Lesson 11-10 Secant-Secant, Secant/Tangent Relationships in Circles

**AGENDA:**
- Check & Review Homework 11-9
- Notes and Guided Practice

**HOMEWORK:**
- p. 796 # 17-21, 25, 27

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Use the diagram for Exercises 22 and 23.
22. M is the midpoint of PQ. RM = 10 cm, and PQ = 24 cm.
   a. Find MS. 14.4 cm
   b. Find the diameter of O. 24.4 cm
23. M is the midpoint of PQ. The diameter of O is 13 in., and RM = 4 in.

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13. \( x = 4.2 \), LJ = 14.2, MN = 13

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p. 796-797: #12, 13, 22, 23, 25

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25. \( x = 8; y = 6\sqrt{3} \)
Arc-angle relationships are measured in **DEGREES**.

Segments and chords are measured in **LINEAR UNITS**, such as inches or centimeters.

In each problem, identify whether you are asked to find an angle measure, an arc measure or a segment length.

1.) \(63^\circ\) \(84^\circ\) \(84^\circ\)
2.) \(2^\circ\)
3.) \(27\text{cm}\) \(3\text{cm}\) \(x\)
4.) \(204^\circ\)

Try These:

2. \(4\) \(2\) \(10\)
3. \(6\) \(6\) \(4\) \(4\)
1. SECANT-SECANT PRODUCT THEOREM

Consider an exterior angle formed by two secants. What relationship can you find among the chords and other segments? Hint: Draw in other chords to form triangles ADE and CBE... What could you prove about them?

![Diagram of Secant-Secant Product Theorem]

\[ \Delta ADE \sim \Delta CBE \]

![Whole Outer Formula]

\[ \frac{AE}{CE} = \frac{ED}{EB} \]

![Reflexive Property]

\[ \Delta s \sim \Delta s \]

![Proportion Sides]

\[ AE \cdot EB = CE \cdot ED \]

![Cross Products]

\[ \text{Whole} \cdot \text{Outer} = \text{Whole} \cdot \text{Outer} \]

\[ W_1 \cdot O_1 = W_2 \cdot O_2 \]

2. SECANT-TANGENT PRODUCT THEOREM

Consider an exterior angle formed by a secant and a tangent. What relationship can you find among the chord and other segments? Hint: Draw in other chords to form triangles ACE and CBE... What could you prove about them?

![Diagram of Secant-Tangent Product Theorem]

\[ \Delta AEC \sim \Delta CEB \]

![Whole Outer Formula]

\[ \frac{AE}{CE} = \frac{EC}{EB} \]

![Whole Outer Formula]

\[ AE \cdot EB = CE \cdot EC \]

\[ W_1 \cdot O_1 = W_2 \cdot O_2 \]
If 2 secants intersect in the exterior of a circle, then product of the lengths of one secant and its external segment equals the product of the other secant and its external segment.

\[ AE \cdot BE = CE \cdot DE \]

Extra Example

6.
3) SECANT – TANGENT PRODUCT THM:
If a secant & tangent intersect outside a circle, then the product of the lengths of the secant and its external segment = the length of tangent squared.

\[ AC \cdot BC = DC^2 \]

Secant \( AC \) and tangent \( DC \) intersect at \( C \).

**PRACTICE:** Find the Value of \( x \):

C) \[ \overline{WO}_1 = \overline{WO}_2 \]
\[ (SQ)(RQ) = (PQ)(PA) \]
\[ (9)(4) = (\overline{W}_1)(\overline{W}_1) \]
\[ 36 = x^2 \]
\[ x = \sqrt{36} = 6 \]
6 units = \( x \)

Practice 1 - Solve for \( x \)

\[ \overline{WO}_1 = \overline{WO}_2 \]
\[ (20)(5) = (x+21)(x) \]
\[ 100 = x^2 + 21x \]
\[ 0 = x^2 + 21x - 100 \]
\[ 0 = (x+25)(x-4) \]
\[ x+25 = 0 \]
\[ x = -25 \]
\[ x-4 = 0 \]
\[ x = 4 \]

\( x = 4 \) units.
2) Solve for $x$ and $y$.

\[
X: \quad W_0 = W_0 \quad \begin{array}{c}
11 \\
22
\end{array}
\]
\[
(X + 30) \cdot X = (20)(20)
\]
\[
X^2 + 30X = 400
\]
\[
X^2 + 30X - 400 = 0
\]
\[
(X + 40)(X - 10) = 0
\]
\[
x + 40 = 0 \quad \Rightarrow \quad x = -40
\]
\[
x - 10 = 0 \quad \Rightarrow \quad x = 10 \text{ cm}
\]

\[
Y: \quad W_{01} = W_{02}
\]
\[
(4 + 8) \cdot 8 = (20)(20)
\]
\[
8Y + 64 = 400
\]
\[
8Y = 336
\]
\[
Y = 42 \text{ cm}
\]

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**REGENTS QUESTIONS**

In the diagram below of $\odot O$, $\overline{AB}$ intersects $\odot O$ at $D$, secant $AOC$ intersects $\odot O$ at $E$. If $AE=4$, $AB=12$, and $DB=6$, find $OC$.

\[
\begin{align*}
W_{01} &= W_{02} \\
(AB)(AD) &= (AC)(AE) \\
(12)(6) &= (4+2x)(4) \\
72 &= 16 + 8x \\
56 &= 8x \\
7 &= x = OC \text{ units}
\end{align*}
\]
Find the diameter of the circle (not drawn to scale). \( BC = 18 \), and \( DC = 21 \). Round your answer to the nearest tenth.

\[
\frac{W_1}{O_1} = \frac{W_2}{O_2}
\]

\[
(A)(BC) = (CD)(CD)
\]

\[
(x+18)(18) = (21)(21)
\]

\[
x + 324 = 441
\]

\[
x = 117
\]

\[
Diam = 6.5 \text{ units}
\]
Fill in your graphic organizer if you have not done so already:

<table>
<thead>
<tr>
<th>Segment Length Relationships</th>
<th>2 Tangents</th>
<th>Same External Point: Secant – Tangent</th>
<th>Secant - Secant</th>
<th>Same Internal Pt: Chord-Chord</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td><img src="example_image1.png" alt="Diagram" /></td>
<td><img src="example_image2.png" alt="Diagram" /></td>
<td><img src="example_image3.png" alt="Diagram" /></td>
<td><img src="example_image4.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Algebraic Equation</td>
<td>Tangent segment₁ = Tangent segment₂</td>
<td>W₁O₁ = W₁O₂</td>
<td>W₁O₁ = W₁O₂</td>
<td>P₁R = P₁P₂</td>
</tr>
<tr>
<td>Derived from</td>
<td>RHL ≤ AA~</td>
<td>Similar Triangles (AA~)</td>
<td>AA~</td>
<td>AA~</td>
</tr>
</tbody>
</table>

\[ \angle \triangle \rightarrow \text{PROPORTIONAL SIDES} \]

Fill in your graphic organizer before beginning your homework:

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