Lesson 11-2 & 3 ORANGE
Arcs & Chords / Arc Ratios

AGENDA
- Check HW 11.1
- Lesson Notes & Guided Practice (fill in graphic org)

HOMEWORK
p 761 # 15, 25-30, 38, 39, 48
Arc Ratio Problems on the bottom of 11-3 Notes
QUIZ - NEXT CLASS!

p 752-53: #11,13, 16-27

11. chords: \( RS, VW \); secant: \( VW \); tangent: \( \ell \); diam.: \( VW \); radii: \( PV, PW \)

16. \( AB \)
\[
2x^2 = 8x \\
x = 4
\]

17. \( RT \)
\[
y^2 = y \\
y = 7
\]
18. Two circles with the same center are congruent. S
19. A tangent to a circle intersects the circle at two points. N
20. Tangent circles have the same center. N
21. A tangent to a circle will form a right angle with a radius that is drawn to the point of tangency. A
22. A chord of a circle is a diameter. S

**Graphic Design** Use the following diagram for Exercises 23–25.
The peace symbol was designed in 1958 by Gerald Holtom, a professional artist and designer. Identify the following.
23. diameter \( AC \)
24. radii \( PA, PB, PC, PD \)
25. chord \( AC \)

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**Arc Vocabulary**

Fill in the definitions and an example of each using \( \odot M \).

- **Central Angle** – an \( \angle \) whose vertex is the \( \text{center} \) of the circle. Ex: \( \angle AMC \)
  - Note: a central angle \( \text{intercepts} \) an arc (the endpoints and all points on the circle between them) of the circle. The measure of the intercepted arc is \( \text{equal to} \) the degree measurement of its central angle.
  - Ex: \( \text{m} \angle AMC = \text{m} \angle AC \)

- **Major Arc** – the arc on the \( \text{exterior} \) of a central angle.
  - Named by \( 3 \) points.
  - Ex: \( \text{---} \)

- **Minor Arc** – the arc on the \( \text{interior} \) of a central angle.
  - Named by \( 2 \) points.
  - Ex: \( \text{---} \)
• Semicircle – an arc whose endpoints are a diameter of the circle.
  o Named by 3 points.
  o Ex: ________________
  o Note: a central angle that intercepts a semicircle (180°) is a ___

• Adjacent Arcs – arcs on the same circle that share a COMMON ENDPOINT
  o Ex: \(\overline{AC} \& \overline{CD}\)

• Congruent Arcs – arcs which have EQUAL MEASURE
  o Ex: ________________
  o Ex: ________________

\[\overline{AC} \cong \overline{DB}\]
\[\overset{\sim}{\overarc{AEB}} \cong \overset{\sim}{\overarc{ACB}}\]
\[\overset{\sim}{\overarc{CD}} \cong \overset{\sim}{\overarc{CD}}\]
\[\overset{\sim}{\overarc{AD}} \cong \overset{\sim}{\overarc{BC}}\]
Arc Addition Postulate
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example. Find the measure of each arc.

1) $\widehat{AD} = \widehat{AB} + \widehat{BC}$
   $47^\circ + 110^\circ = 157^\circ$

2) $\widehat{AD}$

3) $\widehat{ADC} + \widehat{AC} = 360^\circ$
   $\widehat{ADC} = \frac{360^\circ}{2} = 180^\circ$

4) $\widehat{DCB}$
   $47^\circ + 110^\circ = 157^\circ$

$\widehat{MAD} + 47^\circ = 203^\circ$
$\widehat{MAD} = 156^\circ$

Fill in graphic organizer (front)

$\angle$ and arc relationships

last column

Center of circle

CENTRAL $\angle$

(EXTENDED) RADIERS

$\Theta = \widehat{ARC}$
**Theorem 11-2-2**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Congruent central angles have congruent chords.</td>
<td>( \angle EAD \cong \angle BAC )</td>
<td>( DE \cong BC )</td>
</tr>
<tr>
<td>(2) Congruent chords have congruent arcs.</td>
<td>( ED \cong BC )</td>
<td>( DE \cong BC )</td>
</tr>
<tr>
<td>(3) Congruent arcs have congruent central angles.</td>
<td>( ED \cong BC )</td>
<td>( \angle DAE \cong \angle BAC )</td>
</tr>
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</table>

In a \( \odot \) circle or \( 2 \cong \odot \)'s, \( \cong \) \( \leftrightarrow \cong \) \( \leftrightarrow \cong \)

*Fill in graphic organizer*

- \( \cong \) central angles have \( \cong \) chords
- \( \cong \) chords have \( \cong \) arcs
- \( \cong \) arcs have \( \cong \) central angles

---

**Example:** Given \( \odot F \), find the following:

a) \( m\angle CDE = 90^\circ + 18^\circ = 108^\circ \)
b) \( m\angle BC = 360^\circ - (18^\circ + 162^\circ + 18^\circ + 90^\circ) = 72^\circ \)
c) \( m\angle CAE = 360^\circ - m\angle CDE = 252^\circ \)
d) \( m\angle AFE = 162^\circ \)
Practice:

1) \( RS \cong TU \). Find the measure of \( \overline{RS} \).

IN A \( \odot \), \( \cong \) CHORDS \( \rightarrow \cong \) ARCS

\[ 3x = 2x + 27 \]
\[ x = 27 \]
\[ m\angle RST = 81^\circ \]

2) \( \odot B \cong \odot E \). \( \overarc{AC} \cong \overarc{EF} \). Find the measure of \( \angle DEF \).

IN \( \cong \) \( \odot \)'s,
\( \cong \) ARCS \( \rightarrow \cong \) CENTRAL \( \angle \)'S

\[ 7y - 43 = 5y + 5 \]
\[ y = 24 \]
\[ m\angle DEF = 125^\circ \]
3) \( PT \) bisects \( \angle RPS \) in \( \bigcirc P \). Find \( RT \).

\[ \overrightarrow{2} \begin{align*} \angle PRT &= \angle RPS \\
RT &= \angle PRT \\
20-4x &= 6x \\
x &= \frac{20}{10} = 2 \end{align*} \]

IN A \( \bigcirc \), \( \overset{\sim}{\sim} \) CENTRAL \( \angle \)’S \( \overset{\sim}{\sim} \) CHORDS

\[ 6x = 20 - 4x \]

\[ x = 2 \]

\[ RT = 12 \]

4) \( \angle A \approx \angle B \). \( \overline{CD} = \overline{EF} \). Find \( \overarc{mCD} \).

IN \( \sim \) \( \bigcirc \)’S, \( \overset{\sim}{\sim} \) CHORDS \( \overset{\sim}{\sim} \) ARCS

\[ 25y = 30y - 20 \]

\[ y = 4 \]

\[ m\overarc{CD} = 100^\circ \]
Parallelogram Chords

Theorem: In a circle, parallel chords intercept congruent arcs.

*Fill in graphic organizer

Arcs and Chords

- Parallel chords intercept congruent arcs

Example: Given trapezoid ABCD inscribed in circle O, 1) explain why the trapezoid is an isosceles trapezoid and 2) find the measure of $\overarc{AC}$.

\[
\begin{align*}
\text{IN A } &\bigodot \text{, II CHORDS } \implies \overarc{AD} \cong \overarc{BC} \\
&50^\circ \\
5x &= 7x - 20 \\
x &= 10 \\
\overarc{AD} &= 50^\circ = \overarc{BC} \\
\text{IN A } &\bigodot \text{, } \overarc{AC} \cong \overarc{BC} \implies \text{ CHORDS } \overarc{AD} \cong \overarc{BC} \\
&50^\circ \\
&\text{ISOS TRAP ABCD W/ } \overarc{AC} \cong \overarc{BC} \\
\overarc{AC} &= \overarc{AB} + \overarc{BC} \\
&= 80^\circ + 50^\circ \\
\overarc{AC} &= 130^\circ
\end{align*}
\]
Arc Ratios

Extending the arc addition postulate, try the following problems:


\[
5x : 3x : 6x : 1x
\]

\[
5x + 3x + 6x + 1x = 360^\circ
\]

\[
15x = 360^\circ
\]

\[
x = 24
\]

B) In a circle, a chord divides the major and minor arcs into a ratio of 13:5. Find the measure of each arc.

\[
\text{MINOR + MAJOR} = 360^\circ
\]

\[
5x + 13x = 360^\circ
\]

\[
18x = 360^\circ
\]

\[
x = 20
\]

\[
\text{MAJOR} = 260^\circ
\]

\[
\text{MINOR} = 100^\circ
\]

C) Given the two concentric circles centered at \( A \) with radius \( AC = 3 \) and \( AF = 5 \), the ratio of \( EF \) to \( EF \) is 1:7. Find the measure of \( \angle CAB \) and the measure of \( CB \).

\[
\text{MAJOR + MINOR} = 360^\circ
\]

\[
7x + 1x = 360^\circ
\]

\[
8x = 360^\circ
\]

\[
x = 45^\circ
\]

\[
\text{m \angle CAB} = 45^\circ
\]

\[
\text{m \angle CAB} = 45^\circ
\]

\[
\text{m \angle CAB} = 45^\circ
\]

\[
\text{m \angle CAB} = 45^\circ
\]
**Perpendicular Bisector of a Chord**
In a circle, the perpendicular bisector of a chord must include the center of the circle.

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<td>11-2-3</td>
<td>In a circle, if a radius (or diameter) is perpendicular to a chord, then it bisects the chord and its arc.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[Diagram showing CD ⊥ EF]</td>
<td>CD bisects EF and EF.</td>
</tr>
<tr>
<td>11-2-4</td>
<td>In a circle, the perpendicular bisector of a chord is a radius (or diameter).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[Diagram showing JK ⊥ bisector of GH]</td>
<td>JK is a diameter of GH.</td>
</tr>
</tbody>
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**Complete graphic organizer**
- If a radius (or diameter) is ⊥ to a chord, it bisects the chord and the arc.
Attachments

Bridge to Unit 11 KEY.docx