Lesson 11-3 & 11-4: Perpendicular Bisectors of Chords + Sector Area and Arc Length

**AGENDA**
- Check HW 11.2 textbook and 11-3 problems in notes
- Lesson Notes & Guided Practice (fill in graphic org)

**HOMEWORK**
11-3: p 761 # 31, 32, 37, 47, 49
11-4: Worksheet
Quiz next class

1. \(1x + 9x = 360\)
   \(x = 36\) \(\text{major} = 324^\circ\)

2. \(3x + 5x = 180\)
   \(8x = 180\)
   \(x = 22.5^\circ\)
HW p 761 # 15,25-30,38,39,48

15. $\angle QPR \equiv \angle RPS$. Find $QR$.

25. $\text{m}\overline{MP} = 152^\circ$

26. $\text{m}\overline{QNL} = 208^\circ$

27. $\text{m}\overline{TV} = 155^\circ$

28. $\text{m}\overline{TV} = 235^\circ$

29. $\overline{OA} \equiv \overline{OB}$, and $\overline{CD} \equiv \overline{EF}$.

   Find $\text{m}\angle CAD = 147^\circ$

30. $\overline{JK} \equiv \overline{LM}$. Find $\text{m}\angle K = 85^\circ$

38. $\text{m}\overline{EF} = 136^\circ \quad (6x - 2)$

39. $\text{m}\angle SPT = 105^\circ$

48. Which of these arcs of $\odot O$ has the greatest measure?

   - $\overline{WY}$
   - $\overline{UV}$
   - $\overline{TV}$

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- Central angles have congruent chords
- Congruent chords have congruent arcs
- Congruent arcs have congruent central angles
- If a radius (or diameter) is $\perp$ to a chord, it bisects the chord and the arc
- In a circle, all radii are congruent
- A tangent is $\perp$ to the radius at the point of tangency
- 2 segments, tangent to circle from the same external point $\Rightarrow$ segments $\equiv$

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Center of circle

Central Angle

radii - (extended)

$m< = m \text{ ARC}$
### Perpendicular Bisector of a Chord

In a circle, the perpendicular bisector of a chord must include the center of the circle.

#### Theorems

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>11-2-3</strong></td>
<td>In a circle, if a radius (or diameter) is perpendicular to a chord, then it bisects the chord and its arc.</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>11-2-4</strong></td>
<td>In a circle, the perpendicular bisector of a chord is a radius (or diameter).</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
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</table>

#### Arrows and Chords

- If a radius (or diameter) is perpendicular to a chord, it bisects the chord and the arc.

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**Example 1:** Find $QR$ in simplest radical form. **Watch your final answer**

$$QR = 2x$$

$$QR = 2(10\sqrt{3})$$

$$= 20\sqrt{3}$$

$$x^2 + 10^2 = 20^2$$

$$x^2 = 300$$

$$x = \pm \sqrt{300} \sqrt{3}$$

$$x = 10\sqrt{3}$$
Example 2: Find CE.

Example 3: Find LX.

\[ WX \cong YZ \rightarrow WX \cong YZ \]

\[ WX = YZ \]

\[ WX = 12 \]

\[ LX = 6 \] HALVES OF SEGMENTS ARE \( \cong \)
Example 3:

\( \angle X \cong \angle Y \) and \( RQ = 35.6 \). Identify \( ED \) rounded to the nearest tenth.

\( \angle \text{SUPP \ THM} \)
\( \angle \text{CENTRAL \ \angle S} \rightarrow \angle \text{CHORDS} \)
\( \angle \text{IN} \ \angle \text{\ O \ \ S} \)

\[ XD = 26 = XB \]
\[ RADIUS \]
\[ XD = XE + ED \]
\[ 26 = a + ED \]

Regents Question: In the diagram below, circle \( O \) has a radius of 5, and \( CE = 2 \). Diameter \( AC \) is perpendicular to chord \( BD \) at \( E \).
What is the length of \( BD \)?

(1) 12
(2) 10
(3) 8
(4) 4

\[ BD = 2 \times = 2(4) = 8 \]
11-4 Notes: Sector Area and Arc Length; Radian Measure

Recall Circle Formulas: Area = \( \pi r^2 \) and Circumference = \( 2\pi r \)

An informal dissection method can be used to determine the area of a circle. Consider the following circles cut into smaller and smaller sectors and the polygon they compose (connect the vertices on the circle to draw each inscribed polygon):

- **Circle #1** (4 Sectors of 90° each)
  - \( \frac{90°}{360°} = \frac{1}{4} \)
- **Circle #2** (6 Sectors of 60° each)
  - \( \frac{60°}{360°} = \frac{1}{6} \)
- **Circle #3** (12 Sectors of 30°)
  - \( \frac{1}{12} = \frac{x}{360°} \)
- **Circle #4** (24 Sectors of 15°)
  - \( \frac{15°}{360°} = \frac{1}{24} \)

What does a polygon start to approximate as the number of sides gets very large?
A **CIRCLE** where the radius and the apothem approach coincidence.

Strictly as dissection, the sectors begin to approximate the area of a parallelogram or a rectangle:

- \( \frac{1}{2} \text{ circumference} = \frac{1}{2} (2\pi r) = \pi r \)
- **Area of a Circle**

\[ A = \pi r^2 \]
Examining Arc Lengths and Radii

1. Consider the diagram at the right showing two circles with the same center, \( O \).
2. When circles are concentric, it can be shown that the two circles are similar by using a single transformation called a **DILATION**.
3. Represent the ratio of similitude of the smaller circle to the larger circle in this diagram.

\[
\frac{\text{smaller circle}}{\text{larger circle}} = \frac{r_1}{r_2}
\]

4. The same dilation which maps the smaller circle onto the larger circle will also map the slice (sector) of the smaller circle with an arc length of \( s_1 \) onto the slice (sector) of the larger circle with an arc length of \( s_2 \).

Such a mapping of the slices (sectors) will only occur when ... **CENTRAL \( \theta \) IS MAINTAINED**

Such a mapping tells us that these slices (sectors) are **SIMILAR**.

5. Explain why the following proportion is true.

\[
\frac{r_1}{r_2} = \frac{s_1}{s_2}
\]

6. Based upon the proportion in #5, complete:

\[
\frac{s_1}{r_1} = \frac{s_2}{r_2}
\]

Explain mathematically how you arrived at your answer.

**PROPORTIONAL LENGTHS**

**CROSS PRODUCTS PROPERTY**

7. The equation in #6 shows that the ratio of the arc length intercepted by a central angle to the radius of the circle will always yield **CONSTANT RATIO**.

**CONCLUSION:** The length of an arc will be proportional to the central angle and depends upon the radius. The radius determines the total circumference and area of the circle.

We can see that the arc length is a fractional part of the circumference of the circle. Let’s try to put this information into a formula which will support our conclusion.

Will this also work for the area of fractional part of a circle? **YES**

\[
\frac{\theta}{360^\circ} = \frac{\text{sector area}}{\text{area of \( \theta \)}}
\]

\[
\frac{\theta}{360^\circ} = \frac{\text{arc length}}{\text{circumference}}
\]
If the area of the circle is $8\pi$, what is the area of the green sector?

If the radius of the circle is 3, what is the area of the green sector?
What two things did the area depend upon?

Extend: the general formula for finding the area of a sector is

Central Angle Proportions: Sector / Arc Length and Radian Measure

\[ \frac{\theta^\circ}{360^\circ} = \frac{\text{Area}_{\text{sector}}}{\text{Area}_{\text{circle}}} \]

Note: the area is in square units, not degrees!
How do we do this with a central angle of any degree measure? Find the area of the sector, in terms of $\pi$ and to the nearest hundredth.

Practice: Find the area of the sector, in terms of $\pi$ and to the nearest hundredth.
A segment of a circle is a region bounded by an arc and its chord.

How would you find the area of segment RPQ?

\[
\text{area of segment} = \text{area of sector} - \text{area of triangle}
\]

Example: Find the area of the segment LNM to the nearest hundredth.

\[
\text{AREA}_{SEG} = \text{AREA}_{SECTOR} - A_{\triangle LNM}
\]

\[
\text{LNM} = \frac{1}{2}bh
\]

\[
\frac{120^\circ}{360^\circ} = \frac{x}{81\pi}\text{ cm}^2
\]
Consider what we explored about finding the area of a sector.
How could we find the length of the arc \( \overline{AB} \)?

Definition: Arc length is the distance along an arc measured in linear units.

What will the arc length depend upon?

Central Angle Proportions: Sector / Arc Length and Radian Measure

\[
\frac{\theta^\circ}{360^\circ} = \frac{\text{ArcLength}}{\text{Circumference of circle}}
\]
Practice: Find the length of $\overline{JL}$ in terms of $\pi$ and to the nearest hundredth.

\[ C = 2\pi r = 2\pi (16) \]
\[ \theta^\circ = \frac{360^\circ}{360^\circ} \cdot \frac{x}{32\pi} \]
\[ \frac{10}{32\pi} = \frac{x}{32\pi} \]
\[ 32\pi = 10x \]
\[ \frac{32\pi}{10} = x \]
\[ \frac{16\pi}{5} \text{ in} \]
\[ \approx 10.05 \text{ in} \]

**Radian Measure**

A radian is defined as the ratio of the arc length to the radius of the circle.

**Practice:** What is the measure of the central angle in radians?

\[ 10 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \]
\[ 80^\circ \cdot \frac{\pi \text{ radians}}{180^\circ} = \frac{80\pi}{180} \]
\[ \frac{8\pi}{18} = \frac{4\pi}{9} \text{ radian} \]
Comparing Arc Length and Arc Measure

The ratio of the measure of the minor arc to the major arc in a circle with a radius of 7 inches is 5:31. Find the length of the minor arc. Answer to the nearest whole unit.