x - 17$
$29 = 2x$
$14.5 = x$
So $KG = 3(14.5) - 17 = 26.5$.

14. Since $GJ = GK$ and $t \perp \overline{JK}$, t is the $\perp$ bisector of $\overline{JK}$ by the Conv. of the $\perp$ Bisector Thm.
$JK = 2JH$
$JK = 2(26.5) = 53$

15. $RQ = TQ$
$RQ = 1.3$

16. Since $RQ = TQ$, $\overline{RQ} \perp \overline{RS}$, and $\overline{TQ} \perp \overline{TS}$, $\overrightarrow{SQ}$ bisects $\angle RST$ by the Conv. of the Bisector Thm.
$m\angle RST = 2m\angle RSQ$
$m\angle RST = 2(58°) = 116°$

17. $m\angle QSR = m\angle QST$
$9a + 48 = 6a + 50$
$3a + 48 = 50$
$3a = 2$
$m\angle QST = 6\left(\frac{2}{3}\right) + 50 = 54°$

19. Step 1 Graph $\overline{EF}$.
The $\perp$ bisector of $\overline{EF}$ is $\perp$ to $\overline{EF}$ at its midpoint

Step 2 Find the midpoint of $\overline{EF}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

midpoint of $\overline{EF} = \left(\frac{-4 + 0}{2}, \frac{-7 + 1}{2}\right) = (-2, -3)$

Step 3 Find the slope of the perpendicular bisector.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

slope of $\overline{EF} = \frac{1 - (-7)}{0 - (-4)} = \frac{8}{4} = 2$

Since the slopes of $\perp$ lines are opposite reciprocals, the slope of the $\perp$ bisector is $-\frac{1}{2}$.

Step 4 Use point-slope form to write an equation.
The $\perp$ bisector of $\overline{EF}$ has slope $-\frac{1}{2}$ and passes through $(-2, -3)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = -\frac{1}{2}[x - (-2)]$$

$$y + 3 = -\frac{1}{2}(x + 2)$$

20. Step 1 Graph XY.
The $\perp$ bisector of $\overline{XY}$ is $\perp$ to $\overline{XY}$ at its midpoint

Step 2 Find the midpoint of $\overline{XY}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

midpoint of $\overline{XY} = \left(\frac{-7 + (-1)}{2}, \frac{5 + (-1)}{2}\right) = (-4, 2)$

Step 3 Find the slope of the perpendicular bisector.

slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

slope of $\overline{XY} = \dfrac{-1 - 5}{-1 - (-7)} = \dfrac{-6}{6} = -1$

Since the slopes of $\perp$ lines are opposite reciprocals, the slope of the $\perp$ bisector is 1.

Step 4 Use point-slope form to write an equation.
The $\perp$ bisector of $\overline{XY}$ has slope $-\frac{1}{2}$ and passes through $(-2, -3)$.

$y - y_1 = m(x - x_1)$
$y - 2 = 1[x - (-4)]$
$y - 2 = x + 4$

21. Step 1 Graph $\overline{MN}$.
The $\perp$ bisector of $\overline{MN}$ is $\perp$ to $\overline{MN}$ at its midpoint

Step 2 Find the midpoint of $\overline{MN}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

midpoint of $\overline{MN} = \left(\frac{-3 + 7}{2}, \frac{1 + (-5)}{2}\right) = (2, -3)$

Step 3 Find the slope of the $\perp$ bisector.

slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

slope of $\overline{MN} = \dfrac{-5 - (-1)}{7 - (-3)} = \dfrac{-4}{10} = -\dfrac{2}{5}$

Since the slopes of $\perp$ lines are opposite reciprocals, the slope of the bisector is $\frac{5}{2}$.

Step 4 Use point-slope form to write an equation.
The bisector of $\perp$ has slope $-\frac{1}{2}$ and passes through $(-2, -3)$.

$y - y_1 = m(x - x_1)$

$y - (-3) = \frac{5}{2}(x - 2)$

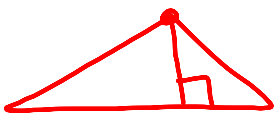
$y + 3 = \frac{5}{2}(x - 2)$

22.

$$
\begin{aligned}
PS &= PT \\
3m + 9 &= 5m - 13 \\
9 &= 2m - 13 \\
22 &= 2m \\
11 &= m
\end{aligned}
\qquad
\begin{aligned}
QS &= QT \\
6n - 3 &= 4n + 14 \\
2n - 3 &= 14 \\
2n &= 17 \\
n &= 8.5
\end{aligned}
$$

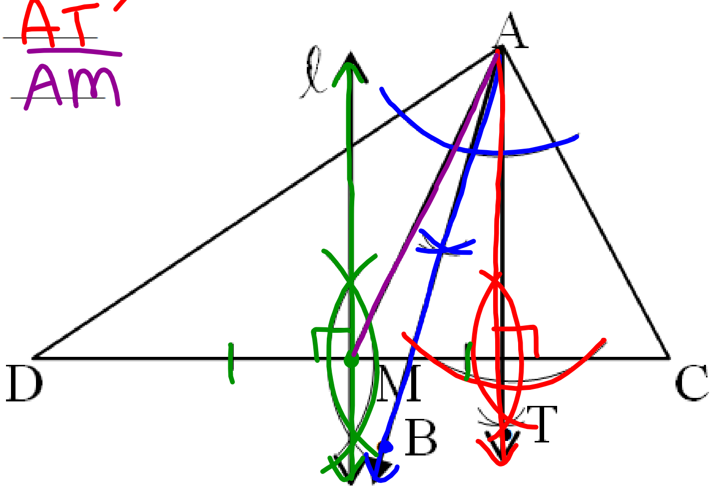
Warm Up

Are medians, perpendicular bisectors, altitudes, and angle bisectors the same segment, ray, or line for all triangles? Given ΔACD, precisely name the



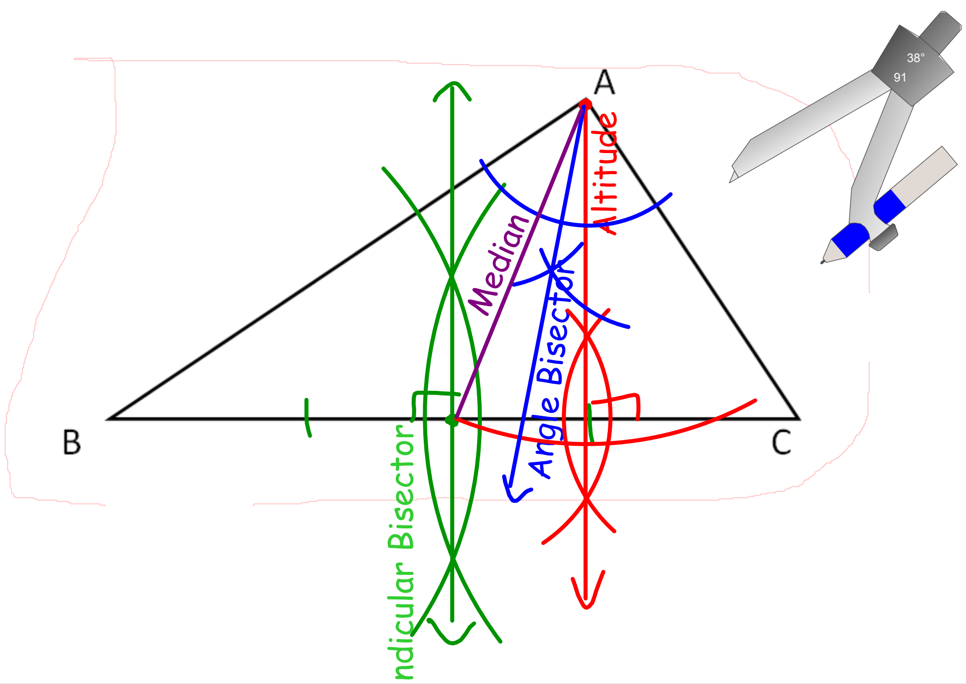
Angle Bisector $\overrightarrow{AB}$ Altitude $\overrightarrow{AT}$

Perpendicular Bisector ℓ Median $\overline{AM}$



5-2 Warm Up Activity

1. Draw the perpendicular bisector of base $\overline{BC}$ using a compass and straightedge OR by folding your triangle (you can also use your universal angle maker).
2. Draw the altitude from vertex A to base $\overline{BC}$ using your compass OR by folding your triangle. ⊥ FROM PT OFF
3. Draw the angle bisector of angle A using your universal angle maker OR a compass OR by folding your triangle.
4. Draw the median between vertex A and the base $\overline{BC}$. (You already found the midpoint of $\overline{BC}$ in part 1.)
5. Write your conclusions in the space below the triangle based on what you see.



EXPLORATION

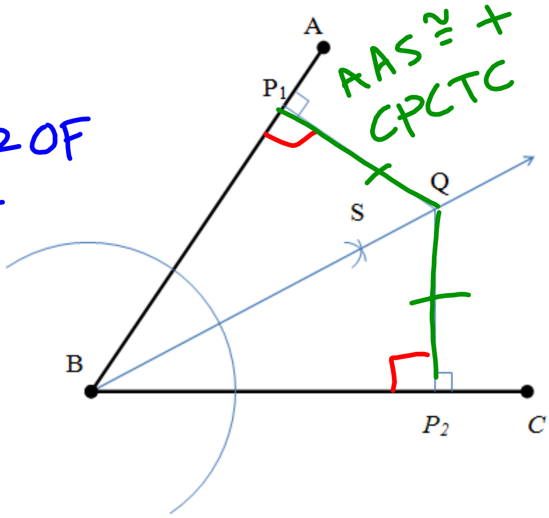
1. Using the construction at right,

a. What would you call $\overrightarrow{BS}$? ∡ BISECTOR OF ∡ABC

b. What is the relationship between $\overline{QP_1}$ and $\overline{AB}$?

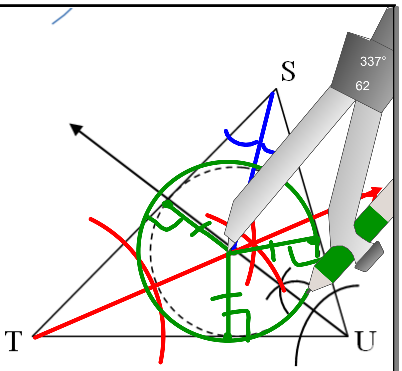
Between $\overline{QP_2}$ and $\overline{BC}$? ⊥

c. What is the relationship between lengths QP_1 & QP_2? ≅



2. Using the drawing,

a. Draw a segment connecting point I to vertex S.
What does it look like the segment might do to angle S? Make a conjecture: $\overline{SI}$ BISECTS $\measuredangle S$



b. Confirm your conjecture by using your compass to construct the bisector of angle ~~T~~ S. IS $\overrightarrow{SI}$ coincident with the bisector of $\measuredangle S$?

c. Where do you think the third angle bisector would be?
(Hint: What does any of the three segments connected to I become?)

d. What other segments can you draw that might be significant? Why? (Hint: consider what we just discussed about points on angle bisectors and where the equidistance is).
FROM I $\perp$ TO $\measuredangle$ SIDE RAYS
SIDES OF $\triangle$

e. Where is the circle located? I = INCENTER

f. What could you call point I in relation to the circle? CENTER

- Try drawing an obtuse triangle and repeating drawing all 3 bisectors. What happened? Were any of your conjectures confirmed?
- Try drawing a right triangle and repeating drawing all 3 bisectors. What happened? Were any of your conjectures confirmed?

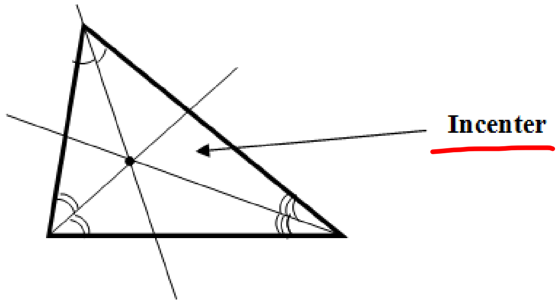
Defn: When 3 or more lines intersect in the same point, they are CONCURRENT. The point of intersection is called the point of CONCURRENCY.

The 4 special points of concurrency in triangles that we will study are:

Point of Concurrency	Lines that meet to form the point
Incenter *	Angle Bisectors TODAY
Circumcenter	Perpendicular bisectors
Orthocenter	Altitudes
Centroid	Medians

Incenter

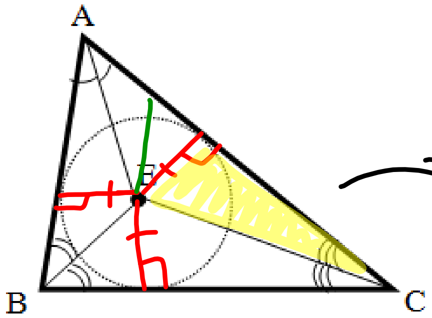
Defn: The point of concurrency of the angle bisectors of a triangle is called the INCENTER of the triangle.



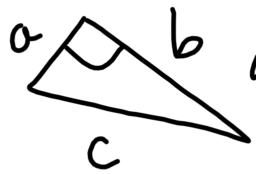
Theorem 5-2-2

The incenter of a triangle is equidistant from the SIDES of the triangle.

Remember, these segments (congruent radii of the inscribed circle) must be $\perp$ to the sides!



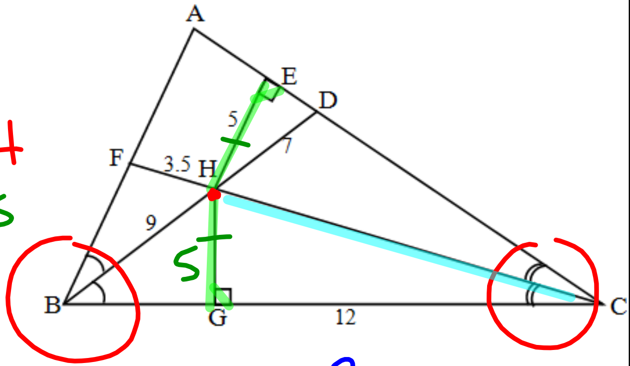
NOTE: The incenter of a triangle is the center of the circle that would be inscribed inside the triangle.



$a^2+b^2=c^2$

Example 1:

A) In the following triangle, what is the length of $\overline{GH}$?



INCENTER = H

H ⊥ TO SIDES

B) Find the length CH.

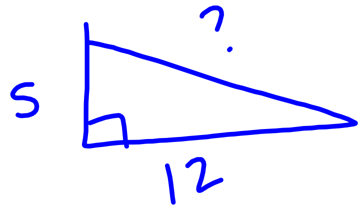
$5^2+12^2=x^2$

$25+144=x^2$

$169=x^2$

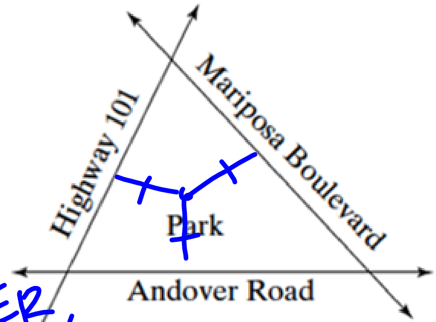
$\pm\sqrt{169}=x$

$13=x=CH$



Example 2:

City planners want to locate a fountain equidistant from three straight roads that enclose a park. Explain how they can find the location.



CONSTRUCT 2 OR 3 ∡ BISECTORS TO LOCATE THE INCENTER, WHICH IS EQUIDISTANT TO THE ROADS.

Construction:

1. Construct the incenter of the triangle using at least two angle bisectors.
2. Draw in a radius (perpendicular to the triangle side) using your universal angle maker.
3. Construct the circle that is inscribed in the triangle using the radius you drew.