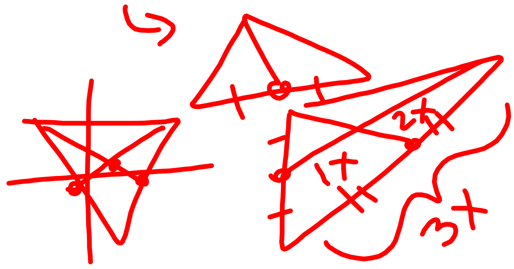
Lesson 5.6 - Midsegments of Triangles

AGENDA
- Homework Check & Review
- Lesson notes & guided practice

QUIZ 2 -
ORTHOCENTER
CENTROID

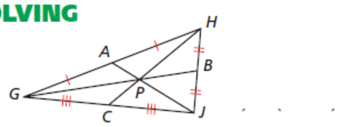


HOMEWORK:
- p. 324-25: #11-16,18-20, 24,26
- Proofs E and F

Day 5 HW Answers: p. 318: #12-16, 21-26,29-32, 37, 43

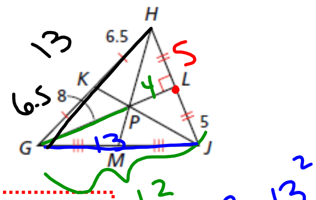
PRACTICE AND PROBLEM SOLVING

$PA = 2.9$, and $HC = 10.8$. Find each length.



12. PC 3.6
13. HP 7.2
14. JA 8.7
15. JP 5.8

Find each measure.



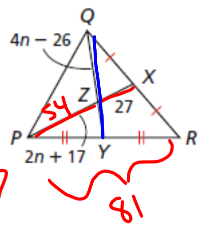
21. GL 12 ✓
22. PL 4 ✓
23. HL 5 ✓
24. GJ 13
25. perimeter of $\triangle GHJ$ 36 units
26. area of $\triangle GHJ$ 60 square units

12
$12^2+5^2=13^2$

Algebra Find the centroid of a triangle with the given vertices.

28. $X(8, -1)$, $Y(2, 7)$, $Z(5, -3)$ (5, 1) Details on slide 3

Find each length.

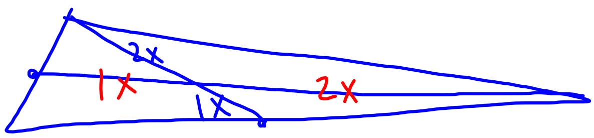


29. PZ 54 ✓
30. PX 81 ✓
31. QZ 48
32. YZ 24

QY = 72 by subbing in n=$^{37}/_2$ value from PZ

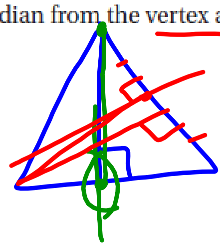
$54 = 2n+17$

Q
4n-26
Z
?
Y



16. Design In the plan for a table, the triangular top has coordinates (0, 10), (4, 0), and (8, 14). The tabletop will rest on a single support placed beneath it. Where should the support be attached so that the table is balanced? (4, 8)

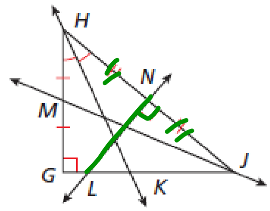
37. In an isosceles triangle, the altitude and median from the vertex angle are the same line as the bisector of the vertex angle. A



43. In the diagram, which of the following correctly describes $\overline{LN}$?

Ⓐ Altitude Ⓒ Median

Ⓑ Angle bisector Ⓓ Perpendicular bisector

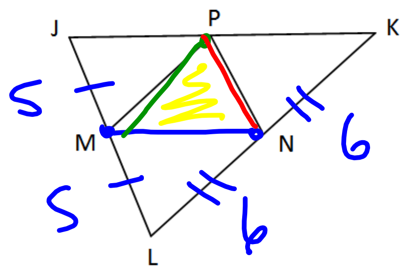


$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

BIS OF $\overline{HJ}$

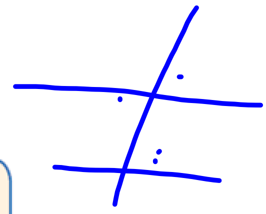
Defn: A **midsegment** is the segment that joins the midpoints of 2 sides of a triangle. Example: $\overline{MN}$

A **midsegment triangle** is the triangle formed by all three midsegments. Example: $\triangle MNP$



Triangle Midsegment Theorem

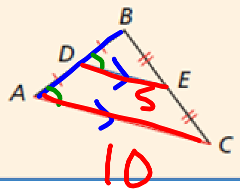
A midsegment of a triangle is parallel to its opposite side & it is ½ the length of that side.



Theorem 5-4-1 Triangle Midsegment Theorem

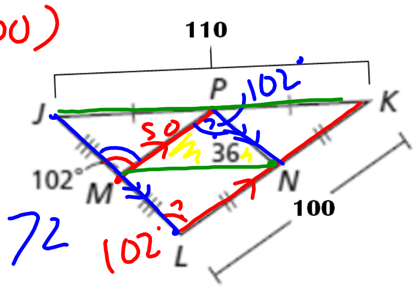
A midsegment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.

$\overline{DE} \parallel \overline{AC}$, $DE = \frac{1}{2}AC$



∠ PAIRS

Example 1: Given the drawing at right, find



a) PM

$PM = \frac{1}{2}LK = \frac{1}{2}(100)$
$= 50$

b) $m\angle MLK$

$\parallel \to$ CORR ∠'s ≅
102°

c) JL

$2\,PM = JL$
$2(36) = JL = 72$

d) $m\angle NPM$

$\parallel \to$ ALT INT ∠'s ≅ 102°

e) Perimeter of ΔJKL

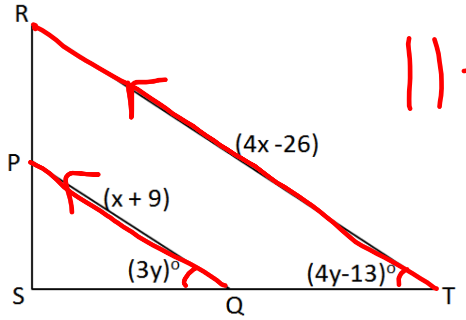
$110 + 100 + 72 = 282$

f) Perimeter of ΔNMP

$55 + 50 + 36 = 141$

$= \frac{1}{2} P_{\Delta JKL}$

Example 2: $\overline{PQ}$ is the midsegment of $\triangle RST$. Find x & y.



$\parallel \to$ CORR $\measuredangle$'s $\cong$

$3y = 4y - 13$

$\boxed{13 = y}$

$PQ = \frac{1}{2}RT$

$2PQ = RT$

$2(x+9) = 4x-26$

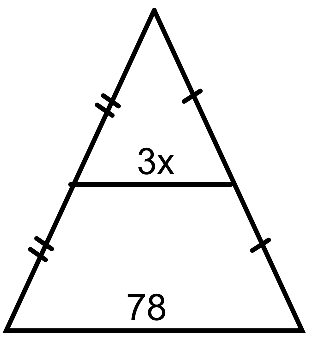
$2x+18 = 4x-26$

$44 = 2x$

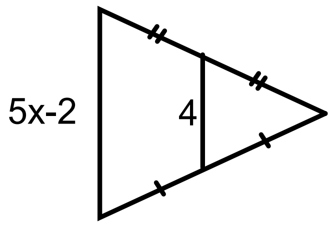
$\boxed{22 = x}$

TRY THESE TWO

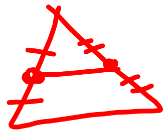
1)



2)



Midsegment Proof You may prove a segment is a midsegment by one of two methods:



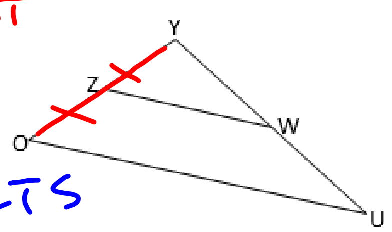
- **Definition:** Prove the endpoints are midpoints of the triangle's sides.
  - Ways to know you have a midpoint:
    - 2 ≅ HALVES
    - MEDIANS
    - BISECTOR
    - GIVEN
- **Properties:** Prove the segment is **half the length** of the triangle side that it **parallel to** – this is usually in the coordinate plane

DISTANCE   SLOPE

→ MIDPOINT → MIDPOINT

Ex1) Given: W bisects $\overline{YU}$; $\overline{YWU}$; $\overline{YZ} \cong \overline{ZO}$; $\overline{YZO}$

Prove: $\overline{WZ}$ is a midsegment in ΔYOU



SINCE IT'S GIVEN W BISECTS $\overline{YU}$ & $\overline{YWU}$, THEN W IS THE MIDPOINT OF $\overline{YU}$.

SINCE IT'S GIVEN $\overline{YZ} \cong \overline{ZO}$ & $\overline{YZO}$, THEN Z IS THE MIDPOINT OF $\overline{YO}$.

BY DEFINITION, $\overline{WZ}$ IS MIDSEGMENT OF ΔYOU.

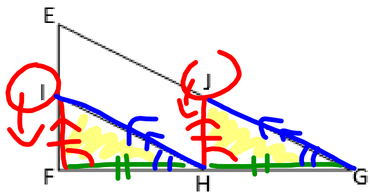
MIDPOINTS 1:2

||

Ex 2) Given: $\overline{IH}$ is a midsegment of ΔEFG; $\overline{FGH}$

$\overline{IF} \cong \overline{JH}$; $\overline{EIF} \parallel \overline{JH}$ CORR ∡'S ≅

Prove: $\Delta IFH \cong \Delta JHG$



S

$\boxed{\overline{IF} \cong \overline{JH}}$

GIVEN

A

$\boxed{\overline{EIF} \parallel \overline{JH}}$

GIVEN

$\boxed{\measuredangle F \cong \measuredangle JHG}$

|| → CORR ∡'S ≅

S

$\boxed{\overline{IH} \text{ MIDSEGMENT } \Delta EFG}$

GIVEN

$\boxed{H \text{ MIDPOINT } \overline{FG}}$

DEFN OF MIDSEGMENT

$\boxed{\overline{FH} \cong \overline{GH}}$

MIDPOINT → 2 ≅ HALVES

△IFH ≅ △JHG

BY SAS ≅ SAS

Midsegment Proof You may prove a segment is a midsegment by one of two methods:

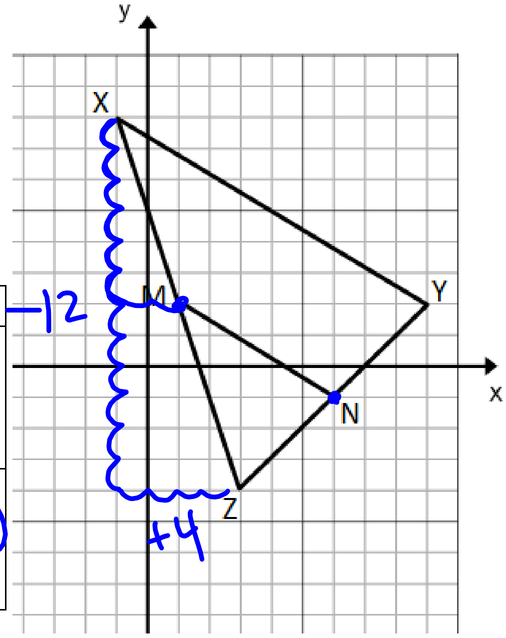
- **Definition:** Prove the endpoints are midpoints of the triangle's sides. MIDPOINT FORMULA
- **Properties:** Prove the segment is **half (using distance formula) the length** of the triangle side that it is **parallel** to (slope = → parallel). This is a conjunction.

Ex: Given: X(-1,8), Y(9,2), Z(3,-4)
M(1,2) and N(6,-1)

Prove: $\overline{MN}$ is a midsegment of ΔXYZ

Calculations:

	Midpoint
$\overline{XZ}$	m(1,2) BY COUNTING
$\overline{ZY}$ Y(9,2) Z(3,-4)	$\left(\frac{9+3}{2}, \frac{2+-4}{2}\right) = (6,-1)$



Statements:

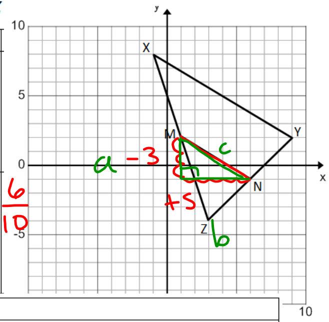
- **Defn:** Since the midpoint of $\overline{XZ}$ is (1,2) and M is given as (1,2), then M is the midpoint of $\overline{XZ}$. Since the midpoint of $\overline{ZY}$ is (6,-1) and N is given as (6,-1) then N is the midpoint of $\overline{ZY}$. Since the segment $\overline{MN}$ joins the midpoints M and N, then $\overline{MN}$ is a midsegment of ΔJKL by DEFN OF MIDSEGMENT.

Given: X(-1,8), Y(9,2), Z(3,-4)
M(1,2) and N(6,-1)

Calculations:

Prove: $\overline{MN}$ is a midsegment of ΔXYZ

	Slope
$\overline{MN}$	$\frac{\Delta y}{\Delta x} = \frac{-3}{5}$ BY COUNTING
$\overline{XY}$	$\frac{\Delta y}{\Delta x} = \frac{2-8}{9-(-1)} = \frac{-6}{10} = \frac{-3}{5}$



Length (distance)
MN -3, 5, c: $(-3)^2 + 5^2 = c^2$; $9 + 25 = c^2$; $34 = c^2$; $\pm\sqrt{34} = MN$
$XY = \sqrt{(9-(-1))^2 + (2-8)^2} = \sqrt{10^2 + (-6)^2} = \sqrt{100+36} = \sqrt{136} = \sqrt{4}\sqrt{34} = 2\sqrt{34}$

Statements:

- **Properties:** Since the slopes $m\overline{MN} = \frac{-3}{5} = m\overline{XY}$, then $\overline{MN} \parallel \overline{XY}$. Since MN= $\sqrt{34}$ and XY= $2\sqrt{34}$ then $MN = \frac{1}{2}XY$ Since $\overline{MN}$ is **half the length** of the segment it is **parallel to**, then $\overline{MN}$ is midsegment of ΔXYZ.