Lesson 11-12: Circles Equations and Completing the Square

AGENDA:
- Check & Review Homework 11-11
- Lesson Notes & Guided Practice

HOMEWORK:
• Worksheet 11-12

Write the equation of each circle.
10. \( \odot R \) with center \( R(-12, -10) \) and radius \( 8 \) \( (x + 12)^2 + (y + 10)^2 = 64 \)
11. \( \odot S \) with center \( S(1.5, -2.5) \) and radius \( \sqrt{7} \) \( (x - 1.5)^2 + (y + 2.5)^2 = 7 \)
12. \( \odot C \) that passes through \( (2, 2) \) and that has center \( C(1, 1) \) \( (x - 1)^2 + (y - 1)^2 = 2 \)

20. \( x^2 + (y - 2)^2 = 9 \)
21. \( x^2 + y^2 = 100 \)

28. Consider the circle whose equation is \( (x - 4)^2 + (y + 6)^2 = 25 \). Write, in point-slope form, the equation of the line tangent to the circle at \( (1, -10) \). \( y + 10 = \frac{3}{4}(x - 1) \)

Find the center and radius of each circle.
30. \( (x - 2)^2 + (y + 3)^2 = 81 \)
31. \( x^2 + (y + 15)^2 = 25 \)

(2, -3); 9
(0, -15); 5

34. circle with equation \( (x - 8)^2 + (y + 5)^2 = 7 \) \( A = 7\pi; C = 2\sqrt{7}\pi \)
35. circle with center \((-1, 3)\) that passes through \((2, -1)\) \( A = 25\pi; C = 10\pi \)

39. \( \odot A \) has a diameter with endpoints \((-3, -2)\) and \((5, -2)\). Write the equation of \( \odot A \).

\( (x - 1)^2 + (y + 2)^2 = 16 \)
Completing the Square for Center-Radius Form for a Circle: In order to get an equation like $x^2 + 10x + y^2 - 8y - 9 = 0$ into center-radius form, we need to complete the square for both $x$ and $y$.

Recall: $ax^2 + bx$

1. Be sure that the coefficient of the highest power is one. If it is not, divide each term by that value to create a leading coefficient of one.

2. Move the constant term to the right hand side.

3. Prepare to add the needed value to create the perfect square trinomial. Be sure to balance the equation. The boxes may help you remember to balance.

4. To find the needed value for the perfect square trinomial, take half of the coefficient of the middle term ($x$-term), square it, and add that value to both sides of the equation.

   Take half and square

   $x^2 + 8x + \square = 4 + \square$
Practice: put each of the following equations into center-radius form for a circle. Then determine the center and radius.

A) \( x^2 - 12x + y^2 = 64 \)

\[
(x - \frac{-12}{2})^2 + y^2 = (\frac{-12}{2})^2 + 64
\]

Center \((6, 0)\) \(r = 10\)

B) \( x^2 + 10x + y^2 - 2y + 1 = 0 \)

\[
x^2 + 10x + \square + y^2 - 2y + \square = -1 + \square + \square
\]

\[
\left(\frac{10}{2}\right)^2 = (5)^2 = 25 \quad \left(-\frac{2}{2}\right)^2 = (-1)^2 = 1
\]

\[
(x + 5)^2 + (y - 1)^2 = 25 \quad \text{Center} (-5, 1) \quad r = 5
\]

\((x + 5)(x + 5) = x^2 + 10x + 25\)
ADDITIONAL APPLICATIONS OF CIRCLES IN THE COORDINATE PLANE

FINDING AREA AND CIRCUMFERENCE FROM AN EQUATION OF A CIRCLE

Find the area and circumference of \( \odot G \) with the equation \( x^2 + 6x + y^2 - 8y = 10 \). Express your answer in terms of \( \pi \).

\[
\begin{align*}
x^2 + 6x + \left(\frac{6}{2}\right)^2 + y^2 - 8y + \left(\frac{8}{2}\right)^2 &= 10 + \left(\frac{9}{2}\right) + \left(\frac{16}{2}\right)\\
(x + 3)^2 + (y - 4)^2 &= 35
\end{align*}
\]

\[
C = 2\pi r = 2\pi \sqrt{35} \text{ units, } r = \sqrt{35} \\
A = \pi r^2 = \pi (\sqrt{35})^2 = 35\pi \text{ units}^2
\]

\[R = \sqrt{35}\]

DETERMINING IF A POINT IS ON A CIRCLE

Is the point \((7,8)\) on the circle with the equation \((x + 2)^2 + (y - 5)^2 = 100\)?

\[
(7+2)^2 + (8-5)^2 = 100 \\
9^2 + 3^2 = 100 \\
9 + 9 = 100 \quad \text{NO (7,8) does not satisfy the equation of the circle.}
\]
**WRITING AN EQUATION GIVEN THE ENDPOINTS OF A DIAMETER**

Write the equation for \( \bigcirc C \) that has endpoints of the diameter at \( A(-4,2) \) and \( B(4,4) \). Use of the grid is optional.

\[
(x-h)^2 + (y-k)^2 = r^2
\]

- **CENTER** \((h,k)\)
  - \((-4+4)/2, (2+4)/2\) = \((0,3)\)
- **MIDPOINT** \((x_m, y_m)\)
  - \((x_1 + x_2)/2, (y_1 + y_2)/2\) = \((0,3)\)
  - **Equations**
    - \((x-h)^2 + (y-k)^2 = r^2\)
    - \((x-0)^2 + (y-3)^2 = 5^2\)
    - \(x^2 + (y+1)^2 = 25\)

**FINDING THE EQUATION OF A TANGENT LINE**

Determine the equation of the line tangent to the circle with equation \((x - 3)^2 + (y - 1)^2 = 25\) at the point \((6,5)\). Use of the grid is optional.

Recall: \(y - y_1 = m(x - x_1)\)

- \((3,1)\)
- \((6,5)\)
- \(m = \frac{5-1}{6-3} = \frac{4}{3}\)

**SLOPE OF RADIUS**

- \(\frac{4}{3}\)
- \(-\frac{3}{4}\)

**EQUATION OF THE LINE**

\((y-5) = -\frac{3}{4}(x-4)\)
FINDING POINTS OF INTERSECTION ON A GRAPH - Find the points of intersection of the equations:

A) \[ x^2 + y^2 = 16 \] \& \[ y = 4 \]

[Graph of a circle with center at (0,0) and radius 4, intersecting a line at \((0,4)\).]

SOLN: \((0,4)\)

B) \[ (x - 3)^2 + (y + 2)^2 = 4 \] \& \[ y + 6 = 1(x - 1) \]

\[ m = \frac{1}{1} \]

POINT \((1, -6)\)

\[ (x - 3)^2 + (y + 2)^2 = 4 \]

\((5, -2), (3, -4)\)

SOLUTIONS
1) Determine the area and circumference for a circle with the equation \( x^2 - 4x + y^2 - 6y = 0 \)

2) Is the point \((-2, 3)\) on the circle centered at \((0, 5)\) with a radius of 3? Explain your reasoning.

3) The diameter of a circle has endpoints at \((-2, 3)\) and \((6, 3)\). What is an equation of the circle?
   \[
   \begin{align*}
   [1] \quad (x - 2)^2 + (y - 3)^2 &= 16 \\
   [2] \quad (x - 2)^2 + (y - 3)^2 &= 4 \\
   [3] \quad (x + 2)^2 + (y + 3)^2 &= 16 \\
   [4] \quad (x + 2)^2 + (y + 3)^2 &= 4
   \end{align*}
   \]

4) Write an equation of the line \( \hat{h} \) that is tangent to circle \( A \) whose equation is \((x + 3)^2 + y^2 = 25\) at the point \(B(4, 2)\) in point-slope form. Use of the grid is optional.
5) Determine and state the points of intersection of the equations:
   a. \((x - 4)^2 + (y + 3)^2 = 9\)
   b. \(y = x - 4\)

Use circle vocabulary to describe the line as it relates to the circle: ________________

PUTTING IT ALL TOGETHER

*Graphic Organizer*

Circle Equation \(\rightarrow\) Graph: \(\sqrt{(x-h)^2 + (y-k)^2} = r^2\)

1. **Determine the center and Plot**
2. **Determine the radius, r**
3. **Count r- 4 times and plot 4 points**
4. **Connect points with ARCs and label**

Given Diameter Endpoints \(\rightarrow\) Equation or Graph

1. Calculate center using ___________ midpoint formula
2. Calculate radius using ___________ distance formula w/center & pt
3. **Equation:** Plug in center and radius
   **Graph:** Plot center, count r to plot 4 pts, connect pts w/arcs